Human Capital Investment and Work Incentives*

Matthias Kräkel, University of Bonn**

Abstract

Traditional human capital theory based on the work by Gary Becker shows that firms do not invest in general human capital but offer firm-specific training that is only useful for the training firm. I extend the traditional approach by adding two natural assumptions – workers cannot be forced to acquire new knowledge, and they exert unobservable effort to produce valuable output for their employer. I show under which conditions firms do not offer firm-specific training but invest in general human capital, which increases the workers’ outside option. This investment behavior is well in line with the documented prevalence of industry-specific and occupation-specific human capital over firm-specific human capital.

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** Matthias Kräkel, Department of Economics, University of Bonn, Adenauer-allee 24–42, 53113 Bonn, Germany, phone: +49-228-739211, fax: +49-228-739210, e-mail: m.kraekel@uni-bonn.de.
1 Introduction

In his seminal article, Becker (1962) shows that a firm does not invest in the knowledge of its workers if this knowledge is also useful for other firms (general human capital). However, in case of firm-specific human capital, which can only be used in the training firm, investment costs and returns are shared between the worker and the firm (see also Hashimoto 1981), resulting in a sharing of the respective quasi-rent (Hutchens 1989, p. 52).

In this paper, I extend traditional human capital theory by including two assumptions that do not seem to be unrealistic – workers cannot be forced to acquire human capital, and they choose unobservable effort to produce valuable output for their employers. In the basic model, I show that in this extended setting firms do not invest in purely firm-specific human capital but are often willing to invest in general training.\(^1\)

Traditional human capital theory assumes that a worker has no choice whether to acquire knowledge or not. When a firm decides to invest in human capital, a worker is considered more like a robot to be programmed rather than a human being who is free to learn or not. Often, however, such programming is not possible. For example, teachers typically do their best to transfer knowledge to their students, but the latter do not always appreciate the teachers’ efforts. In the following, I consider a situation in which firm and worker need to cooperate for human capital investment to be successful. As a second extension, I add a moral hazard problem to the traditional approach. The worker has to choose between working hard or not working hard, which is not directly observable by the firm. As the worker is protected by limited liability, creating incentives is costly for the firm.

\(^1\)Empirical studies also show the prevalence of general over firm-specific human capital; see Schönberg (2007).
In the extended setting, I show that the firm does not invest in purely firm-specific training but may prefer investment in general human capital. The intuition is the following. Additional human capital helps a worker to carry out his task. As a consequence, the firm only needs lower-powered incentives to motivate the worker in case of moral hazard, resulting in a lower rent for the worker. Firm-specific human capital solely leads to this rent reduction without affecting the worker’s outside option. Hence, the worker will always disagree to make use of the offered training. The firm anticipates this behavior and decides against training to save cost. If, however, human capital is sufficiently general, it can lead to a significant increase in the worker’s outside option with the result that the worker is now willing to learn.\textsuperscript{2} In turn, to make the firm willing to offer training, the outside option should not be allowed to become prohibitively large since the worker has to be compensated for it when staying with the training firm.

To check the robustness of the main finding, I discuss four variants of the basic model. First, I consider the possibility that the firm may commit to a long-term wage profile offered to the worker. If such commitment is possible, investment in purely firm-specific human capital will be successful. However, long-term commitment can also lead to considerable disadvantages for a firm in an uncertain environment. Second, I deviate from the basic model by assuming that the firm does not have all the bargaining power. Instead, wages are determined via the Nash bargaining solution. In this setting, purely firm-specific training will be successful if the efficiency gains from training are sufficiently large and the worker’s bargaining power is not too high. Third, I modify the assumption of the basic model that human capital lowers a

\textsuperscript{2}For example, general human capital increases the worker’s income by the worker becoming self-employed.
worker’s effort cost. In that case, additional human capital makes completion of the task easier for the worker, enabling him to save effort costs (e.g., the worker needs less time to complete the task). As an alternative, I consider a scenario in which human capital increases the worker’s success probability. For example, we can think of knowledge that makes the machine or technique used by the worker more effective but leaves his effort costs unchanged (e.g., his working time). I show that a result similar to that of the basic model will hold if human capital and effort are complements. If, however, human capital and effort are substitutes, the worker’s rent will always increase with training, making any kind of human capital investment successful. Finally, I let the firm’s training decision be described by a continuous variable. As in the basic model with discrete training, purely firm-specific human capital investment will not be successful.

The major finding of the paper fits well with the empirical human capital literature. Several empirical studies document that often human capital is neither purely firm specific nor purely general.\(^3\) Instead, it is productive in several but not all firms and can often be classified as either occupation specific (i.e., knowledge is valuable in a certain type of job that can be found in different firms) or industry specific (i.e., knowledge is only valuable in firms that belong to a certain industry). Neal (1995) uses data from the Displaced Worker Surveys and shows that the wage losses of displaced workers switching industries are substantially larger than the wage losses of workers staying in their predisplacement industries. Hence, workers’ human capital is for the most part industry specific. Neal (1995, p. 653) concludes: "Workers apparently receive compensation for some skills that are neither completely

\(^3\)See, e.g., Poletaev and Robinson (2008), Zangelidis (2008), and Suleman and Lagoa (2013).
general nor firm-specific but rather specific to their industry or line of work."

The parameter condition for the main result in Proposition 1 below describes the same kind of observation: firms will only invest in human capital if it is not purely firm specific but to some extent general, thus increasing a worker’s outside option. The importance of industry-specific human capital relative to firm-specific human capital is also supported by subsequent empirical studies, for example the paper by Parent (2000), which uses data from the National Longitudinal Survey of Youth and the Panel Study of Income Dynamics.

As Neal (1995, pp. 669–670) already conjectures, as an alternative to industry-specific human capital, workers may primarily be provided with occupation-specific knowledge. This conjecture is supported by the empirical analysis of Kambourov and Manovskii (2009). They show that five years of occupational tenure lead to wage increases of 12 to 20 percent. When controlling for occupation-specific human capital, both firm-specific and industry-specific knowledge turn out to be rather unimportant. Sullivan (2010) shows that industry-specific and occupation-specific human capital have the highest impact on workers’ wages. Whereas for craftsmen occupation-specific human capital is most important, managerial wages are mainly determined by industry-specific human capital. For professional employment both types of human capital are crucial. The results of the basic model presented in this paper are in line with both strands of the empirical literature – one highlighting the importance of industry-specific human capital and the other promoting occupation-specific human capital – as they

\footnote{See also Shaw (1984, 1987), Toda (2011), and Nawakitphaitoon (2014). Gathmann and Schönberg (2010) consider task-specific human capital. They show that some skills are transferable across different occupations, which qualifies the concept of occupation-specific human capital. Kwon and Meyersson Milgrom (2014) report that firm-specific and occupation-specific human capital are equally valuable.}
show that a necessary condition for successful human capital investment is its applicability outside the training firm.

Besides the empirical human capital literature, my paper is also related to three fields in the theoretical literature. First, there are other theoretical papers that also extend the traditional human capital approach. In particular, Acemoglu and Pischke (1999a, 1999b) and Kessler and Lülfesmann (2006) modify Becker’s assumption of competitive markets and show that firms may invest in general training if labor markets are imperfect. Lazear (2009) suggests an alternative concept of human capital specificity. According to his approach, skills are typically general in the sense that they are used in several firms, with each firm choosing specific weights for different skills. Second, my paper is related to the existing moral hazard literature. Moral hazard can lead to inefficient outcomes in labor relationships if workers are risk averse or protected by limited liability. My paper follows the strand of the literature that considers limited liability as contractual friction (see, among many others, Sappington 1983, Che and Yoo 2001, Schmitz 2005a, Poblete and Spulber 2012, Chen and Chiu 2013). Third, if a contracting party undertakes a relation-specific investment within an incomplete-contract setting, this party might be held up by the other contracting party. As both parties can anticipate the hold-up problem, they may end up in a situation with underinvestment (see, e.g., Klein et al. 1978, Hart and Moore 1988, Schmitz 2001, Hoppe and Schmitz 2011). My paper is related to this literature because I consider investments in firm-specific human capital and analyze the implications of the degree of specificity of investments for optimal contracts. However, the rent left to the worker is due to the underlying moral hazard problem.

The paper is organized as follows. Sections 2 and 3 introduce the basic
model and show why it is necessary for successful human capital investment that training is also productive outside the training firm. Section 4 discusses the four variants of the basic model outlined above. Section 5 concludes.

2 The Basic Model

A firm has to hire an additional worker to run a business. Both parties are assumed to be risk neutral. The worker has an economic life of two periods, $t = 1, 2$. In $t = 1$, the worker can acquire human capital, in $t = 2$ the worker can choose effort to produce valuable output for the firm. After $t = 2$ the worker retires.

The firm’s reservation value for each period is zero. In the training period, $t = 1$, the worker’s reservation value is $\bar{u}_1 = 0$, but it may rise in $t = 2$ due to increased human capital. Successful human capital investment requires both training offered by the firm and the worker’s willingness to learn. Let $\tau \in \{0, 1\}$ denote the firm’s training decision. $\tau = 1$ means that the firm offers training, whereas $\tau = 0$ indicates that the firm does not offer training. The firm’s training costs are $\tau \cdot \kappa$ with $\kappa > 0$. Costs arise for the firm, for example, when assigning experienced employees to the worker to share their valuable knowledge, thus reducing their effective working time. The worker’s learning decision is denoted by $\ell \in \{0, 1\}$ with $\ell = 1$ ($\ell = 0$) indicating that the worker learns (does not learn). Hence, the paper departs from the previous literature on human capital by assuming that a worker cannot be forced to acquire new knowledge. Human capital investment influences the worker’s effort cost in $t = 2$, where the worker has to decide between working hard or not. When the worker chooses effort $e \in \{0, 1\}$ in $t = 2$, his effort cost amounts to $e \cdot (c - \delta \cdot \tau \cdot \ell)$ with $c > 0$ and $\delta \in (0, c)$. The parameter $\delta$ denotes
the effort cost reduction through successful human capital investment. I assume that human capital investment is efficient, i.e., $\kappa < \delta$.

