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Less fighting than expected

Experiments with wars of attrition and all-pay auctions

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Abstract While all-pay auctions are well researched experimentally, we do not have much laboratory evidence on wars of attrition. This paper tries to fill this gap. Technically, there are only a few differences between wars of attrition and all-pay auctions. Behaviorally, however, we find striking differences: As many studies, our experiment finds overbidding in all-pay auctions. In contrast, in wars of attrition we observe systematic underbidding.

We study bids and expenditures in different experimental frames and matching procedures and tie in with the literature on stepwise linear bidding functions.

Keywords: War of attrition, dynamic bidding, all-pay auction, stabilization, volunteer's dilemma, experiment
JEL classification C72, C92, D44, E62, H30

1 Introduction