In the working period, $t = 2$, the worker’s reservation value is given by $\tilde{u}_2$. Without successful human capital investment (i.e., $\tau \cdot l = 0$), we have $\tilde{u}_2 = \tilde{u}_1 = 0$. If investment is successful (i.e., $\tau \cdot l = 1$), the worker’s reservation value will be $\tilde{u}_2 = \tilde{u} \geq 0$. In case of purely firm-specific human capital, we have $\tilde{u} = 0$ as the acquired knowledge is not productive outside the training firm. If this knowledge can, at least partly, be used in other economic activities outside the firm, we will have $\tilde{u} > 0$. The higher $\tilde{u}$, the larger is the degree of generality of the acquired human capital. I assume that successful human capital investment cannot be verified by courts, but the worker can either become self-employed and use the general human capital or, with positive probability, can convince another employer that he has acquired general knowledge. In either case, human capital increases the worker’s outside option.

When choosing effort $e \in \{0, 1\}$ in $t = 2$, the worker produces valuable output $\pi > 0$ for the firm with probability $p_e$ and zero output with probability $1 - p_e$. Effort is only observable by the worker, which leads to a moral hazard problem for the firm. I assume that $1 > p_1 > p_0 > 0$. Hence, more effort makes successful production more likely. Realized output is verifiable so that second-period wages can be made contingent on it. Let $w_H (w_L)$ denote the wage paid to the worker if realized output is $\pi$ (zero). To exclude the discussion of additional cases that do not really add to the analysis, I assume that $\pi$ is sufficiently large so that the firm always wants to implement high effort $e = 1$ (see similarly Schmitz 2005b, p. 322, Schmitz 2013, p. 110, Kaya

\[5\]For example, another employer might check the worker’s skills and form subjective beliefs about his productivity.
This condition implies that second-period employment of the worker is efficient, given that he works hard:

\[
p_1\pi - (c - \delta \tau l) > \bar{u}_2 \iff \pi > \frac{c - \delta \tau l + \bar{u}_2}{p_1}. \tag{2}
\]

The worker maximizes expected net income, i.e., expected wage payment minus effort cost. The employment relation is governed by spot contracts. Hence, the worker is free to leave the firm in each period. He is assumed to be wealth constrained or protected by limited liability so that wages must be non negative.

The timing of the model is the following. In \( t = 1 \), the firm offers a non negative wage as contract, which the worker either accepts or rejects. If he rejects, he will receive \( \bar{u}_1 = 0 \) and the game ends. Otherwise, the firm chooses \( \tau \), followed by the worker’s decision on \( l \). In \( t = 2 \), the firm offers the contract \((w_L, w_H)\) and the worker either accepts or rejects. If he rejects, he will receive \( \bar{u}_2 \) and the game ends. Otherwise, the worker decides on effort, \( e \). Finally, output is realized and payments are made. I apply the usual tie-breaking rules, namely that the worker accepts a contract when indifferent between accepting and rejecting and that he chooses \( e = 1 \) when indifferent between \( e = 1 \) and \( e = 0 \). For the given context, I add a third tie-breaking rule, namely that the worker chooses \( l = 1 \) when indifferent between \( l = 0 \) and \( l = 1 \).

The analysis below shows that the firm can always choose an optimal contract with \( w_L = 0 \). The firm prefers the implementation of high effort if \( p_1(\pi - w_H) > p_0 \pi \iff \pi > \frac{p_1}{p_1 - p_0} w_H \) with \( w_H \) denoting the incentive compatible high wage either under a non binding (see (5)) or binding (see (7)) participation constraint.

The tie-breaking rules can be justified from a Nash equilibrium perspective: if a player
3 Solution to the Basic Model

I solve the game by backward induction, starting with the worker’s effort decision in $t = 2$. He will choose high instead of low effort if the incentive constraint

$$ w_H p_1 + w_L (1 - p_1) - (c - \delta \tau l) \geq w_H p_0 + w_L (1 - p_0) $$

$$ \iff w_H - w_L \geq \frac{c - \delta \tau l}{p_1 - p_0} \tag{3} $$

is satisfied. Participation of the worker in $t = 2$ is guaranteed by

$$ w_H p_1 + w_L (1 - p_1) - (c - \delta \tau l) \geq \bar{u}_2 $$

$$ \iff w_L + (w_H - w_L) p_1 \geq \bar{u}_2 + c - \delta \tau l. \tag{4} $$

The firm’s optimal second-period contract $(w_L^*, w_H^*)$ minimizes expected labor costs for implementing $e = 1$, subject to the incentive constraint (3), the participation constraint (4), and the limited-liability constraint $w_L, w_H \geq 0$.

There are two possible cases. First, if the incentive constraint (3) already implies the participation constraint (4) so that

$$ \frac{c - \delta \tau l}{p_1 - p_0} p_1 \geq \bar{u}_2 + c - \delta \tau l \iff \frac{p_0}{p_1 - p_0} (c - \delta \tau l) \geq \bar{u}_2, $$

then the limited-liability constraint for $w_L$ will be binding and the optimal contract will be described by

$$ w_L^* = 0 \quad \text{and} \quad w_H^* = \frac{c - \delta \tau l}{p_1 - p_0}. \tag{5} $$

is indifferent between deviating from a binary action or not, it will be a best response and can thus be part of an optimal solution. If the worker does not choose the firm’s preferred action with probability 1 when indifferent, then the firm can induce the action by infinitesimally increasing the worker’s payoff from the action.
In that case, the worker earns a non negative rent and his expected utility (or expected net income) amounts to

\[ EU := \frac{p_0}{p_1 - p_0} (c - \delta \tau l). \] (6)

Second, if (3) does not already imply (4) (i.e., \( p_0 (c - \delta \tau l) / (p_1 - p_0) < \tilde{u}_2 \)), the firm will choose \( w_L \) and \( w_H \) to make the participation constraint just binding in order to minimize expected labor costs. Again, \( w_L \) and \( w_H \) have to satisfy the incentive constraint (3). Due to the risk-neutrality of both parties, there is a continuum of optimal contracts that differ in how the risk is shared, for example,

\[ w_L^* = 0 \quad \text{and} \quad w_H^* = \frac{c - \delta \tau l + \tilde{u}_2}{p_1}. \] (7)

In case of all these contracts, the worker’s expected net income is \( \tilde{u}_2 \).

Since, for \( t = 2 \), I focus on the worker’s expected net income from the optimal contract, the previous main findings can be summarized as follows:

**Lemma 1** For given \( \tau \) and \( l \), the optimal contract \((w_L^*, w_H^*)\) leads to expected net income \( \max\left\{ \frac{p_0}{p_1 - p_0} (c - \delta \tau l), \tilde{u}_2 \right\} \) of the worker in \( t = 2 \).

The derivation of Lemma 1 has shown that there are two possible outcomes for the optimal contract in \( t = 2 \) – either the incentive constraint (3) and the limited-liability constraint \( w_L \geq 0 \) are binding, or the participation constraint (4) is binding. In the first case, the second-period reservation value \( \tilde{u}_2 \) is so low that the worker earns a non negative rent. This rent will be strictly positive if the worker has not acquired human capital in \( t = 1 \), or if human capital investment was successful but the acquired knowledge is purely firm specific because in both cases we have \( \tilde{u}_2 = 0 \). The worker will also earn a positive rent if he has successfully acquired knowledge, but only
a small part of the human capital is general (i.e., $\bar{u}_2 = \bar{u} > 0$ but $\bar{u}$ is small).
In the second case, where (4) is binding, $\bar{u}_2$ and thus the acquired general human capital are sufficiently large.

In the training period, $t = 1$, the firm’s optimal wage for the worker is zero, since $\bar{u}_1 = 0$ and no verifiable performance measure exists that can be used to control the worker’s behavior. The firm has to choose its optimal training decision, $\tau^* \in \{0, 1\}$, and thereafter the worker his optimal learning decision, $l^* \in \{0, 1\}$:

**Proposition 1** Human capital investment will be successful (i.e., $\tau^* = l^* = 1$) if and only if

$$\frac{p_0}{p_1 - p_0} c \leq \bar{u} \leq \frac{p_0}{p_1 - p_0} c + \delta - \kappa;$$

otherwise the firm will not invest in human capital.

**Proof.** See the appendix. 

The proposition shows that there exists a non empty set of values for $\bar{u}$ that lead to successful human capital investment. The case of purely firm-specific human capital (i.e., $\bar{u} = 0$), however, does not belong to this set. The intuition is the following. The firm will not invest in firm-specific training since it anticipates that the worker will not make use of the training offer, because training would not affect his outside option but reduce his second-period rent from $\frac{p_0}{p_1 - p_0} c$ to $\frac{p_0}{p_1 - p_0} (c - \delta)$. In other words, the firm has to offer incentive pay to induce the worker to choose high effort in $t = 2$. After successful training, it is less hard for the worker to exert high effort. As a consequence, the firm can reduce its optimal incentive pay. Since the new low-powered incentives lead to a reduction of the worker’s rent, the worker will never agree to acquire purely firm-specific human capital.

The condition in Proposition 1 highlights that there is a critical lower and upper bound for the worker’s outside option $\bar{u}$ to guarantee successful
human capital investment. On the one hand, the outside option has to be sufficiently attractive for the worker to induce him to forgo a positive rent by agreeing to learn. On the other hand, the outside option cannot be too attractive, because the firm has to compensate the worker for rejecting this option, causing labor costs to be prohibitively high for sufficiently large values of $\bar{u}$. The condition states that human capital must be general to at least some extent to make both parties agree on human capital investment.

Finally, the condition in Proposition 1 shows that the efficiency gains from training, $\delta - \kappa$, have to be large enough for the firm to be interested in financing human capital investment. This observation results from the fact that in the relevant situation, in which the worker accepts the training offer, the participation constraint is binding. Therefore, all efficiency gains from human capital investment accrue to the firm.

4 Discussion

In the following, I address the robustness of my results by considering four natural variations of the basic model.\footnote{A further variation of the basic model would follow from the introduction of learning costs $l \cdot \kappa_W$ for the worker with $\kappa_W > 0$. Now, the worker would only agree to learn, if $\bar{u} - \kappa_W \geq p_0 c / (p_1 - p_0)$, leading to a straightforward modification of the left-hand side of the condition in Proposition 1.} The previous results have shown that, w.l.o.g., I can restrict the analysis to contracts with $w_L = 0 < w_H$.

4.1 Long-Term Commitment

In Section 2, I assume that the employment relationship is governed by spot contracts and the firm cannot make use of long-term commitment. These
assumptions seem realistic in situations where the firm has only limited foresight and lacks sufficient commitment power. However, there also exist rather stable work environments in which a firm can credibly commit to a long-term compensation of workers. In this subsection, I consider the possibility that the firm is able to offer the wage schedule for the second period, \((w_L, w_H)\), already in \(t = 1\). Even having accepted the long-term wage profile, the worker is nevertheless free to leave the firm after the first period. I distinguish between two cases. First, I assume that the firm has sufficient commitment power to stick to \((w_L, w_H)\) independent of the given situation. Second, I consider the scenario in which the firm and the worker can renegotiate the initial contract \((w_L, w_H)\) in \(t = 2\) if both prefer to do so.

In the basic model, the worker is only willing to learn if the investment in human capital sufficiently boosts his outside option. Thus, purely firm-specific training is not in the worker’s interest, because the worker anticipates that such knowledge increase will entail a cut in his second-period rent. If, however, the firm is able to commit to a second-period contract that is sufficiently attractive for a trained individual, the worker might agree to learn even in situations where human capital investment does not considerably increase his outside option. The following result can be shown:

**Proposition 2** If contracts are not renegotiable and, in \(t = 1\), the firm is able to commit to the wage schedule that is offered in \(t = 2\), human capital investment will be successful if and only if \(\ddot{u} \leq \frac{\rho_0}{p_1 - p_0} c + \delta - \kappa.\)

**Proof.** See the appendix. ■

The proposition shows that the firm will succeed in firm-specific human capital investment (i.e., \(\ddot{u} = 0\)) if it is able to commit to long-term wages already at the beginning of the employment relationship. More generally,
when he decides on \( l \), the worker’s incentive problem is completely eliminated because the firm can guarantee him a sufficiently high income in case of successful human capital investment. This commitment advantage is independent of the human capital’s degree of specificity. The comparison of Propositions 1 and 2 shows that only the firm’s incentive problem with regard to training remains unsolved. If the worker becomes too expensive for the firm due to additional human capital, the firm will prefer not to offer training.

The proof of Proposition 2 shows that, according to the commitment solution, the firm sticks to a contract that is not optimal for either party, given that the worker has chosen \( l = 0 \). In that case, both the firm and the worker would be better off renegotiating the initial contract to restore incentives. The worker would benefit from a higher wage and the firm from higher effort. In the second part of this subsection, I therefore look for the optimal commitment solution when allowing the firm and the worker to renegotiate \((w_L, w_H)\) at the beginning of \( t = 2 \). Note that commitment is still valuable to the firm, because renegotiation requires that both parties have to agree to a new contract so that the firm cannot unilaterally deviate to lower wages in the second period.

If renegotiation is possible, both parties will anticipate that the initial contract is renegotiated at the beginning of \( t = 2 \), given that the worker has chosen \( l = 0 \) and the initial value of \( w_H \) is not incentive compatible in this situation, i.e., \( w_H < c/(p_1 - p_0) \). To avoid renegotiation and, thus, a high wage \( w_H = c/(p_1 - p_0) \), the firm can already offer an initial wage \( w^*_H \) that is high enough to make the worker prefer \( l = 1 \) to \( l = 0 \) in the first period. If this initial wage \( w^*_H \) is lower than \( c/(p_1 - p_0) \), the firm will benefit from avoiding renegotiation and – at the same time – succeed in human capital
investment. It can be shown that $w^*_H$ is necessarily higher than the wage $w_H$ under full commitment – the case considered in Proposition 2 – but that $w^*_H$ is still sufficiently attractive for the firm to choose a commitment-based solution under possible renegotiation instead of optimal spot contracts. In other words, the possibility of renegotiation inevitably increases the firm’s labor costs, but the main result on successful human capital investment will survive. Let $\Delta w_H$ denote the increase of the worker’s wage for high output caused by the possibility of renegotiation (i.e., $\Delta w_H := w^*_H - w_H$). Then, according to the proof of Proposition 3, the expected additional labor costs from renegotiation can be computed as follows:

$$p_1 \cdot \Delta w_H = \begin{cases} \frac{p_0 \delta}{p_1 - p_0} & \text{if } \bar{u} \leq \frac{p_0}{p_1 - p_0} (c - \delta) \\ \frac{p_0 c}{p_1 - p_0} \bar{u} - \bar{u} & \text{if } \frac{p_0}{p_1 - p_0} (c - \delta) < \bar{u} < \frac{p_0}{p_1 - p_0} c \\ 0 & \text{if } \bar{u} \geq \frac{p_0}{p_1 - p_0} c. \end{cases}$$

Thus, if human capital investment is already successful without commitment or if the firm does not want to train the worker with and without commitment, the firm’s additional costs from renegotiation will be zero, but they are strictly positive otherwise. The main result can be summarized as follows:

**Proposition 3** If, in $t = 1$, the firm is able to offer a second-period contract that can be renegotiated at the beginning of $t = 2$, human capital investment will be successful if and only if $\bar{u} \leq \frac{p_0}{p_1 - p_0} c + \delta - \kappa$.

**Proof.** See the appendix. ■

The results of this subsection have shown that long-term commitment allows the firm even to invest in purely firm-specific human capital. However, the commitment solution only works since in the first period there is no uncertainty about second-period states of the world. Imagine, for example, that there exists such uncertainty and that, with some positive probability,
there will be a negative shock, making high-effort implementation unprofitable in the second period. In that case, long-term commitment might lead to an expected loss for the firm, because the worker would accept the initial contract \((w_L, w_H)\) to earn a strictly positive rent. Such an expected loss can be avoided via flexible spot contracts without long-term commitment.

### 4.2 Wage Bargaining

In the basic model, I assume that the firm has all the bargaining power and makes a take-it-or-leave-it offer \((w_L, w_H)\) to the worker in \(t = 2\). To further address the robustness of the previous results, I assume in this subsection that wages are determined by the Nash bargaining solution.\(^9\)

Recall from Section 3 that it suffices to determine \(w_H > 0\) and setting \(w_L\) equal to zero because \(w_H\) can serve two purposes at the same time – creating incentives and transferring enough money to the worker to make him sign the contract (see (5) and (7) above). Let \(\alpha \in (0, 1)\) denote the worker’s bargaining power and \(1 - \alpha\) the firm’s bargaining power. As the firm has a zero outside option concerning the employment of the worker, the Nash bargaining solution, \(w_H^*\), solves

\[
    w_H^* \in \arg\max_{w_H} NP(w_H) := \left( p_1 w_H - (c - \delta \tau l) - \bar{u}_2 \right)^\alpha \cdot \left( p_1 (\pi - w_H) \right)^{1-\alpha} \tag{8}
\]

subject to

\[
    p_1 w_H - (c - \delta \tau l) \geq p_0 w_H, \tag{9}
\]

\[
    p_1 w_H - (c - \delta \tau l) \geq \bar{u}_2, \tag{10}
\]

and

\[
    \pi - w_H \geq 0. \tag{11}
\]

\(^9\)See also Schmitz (2005c), p. 326, on combining a moral-hazard problem with the Nash bargaining solution. The sharing of rents between the firm and the worker subject to different amounts of general and specific human capital has already been discussed in Becker (1962), p. 20.
Thus, the bargaining solution, $w^*_H$, maximizes the Nash product while satisfying the incentive constraint (9), the worker’s participation constraint (10), and the firm’s participation constraint (11).

Rewriting and combining constraints (9) and (10) yields

$$w_H \geq \max \left\{ \frac{c - \delta \tau l}{p_1 - p_0}, \frac{c - \delta \tau l + \bar{u}_2}{p_1} \right\},$$

Together with (11), a feasible solution will exist if

$$\pi \geq \max \left\{ \frac{c - \delta \tau l}{p_1 - p_0}, \frac{c - \delta \tau l + \bar{u}_2}{p_1} \right\},$$

which is guaranteed by (1). The solution to the bargaining problem crucially depends on whether the incentive constraint (9) implies the worker’s participation constraint (10) or vice versa. Define the cutoff value

$$\bar{\alpha}(\tau l) := \frac{p_0}{p_1 - p_0} \frac{(c - \delta \tau l) - \bar{u}_2}{p_1 \pi - (c - \delta \tau l) - \bar{u}_2}.$$  

Then the following result for the optimal wage, $w^*_H$, can be obtained:

**Lemma 2** For given $\tau$ and $l$, the optimal second-period wage, $w^*_H$, is described as follows. If $\bar{u}_2 \geq \frac{p_0}{p_1 - p_0} (c - \delta \tau l)$, then $w^*_H = \alpha \pi + (1 - \alpha) \frac{c - \delta \tau l + \bar{u}_2}{p_1}$; otherwise

$$w^*_H = \begin{cases} 
\alpha \pi + (1 - \alpha) \frac{c - \delta \tau l + \bar{u}_2}{p_1} & \text{if } \alpha \geq \bar{\alpha}(\tau l) \\
\frac{c - \delta \tau l}{p_1 - p_0} & \text{if } \alpha < \bar{\alpha}(\tau l).
\end{cases}$$

**Proof.** See the appendix. ■

Typically, the expected payment will compensate the worker for the forgone outside option ($\bar{u}_2$) and his effort costs ($c - \delta \tau l$) and additionally gives him a share in the surplus $p_1 \pi - (c - \delta \tau l) - \bar{u}_2$, which increases with his bargaining power ($\alpha$):

$$p_1 \cdot \left( \alpha \pi + (1 - \alpha) \frac{c - \delta \tau l + \bar{u}_2}{p_1} \right) = \bar{u}_2 + c - \delta \tau l + \alpha \cdot [p_1 \pi - (c - \delta \tau l) - \bar{u}_2].$$
However, if the worker’s bargaining power is too low \((\alpha < \bar{\alpha}(\tau l))\), the firm has to create additional incentives for the worker to induce him to choose high effort. In that situation, the worker is paid the lowest possible wage just guaranteeing incentive compatibility, i.e., \(w_H^* = (c - \delta \tau l) / (p_1 - p_0)\).

The worker’s expected net income in the second period crucially depends on his wage, \(w_H^*\), and whether human capital investment was successful \((\tau l = 1)\) or not \((\tau l = 0)\). If \(w_H^* = \alpha \pi + (1 - \alpha) \frac{c - \delta \tau l + \bar{u}_2}{p_1}\), the worker will earn
\[
p_1 w_H^* - (c - \delta \tau l) = \bar{u}_2 + \alpha \cdot [p_1 \pi - (c - \delta \tau l) - \bar{u}_2],
\]
i.e., his outside option plus a share in the surplus. If \(w_H^* = \frac{c - \delta \tau l}{p_1 - p_0}\), the worker’s expected net income will amount to
\[
p_1 w_H^* - (c - \delta \tau l) = \frac{p_0}{p_1 - p_0} (c - \delta \tau l),
\]
which is identical with expression (6) of the basic model. In \(t = 1\), the worker has to decide on \(l\). If the firm has chosen \(\tau = 0\), the worker’s best reply will be \(l = 0\). If, however, the firm has chosen \(\tau = 1\), the worker has to compare his expected net income for \(\tau l = 1\) and \(\tau l = 0\) in the given situation. I obtain the following result:

**Lemma 3** Given \(\tau = 1\), the worker will choose \(l = 1\) if and only if
\[
\bar{u} + \alpha \cdot [p_1 \pi - (c - \delta) - \bar{u}] \geq \frac{p_0}{p_1 - p_0} c.
\]

**Proof.** See the appendix.  

Lemma 3 shows that the worker will be interested in successful human capital investment if his share in the surplus is sufficiently large. If the worker’s share is quite small because his bargaining power is negligible \((\alpha \to 0)\), he will nevertheless be interested in human capital investment as long as the increase in his outside option, \(\bar{u}\), is larger than his rent from being employed as an untrained worker, \(\frac{p_0}{p_1 - p_0} c\).
In $t = 1$, the firm has to decide on training $\tau \in \{0, 1\}$. As it can anticipate the worker’s decision on $l$, a necessary condition for offering training is given by condition (15). The proof of Lemma 3 shows that the worker’s wage is $w^*_H = \alpha \pi + (1 - \alpha) \frac{c - \delta + \tilde{u}}{p_1}$ in this case. Thus, the firm’s expected net profits from successful human capital investment read as

$$p_1 (\pi - w^*_H) - \kappa = (1 - \alpha) [p_1 \pi - (c - \delta) - \tilde{u}] - \kappa,$$

which describes the remaining share in the surplus not accruing to the worker minus the firm’s training costs. If the firm does not offer training, the worker will always decide not to learn. According to Lemma 2, the worker’s wage in that case is given by

$$w^*_H = \begin{cases} \alpha \pi + (1 - \alpha) \frac{c}{p_1} & \text{if } \alpha \geq \tilde{\alpha} (0) \\ \frac{c}{p_1 - p_0} & \text{if } \alpha < \tilde{\alpha} (0) \end{cases}$$

and the firm’s expected net profits by

$$p_1 (\pi - w^*_H) = \begin{cases} (1 - \alpha) (p_1 \pi - c) & \text{if } \alpha \geq \tilde{\alpha} (0) \\ p_1 \left( \pi - \frac{c}{p_1 - p_0} \right) & \text{if } \alpha < \tilde{\alpha} (0). \end{cases}$$

To sum up, given $\alpha \geq \tilde{\alpha} (0)$ – which implies condition (15) as can be seen from the proof of Lemma 3 – the firm will prefer $\tau = 1$ to $\tau = 0$ if

$$(1 - \alpha) [p_1 \pi - (c - \delta) - \tilde{u}] - \kappa \geq (1 - \alpha) (p_1 \pi - c) \iff \delta \geq \tilde{u} + \frac{\kappa}{1 - \alpha}.$$

Given $\alpha < \tilde{\alpha} (0)$, the firm will prefer to offer training if

$$\frac{p_0}{p_1 - p_0} c + \delta - \kappa \geq \tilde{u} + \alpha \cdot [\pi p_1 - (c - \delta) - \tilde{u}].$$

The following proposition summarizes the main results:
Proposition 4 Suppose the worker’s wages are determined by the Nash bargaining solution. Human capital investment will be successful if either

\[ \alpha \geq \bar{\alpha}(0) \quad \text{and} \quad \delta \geq \bar{u} + \frac{\kappa}{1 - \alpha}, \]  

(16)
or

\[ \alpha < \bar{\alpha}(0) \quad \text{and} \]

\[ \frac{p_0}{p_1 - p_0} c \leq \bar{u} + \alpha \cdot [p_1 \pi - (c - \delta) - \bar{u}] \leq \frac{p_0}{p_1 - p_0} c + \delta - \kappa. \]  

(17)

Proposition 4 shows that, in case of wage bargaining, successful human capital investment is possible even if human capital is purely firm specific (i.e., \( \bar{u} = 0 \)). In that case, a worker will still agree to learn, given that his bargaining power is high enough for a sufficient share in the surplus, which increases with human capital investment. However, if the efficiency gains from training are not large enough (i.e., \( \delta \) is rather small and \( \kappa \) rather large) or if the worker’s bargaining power, \( \alpha \), is too high, the firm will refrain from investing in human capital.

Condition (16) shows that the worker’s bargaining power sets an upper bound on how general human capital investment can be, i.e., the larger \( \alpha \), the smaller must be \( \bar{u} \) to make condition \( \delta \geq \bar{u} + \frac{\kappa}{1 - \alpha} \) hold. On the other hand, if the worker’s bargaining power, \( \alpha \), tends to zero, condition (17) will become identical with the condition of Proposition 1 for the basic model, where the firm has all the bargaining power. In that case, human capital must be general to at least some degree to induce the worker to choose \( l = 1 \).

4.3 Human Capital and Expected Output

So far, human capital has been considered valuable since it reduces the worker’s effort cost (e.g., it makes completion of a certain task less time consuming). In this subsection, I assume that human capital does not influence
effort cost but increases the worker’s success probability, i.e., of producing π instead of zero.

The model of Section 2 is modified as follows. Irrespective of the worker’s human capital, his effort cost when choosing \( e \in \{0, 1\} \) is given by \( e \cdot c \) with \( c > 0 \). The worker is either trained (indicated by the subscript \( "t" \)) if human capital investment is successful, or untrained (indicated by the subscript \( "u" \)) without human capital investment. Let \( p_{et} \) denote the worker’s probability of producing output \( \pi > 0 \) when choosing effort \( e \) and being trained, whereas \( p_{eu} \) describes the success probability of an untrained worker who chooses effort \( e \). Since both human capital and effort should be productive, it is natural to assume \( p_{et} > p_{eu} \) as well as \( p_{1t} > p_{0t} \) and \( p_{1u} > p_{0u} \). These assumptions do not impose any restriction on the relationship between human capital and effort. Similar to Kaya and Vereshchagina (2014, p. 296) on possible relationships between ability and effort, we can distinguish between two possibilities. If the inequality

\[
p_{1t} - p_{0t} > p_{1u} - p_{0u}
\]

holds, human capital and effort will be complements. That is to say, the additional success probability from exerting higher effort is larger if the worker is trained instead of untrained. This assumption is realistic for situations where training directly raises labor productivity, making exerting effort more effective. However, human capital and effort are substitutes in case of

\[
p_{1t} - p_{0t} < p_{1u} - p_{0u},
\]

where the additional success probability from choosing \( e = 1 \) instead of \( e = 0 \) will be larger if the worker is untrained. In this situation, the production outcome of the worker will be less sensitive to effort if the worker is trained since his performance is mainly determined by additional knowledge. For
example, if the worker has to find and present an innovative solution to a customer, his success may be mainly determined by the degree of innovation, which increases with human capital, rather than by effort invested in the presentation.

Contrary to Section 2, $\pi$ now has to be sufficiently large to ensure that training is efficient. I assume that $\pi \cdot (p_{el} - p_{eu}) > \kappa$, with $\kappa$ again denoting the firm’s training costs. Let $w^*_H$ denote the trained worker’s wage in case of high output with $w^*_H = \max\{\frac{c}{p_{11} - p_{01}}, \frac{\tilde{u} + c}{p_{11}}\}$ and $w^*_L = 0$ the wage paid to the worker in case of low output. I assume that the firm will always prefer successful human capital investment and implementation of high effort, that is,

$$p_{11}(\pi - w^*_H) - \kappa > \max \left\{ p_{01} \pi - \tilde{u} - \kappa, p_{1u} \left( \pi - \frac{c}{p_{1u} - p_{0u}} \right), p_{0u} \pi \right\}. \quad (18)$$

Note that this assumption strengthens the following results since I show that, nevertheless, the firm does not always offer training. All the other assumptions of Section 2, including the time structure, remain the same. I obtain the following result:

**Proposition 5**

(a) If $\frac{p_{11}}{p_{01}} \leq \frac{p_{1u}}{p_{0u}}$, human capital investment will be successful.

(b) If $\frac{p_{11}}{p_{01}} > \frac{p_{1u}}{p_{0u}}$, human capital investment will be successful if and only if

$$\tilde{u} \geq \frac{p_{0u}}{p_{1u} - p_{0u}} c; \text{ otherwise, the firm will not invest in human capital.}$$

**Proof.** See the appendix. ■

Proposition 5 shows that we have to distinguish between two different scenarios. On the one hand, the likelihood ratio of the probability of high output conditional on high versus low effort may (weakly) decrease due to training (i.e., $\frac{p_{11}}{p_{01}} \leq \frac{p_{1u}}{p_{0u}}$). In that case, high output becomes a weaker signal of high effort, therefore making the provision of incentives more expensive for
the firm under human capital investment. In other words, if the worker earns a positive rent under the optimal contract, his rent, given successful training \((EU_t - \bar{u}_2 > 0)\), will be larger than his rent without training \((EU_u - \bar{u}_2 > 0)\):\(^{10}\)

\[
EU_t - \bar{u}_2 = \frac{p_{0t}}{p_{1t} - p_{0t}} c - \bar{u}_2 \geq \frac{p_{0u}}{p_{1u} - p_{0u}} c - \bar{u}_2 = EU_u - \bar{u}_2 \iff \\
\frac{1}{p_{1t} - p_{0t}} - 1 c \geq \frac{1}{p_{1u} - p_{0u}} - 1 c.
\]

Thus, if \(\frac{p_{1t}}{p_{0t}} \leq \frac{p_{1u}}{p_{0u}}\), the worker will always be interested in human capital investment because he will benefit either from an increase in his outside option or his rent. In the latter case, the worker will even benefit from purely firm-specific training.

On the other hand, the likelihood ratio of the probability of high output conditional on high versus low effort may increase due to training (i.e., \(\frac{p_{1t}}{p_{0t}} > \frac{p_{1u}}{p_{0u}}\)). Accordingly, high output becomes a stronger signal of high effort, making the provision of incentives less expensive. Given that the worker earns a positive rent, it will be lower if he is successfully trained as opposed to untrained. Thus, if, under the optimal contract, the worker receives a positive rent instead of his outside option, he will prefer \(l = 0\) to \(l = 1\) – a scenario similar to that discussed in Section 3. Given \(\frac{p_{1t}}{p_{0t}} > \frac{p_{1u}}{p_{0u}}\), the worker will only agree to learn if human capital is sufficiently general and, hence, sufficiently increases his outside option so that \(\bar{u}_2\) exceeds his expected net income when untrained, \(EU_u\), that is, if \(\bar{u} \geq \frac{p_{0u}}{p_{1u} - p_{0u}} c\).\(^{11}\) In this second scenario, purely firm-specific training cannot be successful.

Since

\[
\frac{p_{1t}}{p_{0t}} > \frac{p_{1u}}{p_{0u}} \iff p_{0u} (p_{1t} - p_{0t}) > p_{0t} (p_{1u} - p_{0u})
\]

\(^{10}\)\(EU_m\) \((m = u, t)\) denotes the worker’s expected net income, given that his participation constraint does not bind; see (22) in the appendix.

\(^{11}\)Contrary to Proposition 1, there is no upper bound for \(\bar{u}\) in Proposition 5(b) because, by assumption (18), the firm will always be interested in human capital investment.
human capital and effort must be complements as a necessary condition for
the scenario described in Proposition 5(b). If human capital and effort are
substitutes, the investment in human capital will always be successful,
because training increases the worker’s rent and, in addition, may also increase
his outside option. If, however, human capital and efforts are complements,
training must be sufficiently general to generate an attractive outside option
for the worker to induce him to cooperate with the firm and agree to learn.

4.4 Continuous Human Capital Investment

In this subsection, as in the basic model, I assume that human capital in-
vestment in $t = 1$ reduces the worker’s effort costs in $t = 2$. When high
effort is exerted, these costs are given by $c - \delta l$ with $l \in \{0, 1\}$ and $c > 0$
having the same meaning as in Section 2. Contrary to the basic model, cost
reduction due to human capital investment, $\delta \geq 0$, is now a continuous choice
variable of the firm with the following characteristics: $\delta$ leads to investment
costs $\kappa (\delta) \geq 0$ for the firm, with $\kappa (0) = \kappa' (0) = 0$ and $\kappa' (\delta), \kappa'' (\delta) > 0$ for
$\delta > 0$, and $\lim_{\delta \to 0} \kappa' (\delta) = \infty$. Let $\delta^*$ denote the optimal investment level
chosen by the firm. The first-period outside option is again normalized to
zero. The second-period outside option is given by $\bar{u}_2 (\beta \cdot \delta)$ with $\bar{u}'_2 > 0$.
The parameter $\beta \in [0, 1]$ describes the degree of firm specificity of the human
capital with $\beta = 0$ indicating purely firm-specific training and $\beta = 1$ purely
general training. For simplicity, I assume $\bar{u}_2 (\beta \cdot \delta) = \beta \cdot \delta$ in the following.
All the other assumptions for the basic model, including the timeline, are
kept.

Solving the model by backward induction, I start with the implications
of the optimal second-period contract. The worker’s expected net income in
$t = 2$ can be derived in strict analogy to Lemma 1. It is given by

$$\max \left\{ \frac{p_0}{p_1 - p_0} (c - \delta l), \bar{u}_2 (\beta \cdot \delta) \right\}. $$

In the training period, $t = 1$, the firm offers a zero wage to the worker, who accepts since his outside option is zero and he does not have to bear any costs. Thereafter, the firm has to choose $\delta$ and the worker decides on learning. Again, human capital investment can only be successful if $l = 1$.

Let $\tilde{\delta}$ be defined by

$$(1 - \beta) \tilde{\delta} - \kappa (\tilde{\delta}) + \frac{p_0}{p_1 - p_0} c = 0$$

and assume that $\tilde{\delta} < c$.\(^{12}\) Then the following first-period result holds:

**Proposition 6** Suppose $\delta$ is a continuous choice variable of the firm. Human capital investment will be successful (i.e., $\delta^* > 0$ and $l^* = 1$), if and only if

$$\frac{p_0}{p_1 - p_0} \frac{c}{\beta} \leq \tilde{\delta}. $$

If the firm’s optimal investment $\delta^*$ is positive, it will be decreasing in $\beta$.

**Proof.** See the appendix. □

Proposition 6 shows that, similar to the basic model with discrete human capital investment, the firm will also not invest in purely firm-specific human capital if investment is continuous. Technically, the left-hand side of condition (20) goes to infinity for $\beta \to 0$. In economic terms, the firm faces the same problem as in the basic model: as the worker’s possible rent decreases with human capital investment, the worker will only agree to learn if human

\(^{12}\)As will become clear from the proof of Proposition 6, the critical value $\tilde{\delta}$ defines an upper bound for the firm beyond which human capital investment is not rational. The assumption $\tilde{\delta} < c$ only excludes further cases that do not offer new insights.
capital sufficiently increases his outside option, which is impossible in case of purely firm-specific knowledge. The continuous case shows how the firm will react if human capital becomes more general. Given the firm decides to train the worker \((\delta^* > 0)\), the optimal amount of training will be smaller, the more general the human capital. The reason is that the firm has to compensate the worker for his outside option, which becomes larger, the more general the firm’s training.

If the firm can, in addition to \(\delta\), also choose the degree of training specificity, \(\beta\), it decides on the optimal mixture of human capital to be offered to the worker. Given there are parameter constellations that make human capital investment beneficial for both the firm and the worker, the optimal human capital mixture can be characterized as follows:

**Proposition 7** Suppose \(\tilde{\delta} \geq \frac{p_0}{p_1-p_0} c\). If the firm can choose both the amount of human capital investment, \(\delta\), and the degree of training specificity, \(\beta\), it will prefer \((\delta^*, \beta^*)\) with \(1 = \kappa'(\delta^*)\) and \(\beta^* = \frac{p_0}{p_1-p_0} \frac{c}{\delta^*}\).

**Proof.** See the appendix.

The condition for the upper bound \(\tilde{\delta}\) in Proposition 7 only ensures that there exist parameter constellations in which both parties are willing to invest in human capital. As the worker’s participation constraint for \(t = 2\) is binding in case of successful human capital investment, the firm collects all gains from training. Consequently, it chooses the efficient amount of training, which equates marginal returns and marginal costs of human capital investment (i.e., \(1 = \kappa'(\delta^*)\)). The optimal \(\beta^*\) corresponds to the highest possible degree of training specificity (i.e., the lowest possible value of \(\beta\)) that makes the worker indifferent between \(l = 0\) and \(l = 1\) so that he decides to learn according to the third tie-breaking rule in Section 2. This finding is
quite intuitive because the higher the degree of training specificity, the less expensive will be the hiring of the worker for the firm. However, to induce the worker to accept the training offer, human capital cannot be purely firm specific.

In the setup of this section, $\beta$ denotes the continuous degree of training specificity. In practice, such a continuous measure can be constructed based on the approach suggested by Leping (2009). He suggests a skill-based measure of specificity and argues that the "smaller the number of jobs where the skill is required, the higher is the level of specificity of that particular skill" (Leping 2009, p. 40). The parameter $\beta$ is a relative measure with lower values indicating higher degrees of specificity. Let $S$ be the number of jobs requiring a certain skill, and $N$ the total number of existing jobs. $\beta$ can then be constructed as $\beta = (S - 1)/(N - 1) \in [0, 1]$. Based on data from Estonia, Leping finds that "agriculture, hunting and forestry" and "financial intermediation" are the industries with the highest degree of specific human capital, implying more extensive on-the-job training in these industries than in others.

The construction of $\beta$ in the previous paragraph indicates that, for technological reasons, a firm might face a certain maximum degree of specificity, say $\tilde{\beta}$, so that it can only choose values $\beta \geq \tilde{\beta}$. Suppose that in a given firm a certain set of jobs has to be done that requires a corresponding set of skills. If similar jobs can also be found in other firms (e.g., in those belonging to the same industry), by the very nature of the firm’s production technology its choice of specificity will be constrained by $\beta \geq \tilde{\beta}$. Recall from Proposition 7 that a firm is interested in choosing the efficient total amount of human capital and the corresponding lowest possible value $\beta^*$ that just induces the worker to learn. However, if $\beta^* < \tilde{\beta}$, this solution is no longer
feasible because the firm’s production technology requires a more general training. In that case, the firm chooses $\beta = \tilde{\beta}$. From Proposition 6 we know that the corresponding total amount of human capital investment chosen by the firm is smaller than the efficient amount, because more general human capital increases the worker’s outside option and, thus, makes training more expensive for the firm. Therefore, if a firm is constrained in its choice of training specificity, the findings of this section predict an efficiency loss in form of an underinvestment in human capital. As a possible implication of a binding constraint $\beta \geq \tilde{\beta}$, for some firms occupation-specific or industry-specific human capital might be the most specific human capital that they can offer.

5 Conclusion

In his seminal article, Becker (1962) shows that, in competitive markets, firms do not provide general training but invest in firm-specific human capital. In this paper, I combine human capital investment with a moral hazard problem by assuming that a worker has to choose unobservable effort. Since the worker is protected by limited liability, the implementation of high effort is costly for the firm. I deviate from standard human capital models by allowing the worker to decide whether to learn or not. The major finding of the basic model shows that human capital investment will only be successful if the training offered is also productive outside the training firm. Thus, firms do not invest in firm-specific human capital but offer general training given the hiring of a trained worker is not too expensive. This finding is well in line with empirical studies on human capital, which offer strong support for the prevalence of industry-specific and occupation-specific over firm-specific
human capital.

In this paper, I consider the employment of a single worker to make the main point as clear as possible. In a next step, the issue of human capital investment could then be discussed in a more complex setting that includes the interaction of several workers, for example, interacting in team production or competing in rank-order tournaments (e.g., for job promotion). In such settings, it would be illuminating to analyze how different work organizations foster, or inhibit, firms’ incentives to invest in training. In addition, more complex models could include the competition of different firms for skilled workers to address the problem of employee poaching.

Appendix

Proof of Proposition 1:

From Lemma 1 we know that if the worker’s second-period reservation value after successful training is rather small relative to the worker’s expected net income without training, i.e.,

$$
\bar{u} < \frac{p_0}{p_1 - p_0} c,
$$

the worker will strictly prefer \( l = 0 \) to protect his large second-period rent. The firm anticipates the worker’s decision and chooses \( \tau = 0 \) to save training costs. Hence, the other way round, given \( \tau = 1 \), the worker will only prefer \( l = 1 \) to \( l = 0 \) if the resulting second-period reservation value from successful human capital investment satisfies \( \bar{u} \geq \frac{p_0 c}{(p_1 - p_0)} \). If, under that condition, the firm indeed prefers to train (i.e., \( \tau = l = 1 \)), its total cost for implementing high effort (i.e., \( c - \delta + \bar{u} \) since (4) is binding) and providing training (i.e., \( \kappa \)) will amount to

$$
\kappa + c - \delta + \bar{u}.
$$
If, however, the firm decides against training (i.e., \( \tau = 0 \) and \( l = 1 \)), expected implementation cost will be

\[
p_1 \cdot w^*_H \overset{5}{=} p_1 \cdot \frac{c}{p_1 - p_0}.
\]

Thus, when anticipating \( l = 1 \) as best response to \( \tau = 1 \) since \( \bar{u} \geq p_0 c/ (p_1 - p_0) \), the firm will choose \( \tau = 1 \) if and only if

\[
\kappa + c - \delta + \bar{u} \leq p_1 \frac{c}{p_1 - p_0} \Leftrightarrow \bar{u} \leq \frac{p_0}{p_1 - p_0} c + \delta - \kappa.
\]

**Proof of Proposition 2:**

Suppose that \( \frac{p_0}{p_1 - p_0} (c - \delta) \geq \bar{u} \). From the analysis of the basic model, we know that under this condition the incentive compatible compensation for a trained worker, \((w_L, w_H) = (0, \frac{c - \delta}{p_1 - p_0})\), already implies the worker’s participation constraint so that a trained worker will choose high effort at lowest implementation costs under this contract. Thus, the firm would like to commit in \( t = 1 \) to the contract \((w_L, w_H) = (0, \frac{c - \delta}{p_1 - p_0})\) for \( t = 2 \). If the firm offers training in the first period (i.e., \( \tau = 1 \)), on the one hand the worker can choose \( l = 0 \) so that there will be no successful human capital investment. As a consequence, the worker has high effort costs, implying that \( w_H = \frac{c - \delta}{p_1 - p_0} \) is not incentive compatible. Hence, the worker chooses \( e = 0 \) in \( t = 2 \) and earns expected net income \( p_0 \frac{c - \delta}{p_1 - p_0} \). On the other hand, the worker can choose \( l = 1 \) so that human capital investment is successful. Then, \( w_H = \frac{c - \delta}{p_1 - p_0} \) is incentive compatible and the worker chooses \( e = 1 \) in \( t = 2 \). In that case, the worker earns the expected net income

\[
p_1 \frac{c - \delta}{p_1 - p_0} - (c - \delta) = p_0 \frac{c - \delta}{p_1 - p_0}.
\]

According to the third tie-breaking rule of Section 2, the worker chooses \( l = 1 \).
It has to be checked whether the firm prefers this contractual solution to optimal spot contracts without commitment. Under the commitment solution, the firm’s costs for training the worker and implementing high effort are given by

\[ p_1 \frac{c - \delta}{p_1 - p_0} + \kappa. \]

If the firm does not make use of the commitment solution, it will nonetheless implement high effort via the optimal second-period spot contract. From Proposition 1 we know that under \( \frac{p_0}{p_1 - p_0} (c - \delta) \geq \bar{u} \) successful human capital investment is not possible so that the firm has to offer high-powered incentives via \((w_L, w_H) = (0, \frac{c}{p_1 - p_0})\) in \( t = 2 \). The firm’s expected implementation costs are, therefore,

\[ p_1 \frac{c}{p_1 - p_0}. \]

These costs are higher than the firm’s costs under the commitment solution if and only if

\[ p_1 \frac{c}{p_1 - p_0} > p_1 \frac{c - \delta}{p_1 - p_0} + \kappa \iff \delta \frac{p_1}{p_1 - p_0} > \kappa, \]

which is true because, by assumption, \( \delta > \kappa \).

Now, suppose that \( \frac{p_0}{p_1 - p_0} (c - \delta) < \bar{u} \) – which is equivalent to \( \frac{c - \delta + \bar{u}}{p_1} > \frac{c - \delta}{p_1 - p_0} \) – and that the firm commits to contract \((w_L, w_H) = (0, \frac{c - \delta + \bar{u}}{p_1})\), which will guarantee incentive compatibility and participation if the worker is trained. Again, there are two alternatives for the worker when facing a training offer \((\tau = 1)\) by the firm in \( t = 1 \). First, the worker might choose \( l = 0 \). In that case, the worker has high effort cost and the given contract is either not incentive compatible or unattractive from the firm’s point of view (and, hence, irrelevant as it would contradict the starting point \( \tau = 1 \)).\(^{13}\) Given

\(^{13}\)The contract will be incentive compatible if \( \frac{c - \delta + \bar{u}}{p_1} \geq \frac{c}{p_1 - p_0} \iff \frac{p_0}{p_1 - p_0} c + \delta \leq \bar{u} \), which does not contradict \( \frac{p_0}{p_1 - p_0} (c - \delta) < \bar{u} \). However, the condition \( \frac{p_0}{p_1 - p_0} c + \delta \leq \bar{u} \) violates
that incentive compatibility is violated, the worker chooses $e = 0$ and has expected income $p_0 \frac{c - \delta + \bar{u}}{p_1}$. Second, the worker might choose $l = 1$. In that situation, the worker has low effort cost, chooses high effort and obtains $p_1 \frac{c - \delta + \bar{u}}{p_1} - (c - \delta) = \bar{u}$. The worker prefers $l = 1$ to $l = 0$ because

$$\bar{u} \geq p_0 \frac{c - \delta + \bar{u}}{p_1} \iff \bar{u} \geq \frac{p_0}{p_1 - p_0} (c - \delta)$$

holds.

Finally, we have to check, whether the firm indeed wants to make use of the commitment solution. Given that solution, the firm’s implementation and training costs amount to

$$p_1 \frac{c - \delta + \bar{u}}{p_1} + \kappa = c - \delta + \bar{u} + \kappa.$$

As alternative to the commitment solution, the firm can use spot contracts to implement high effort. From the proof of Proposition 1 we know that for $\bar{u} \geq p_0 c / (p_1 - p_0)$ (and $\bar{u}$ not being too large) the firm also successfully invests in human capital when using spot contracts, leading to identical overall costs $c - \delta + \bar{u} + \kappa$. In that case, the firm is indifferent between the commitment solution and spot contracts. For the parameter constellations

$$\frac{p_0}{p_1 - p_0} (c - \delta) < \bar{u} < \frac{p_0}{p_1 - p_0} c,$$

however, the firm cannot successfully train the worker under spot contracts because the worker prefers $l = 0$. The firm, therefore, chooses $\tau = 0$ in that situation and benefits from a zero outside option of the worker. The firm optimally offers the incentive compatible wage $w_H = \frac{c}{p_1 - p_0}$ in $t = 2$, which also ensures participation because the worker earns a positive expected net the condition of Proposition 2, which describes an upper bound for the worker’s outside option so that human capital investment is beneficial for the firm. This condition is already known from Proposition 1.

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income, which is identical to his rent in that case. As the firm’s implementation costs are
\[ p_1 \frac{c}{p_1 - p_0}, \]
the firm will prefer human capital investment and the commitment solution as long as
\[ c - \delta + \bar{u} + \kappa \leq p_1 \frac{c}{p_1 - p_0} \iff \bar{u} \leq \frac{p_0}{p_1 - p_0} c + \delta - \kappa, \]
which is the condition in Proposition 2, being identical to the upper bound for \( \bar{u} \) in Proposition 1.

Proof of Proposition 3:
Suppose that \( \frac{p_0}{p_1 - p_0} (c - \delta) \geq \bar{u} \) and that the firm lacks sufficient commitment power when announcing the second-period wage schedule \( (w_L, w_H) \) in \( t = 1 \). Consequently, if the firm observes that human capital investment has failed, it will offer the high wage \( w_H = \frac{c}{p_1 - p_0} \) in \( t = 2 \) given that the initial wage was lower. The worker accepts the new wage, because it increases his expected net income. Again, I am looking for the commitment solution that leads to the lowest costs for the firm. Let \( w_H = \tilde{w} \in \left[ \frac{c - \delta}{p_1 - p_0}, \frac{c}{p_1 - p_0} \right] \) be the initial second-period wage that is offered by the firm in \( t = 1 \) being sufficiently large to make the worker choose \( l = 1 \). Suppose, the firm chooses \( r = 1 \). The worker has two alternatives. On the one hand, he can choose \( l = 0 \) so that there is no successful human capital investment, the worker has high effort costs and \( w_H = \tilde{w} \) is not incentive compatible. Thus, the worker and the firm agree to renegotiate the initial contract so that \( w_H = \frac{c}{p_1 - p_0} \), which restores incentives and leads to the expected net income
\[ p_1 \frac{c}{p_1 - p_0} - c = \frac{p_0}{p_1 - p_0} c \]
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for the worker. On the other hand, the worker can choose \( l = 1 \) so that training is successful. Since \( \tilde{w} \in \left[ \frac{c-\delta}{p_1-p_0}, \frac{c}{p_1-p_0} \right) \), the initial contract is incentive compatible and the firm does not offer a higher wage. Thus, the worker chooses \( e = 1 \) and obtains the expected net income \( p_1 \tilde{w} - (c - \delta) \). The lowest wage \( \tilde{w} \) that makes the worker indifferent between \( l = 0 \) and \( l = 1 \) so that he chooses \( l = 1 \) according to the third tie-breaking rule is described by

\[
\frac{p_0}{p_1-p_0} c = p_1 \tilde{w} - (c - \delta) \Leftrightarrow \tilde{w} = \frac{c - \delta}{p_1-p_0} + \frac{\delta p_0}{(p_1-p_0) p_1}.
\]

Although the wage \( \tilde{w} \) is higher than the optimal wage \( w_H = \frac{c-\delta}{p_1-p_0} \) under the previous commitment solution with sufficient commitment power, the firm still benefits from commitment. The firm’s costs are now

\[
p_1 \left( \frac{c - \delta}{p_1-p_0} + \frac{\delta p_0}{(p_1-p_0) p_1} \right) + \kappa,
\]

which are still lower than the expected implementation costs under the optimal spot contract, \( p_1 \frac{c}{p_1-p_0} \).

Now, consider the scenario \( \frac{p_0}{p_1-p_0} (c - \delta) < \bar{u} < \frac{p_0}{p_1-p_0} c \) and let \( w_H = \tilde{w} \in \left( \frac{c-\delta+\bar{u}}{p_1}, \frac{c}{p_1-p_0} \right) \) be the initial second-period wage that is offered by the firm in \( t = 1 \) being sufficiently large to make the worker choose \( l = 1 \).\(^ {14} \) Suppose, the firm offers training \( (\tau = 1) \). If the worker chooses \( l = 0 \), the wage \( w_H = \tilde{w} \) will not be incentive compatible so that the worker and the firm agree to renegotiate the initial contract, leading to \( w_H = \frac{c}{p_1-p_0} \) and expected net income \( \frac{p_0}{p_1-p_0} c \) for the worker. If, however, the worker chooses \( l = 1 \), human capital investment will be successful and \( \tilde{w} \) incentive compatible. In that case, the worker has expected net income \( p_1 \tilde{w} - (c - \delta) \). From above we know that the lowest wage \( \tilde{w} \) that makes the worker indifferent between

\(^ {14} \) Recall that the proof of Proposition 2 shows that commitment-based solutions do not improve spot contracts if \( \bar{u} > \frac{p_0 c}{(p_1 - p_0)} \).
\( l = 0 \) and \( l = 1 \) so that he chooses \( l = 1 \) is given by
\[
\hat{w} \equiv \bar{w} = \frac{c - \delta}{p_1 - p_0} + \frac{\delta p_0}{(p_1 - p_0) p_1},
\]
leading to overall costs
\[
p_1 \left( \frac{c - \delta}{p_1 - p_0} + \frac{\delta p_0}{(p_1 - p_0) p_1} \right) + \kappa
\]
for the firm, which are lower than the implementation costs under the optimal spot contract, \( p_1 \frac{c}{p_1 - p_0} \).

**Proof of Lemma 2:**

Suppose (10) is stricter than (9), i.e.,
\[
\frac{c - \delta \tau l + \bar{u}_2}{p_1} \geq \frac{c - \delta \tau l}{p_1 - p_0} \iff \bar{u}_2 \geq \frac{p_0}{p_1 - p_0} (c - \delta \tau l).
\]
In that case, the Nash product \( NP(w_H) \) takes the value zero at the lower bound of the relevant range \( w_H \in \left[ \frac{c - \delta \tau l + \bar{u}_2}{p_1}, \pi \right] \) and a strictly positive value for wages \( w_H \) that are slightly larger than the lower bound. As
\[
NP'(w_H) = \alpha p_1 \left( p_1 w_H - (c - \delta \tau l) - \bar{u}_2 \right)^{\alpha - 1} \cdot (p_1 (\pi - w_H))^{1 - \alpha}
- (1 - \alpha) p_1 \left( p_1 w_H - (c - \delta \tau l) - \bar{u}_2 \right)^\alpha \cdot (p_1 (\pi - w_H))^{-\alpha}
\]
and
\[
NP''(w_H) = -\alpha (1 - \alpha) p_1^2 \left\{ (p_1 w_H - (c - \delta \tau l) - \bar{u}_2)^{\alpha - 2} \cdot (p_1 (\pi - w_H))^{1 - \alpha}
+ 2 (p_1 w_H - (c - \delta \tau l) - \bar{u}_2)^{\alpha - 1} \cdot (p_1 (\pi - w_H))^{-\alpha}
+ (p_1 w_H - (c - \delta \tau l) - \bar{u}_2)^\alpha \cdot (p_1 (\pi - w_H))^{-\alpha - 1} \right\},
\]
\( NP(w_H) \) is a strictly concave function over the relevant range with an interior maximum being described by the first-order condition \( NP'(w_H) = 0 \):
\[
\bar{w}_H^* = \alpha \pi + (1 - \alpha) \frac{\delta \tau l + \bar{u}_2}{p_1}.
\]
Note that $\tilde{w}_H^* \in (\frac{c-\delta \tau l + \bar{u}_2}{p_1}, \pi)$, because $\alpha \pi + (1 - \alpha) \frac{c-\delta \tau l + \bar{u}_2}{p_1} < \pi \iff \pi > \frac{c-\delta \tau l + \bar{u}_2}{p_1}$ follows from the efficiency condition (2).

Now suppose (9) is stricter than (10), i.e.,

$$\frac{c - \delta \tau l}{p_1} + \bar{u}_2 < \frac{c - \delta \tau l}{p_1} \iff \bar{u}_2 < \frac{p_0}{p_1 - p_0} (c - \delta \tau l),$$

so that the relevant range for $w_H$ is given by $[\frac{c - \delta \tau l}{p_1}, \pi]$. We have to distinguish between two cases. First, it is possible that $\frac{c - \delta \tau l + \bar{u}_2}{p_1} < \frac{c - \delta \tau l}{p_1} \leq \tilde{w}_H^*$ with

$$\frac{c - \delta \tau l}{p_1} \leq \tilde{w}_H^* \iff \alpha \geq \frac{p_0}{p_1 - p_0} (c - \delta \tau l) - \bar{u}_2 = \bar{\alpha} (\tau l).$$

In that case, again the interior solution $w_H = \tilde{w}_H^*$ is obtained. Second, it is possible that $\frac{c - \delta \tau l + \bar{u}_2}{p_1} < \tilde{w}_H^* < \frac{c - \delta \tau l}{p_1 - p_0}$ with

$$\tilde{w}_H^* < \frac{c - \delta \tau l}{p_1 - p_0} \iff \alpha < \bar{\alpha} (\tau l).$$

In this second scenario, the interior solution $\tilde{w}_H^*$ is not feasible and $NP (w_H)$ is maximized by the corner solution

$$w_H = \frac{c - \delta \tau l}{p_1 - p_0}.$$

**Proof of Lemma 3:**

Suppose that the firm has chosen $\tau = 1$. Suppose further that $\bar{u} \geq \frac{p_0}{p_1 - p_0} (c - \delta)$. If the worker chooses $l = 1$ in this situation, his expected income will be $\bar{u} + \alpha [p_1 \pi - (c - \delta) - \bar{u}]$ according to (13). If, however, the worker chooses $l = 0$, we will have $\bar{u}_2 < \frac{p_0}{p_1 - p_0} (c - \delta \tau l)$ and – according to Lemma 2 – his expected income will depend on whether

$$\alpha \geq \frac{p_0}{p_1 - p_0} \frac{c}{p_1 \pi - c} = \bar{\alpha} (0).$$

\(^{15}\bar{\alpha} (\tau l) \leq 1 \iff \frac{c - \delta \tau l}{p_1 - p_0} \leq \pi \text{ is true because } \pi \geq w_H \geq \frac{c - \delta \tau l}{p_1 - p_0}.\)
or not. If $\alpha \geq \tilde{\alpha} (0)$, the worker’s expected net income is again described by (13): $\alpha [p_1 \pi - c]$. If $\alpha < \tilde{\alpha} (0)$, the worker’s expected net income is described by (14): $\frac{p_0}{p_1 - p_0} c$. Altogether, given $\alpha \geq \tilde{\alpha} (0)$, the worker will prefer $l = 1$ to $l = 0$ if

$$\tilde{u} + \alpha [p_1 \pi - (c - \delta) - \tilde{u}] \geq \alpha [p_1 \pi - c] \Leftrightarrow (1 - \alpha) \tilde{u} + \alpha \delta \geq 0,$$

which is true; given $\alpha < \tilde{\alpha} (0)$, the worker will prefer $l = 1$ to $l = 0$ if

$$\tilde{u} + \alpha [p_1 \pi - (c - \delta) - \tilde{u}] \geq \frac{p_0}{p_1 - p_0} c \Leftrightarrow \alpha \geq \frac{\frac{p_0}{p_1 - p_0} c - \tilde{u}}{p_1 \pi - (c - \delta) - \tilde{u}} =: \tilde{\alpha}.$$

Now suppose that $\tilde{u} < \frac{p_0}{p_1 - p_0} (c - \delta)$. Given that the worker chooses $l = 1$, according to Lemma 2 and (13), the worker’s expected net income will be

$$\tilde{u} + \alpha [p_1 \pi - (c - \delta) - \tilde{u}]$$

if, however, $\alpha < \tilde{\alpha} (1)$, then the worker’s expected net income will be $\frac{p_0}{p_1 - p_0} (c - \delta)$ according to (14). Given that the worker chooses $l = 0$, according to Lemma 2 and (13), the worker’s expected net income will be $\alpha [p_1 \pi - c]$ if

$$\alpha \geq \frac{\frac{p_0}{p_1 - p_0} c}{p_1 \pi - c} = \tilde{\alpha} (0);$$

if, however, $\alpha < \tilde{\alpha} (0)$, then the worker’s expected net income will be $\frac{p_0}{p_1 - p_0} c$ according to (14). Note that

$$\tilde{\alpha} (1) = \frac{\frac{p_0}{p_1 - p_0} (c - \delta) - \tilde{u}}{p_1 \pi - (c - \delta) - \tilde{u}} < \frac{\frac{p_0}{p_1 - p_0} c}{p_1 \pi - c} = \tilde{\alpha} (0) \Leftrightarrow$$

$$-\pi \delta p_0 < \left( (p_1 - p_0) \pi - c \right) \tilde{u}$$

is true because $(p_1 - p_0) \pi - c \geq 0$ holds due to (12). Thus, we have to distinguish between three cases. If $\tilde{\alpha} (1) < \tilde{\alpha} (0) \leq \alpha$, then the worker will
prefer \( l = 1 \) to \( l = 0 \) because \( \bar{u} + \alpha [p_1 \pi - (c - \delta) - \bar{u}] > \alpha [p_1 \pi - c] \). If \( \bar{\alpha} (1) \leq \alpha < \bar{\alpha} (0) \), the worker will prefer \( l = 1 \) to \( l = 0 \) if

\[
\bar{u} + \alpha [p_1 \pi - (c - \delta) - \bar{u}] \geq \frac{p_0}{p_1 - p_0} c \iff \alpha \geq \bar{\alpha}.
\]

If \( \alpha < \bar{\alpha} (1) < \bar{\alpha} (0) \), the worker will prefer \( l = 0 \) because \( \frac{p_0}{p_1 - p_0} (c - \delta) < \frac{p_0}{p_1 - p_0} c \).

For summing up the cases in which the worker prefers \( l = 1 \) to \( l = 0 \), it is important to note that

\[
\bar{\alpha} (0) > \bar{\alpha} > \bar{\alpha} (1).
\]

The relation \( \bar{\alpha} > \bar{\alpha} (1) \) can immediately be seen from the two expressions for \( \bar{\alpha} \) and \( \bar{\alpha} (1) \); the relation \( \bar{\alpha} (0) > \bar{\alpha} \) holds because

\[
\frac{p_0}{p_1 - p_0} c > \frac{p_0}{p_1 - p_0} c - \bar{u} \iff (\pi (p_1 - p_0) - c) \bar{u} p_1 > -c \delta p_0
\]

is true according to (12). Therefore, given \( \bar{u} \geq \frac{p_0}{p_1 - p_0} (c - \delta) \), the worker will prefer \( l = 1 \) to \( l = 0 \) if \( \alpha \geq \bar{\alpha} (0) \), or if \( \alpha < \bar{\alpha} (0) \) and \( \alpha \geq \bar{\alpha} \), which can be summarized as \( \alpha \geq \bar{\alpha} \iff \bar{u} + \alpha \cdot [p_1 \pi - (c - \delta) - \bar{u}] \geq \frac{p_0}{p_1 - p_0} c \) – condition (15) in Lemma 3. Given \( \bar{u} < \frac{p_0}{p_1 - p_0} (c - \delta) \), the worker will prefer \( l = 1 \) to \( l = 0 \) if \( \bar{\alpha} (1) < \bar{\alpha} (0) \leq \alpha \), or if \( \bar{\alpha} (1) \leq \alpha < \bar{\alpha} (0) \) and \( \alpha \geq \bar{\alpha} \), which again can be summarized as \( \alpha > \bar{\alpha} \) – condition (15) in Lemma 3.

**Proof of Proposition 5:**

Since, by assumption, the firm is always interested in human capital, it will choose \( \tau = 1 \) if it anticipates that the worker will react by choosing \( l = 1 \). Hence, I only have to check the worker’s willingness to learn. In analogy to Lemma 1, in \( t = 2 \) the worker earns

\[
\max \left\{ EU_m, \bar{u}_2 \right\} \quad \text{with} \quad EU_m := \frac{p_{0m}}{p_{1m} - p_{0m}} c
\]
and \( m = u \) or \( m = t \) depending on whether the worker is untrained or trained. \( EU_m \) denotes the worker’s expected net income under a non binding participation constraint. If

\[
EU_m \leq \tilde{u} \quad (m = u, t),
\]

the worker will prefer \( l = 1 \) to \( l = 0 \) to achieve an attractive outside option. Suppose that the expected net income satisfies

\[
EU_m > \tilde{u} \quad \text{for } m = u \text{ and/or } m = t.
\]

There are four possible constellations:

\[
EU_u < \tilde{u} < EU_t,
\]

\[
\tilde{u} < EU_u < EU_t,
\]

\[
EU_t < \tilde{u} < EU_u,
\]

and \( \tilde{u} < EU_t < EU_u. \)

In two of them – \( EU_t < \tilde{u} < EU_u \) and \( \tilde{u} < EU_t < EU_u \) – the worker prefers \( l = 0 \) to \( l = 1 \). In these two constellations, we have

\[
EU_u = \frac{p_{0u}}{p_{1u} - p_{0u}} c > \frac{p_{0t}}{p_{1t} - p_{0t}} c = EU_t \iff \frac{p_{1t}}{p_{0t}} > \frac{p_{1u}}{p_{0u}}.
\]

In other words, if \( \frac{p_{1t}}{p_{0t}} \leq \frac{p_{1u}}{p_{0u}} \), the worker will always prefer \( l = 1 \) irrespective of whether he earns a positive rent in \( t = 2 \) or not. If, however, \( \frac{p_{1t}}{p_{0t}} > \frac{p_{1u}}{p_{0u}} \) – i.e., the worker’s rent decreases in human capital – he will only prefer \( l = 1 \) if human capital yields an outside option \( \tilde{u} \) that is at least as large as the expected net income \( EU_u \); otherwise, the worker prefers \( l = 0 \) and the firm chooses \( \tau = 0 \) to save training costs.
Proof of Proposition 6:

The firm wants to minimize overall costs for implementing high effort in $t = 2$. If the firm anticipates that the worker learns (i.e., $l = 1$) and if it wants to invest in human capital (i.e., $\delta > 0$), it will minimize

$$c - \delta + \bar{u}_2(\beta \cdot \delta) + \kappa(\delta) = c + \kappa(\delta) - (1 - \beta)\delta. \quad (23)$$

The firm’s objective function (23) takes into account that the worker will only choose $l = 1$ if his second-period participation constraint is binding (see the proof of Proposition 1). Under the binding participation constraint, the firm has to compensate the worker for his effort costs, $c - \delta$, and his foregone outside option, $\bar{u}_2(\beta \cdot \delta) = \beta\delta$. In addition, the firm has to bear its training costs, $\kappa(\delta)$. Recall that, in case of a non-binding participation constraint, human capital investment lowers the worker’s second-period rent. Thus, to make the worker indeed choose $l = 1$ in the first period, human capital investment must yield a second-period outside option for the worker that (weakly) exceeds the worker’s second-period rent without training:

$$\bar{u}_2(\beta \cdot \delta) \geq \frac{p_0}{p_1 - p_0}c \iff (1 - \beta)\delta \geq \frac{p_0}{p_1 - p_0}c. \quad (24)$$

Finally, the firm will only be interested in training the worker if its implementation costs under human capital investment – being described by (23) – do not exceed its implementation costs without human capital investment – being given by the expected wage payment $p_1 \cdot w_H = p_1 \cdot \frac{c}{p_1 - p_0}$ (see (21) in the proof of Proposition 1):

$$c + \kappa(\delta) - (1 - \beta)\delta \leq \frac{p_1c}{p_1 - p_0} \iff (1 - \beta)\delta - \kappa(\delta) + \frac{p_0}{p_1 - p_0}c \geq 0. \quad (25)$$

To summarize, if the firm wants to invest in human capital, it will solve

$$\max_{\delta} (1 - \beta)\delta - \kappa(\delta) - c \quad \text{subject to (24) and (25).}$$
There are three possible outcomes. (i) If \( \hat{\delta} := \arg \max_{\delta} (1 - \beta)(1 - \kappa (\delta) - c) \), being described by the first-order condition \((1 - \beta) = \kappa' (\hat{\delta}) \), satisfies (24) (i.e., \( \hat{\delta} \geq \frac{p_0}{p_1 - p_0} \frac{\zeta}{\hat{\beta}} \)), the firm will choose the interior solution \( \hat{\delta} \) (note that this solution always satisfies (25)). Since \( \partial \hat{\delta} / \partial \beta = -1/\kappa'' (\hat{\delta}) < 0 \), optimal investment decreases in \( \beta \) in this case. (ii) If \( \frac{p_0}{p_1 - p_0} \frac{\zeta}{\hat{\beta}} \in (\hat{\delta}, \bar{\delta}) \) with the upper bound \( \bar{\delta} \) being defined by (19) above, the firm optimally chooses \( \delta = \frac{p_0}{p_1 - p_0} \frac{\zeta}{\hat{\beta}} \) with (25) being slack due to the definition of \( \bar{\delta} \). Again, optimal investment decreases in \( \beta \). (iii) If \( \frac{p_0}{p_1 - p_0} \frac{\zeta}{\hat{\beta}} > \bar{\delta} \), then human capital investment would either harm the firm or the worker.

**Proof of Proposition 7:**

Using the insights from the proof of Proposition 6, the Lagrangian to the firm’s optimization problem reads as follows:

\[
L (\delta, \beta) = (1 - \beta)(1 - \kappa (\delta) - c) + \lambda_1 \cdot \left[ \beta \delta - \frac{p_0}{p_1 - p_0} c \right] + \lambda_2 \cdot \left[ (1 - \beta)(1 - \kappa (\delta)) + \frac{p_0}{p_1 - p_0} c \right]
\]

with \( \lambda_1, \lambda_2 \geq 0 \) as multipliers. The optimality conditions yield

\[
\frac{\partial L}{\partial \delta} = (1 + \lambda_2) [(1 - \beta) - \kappa' (\delta)] + \lambda_1 \beta = 0 \quad (26)
\]

\[
\frac{\partial L}{\partial \beta} = -\delta + \lambda_1 \delta - \lambda_2 \delta = 0. \quad (27)
\]

From condition (27) we obtain \( \lambda_1 = 1 + \lambda_2 > 0 \), which implies that the first constraint – ensuring that the worker learns – is binding. Inserting \( \lambda_1 = 1 + \lambda_2 \) into (26) leads to \( 1 = \kappa' (\delta^*) \). Since the first constraint is binding, optimal training specificity, \( \beta^* \), is described by \( \beta^* \delta^* = \frac{p_0}{p_1 - p_0} c \).
References


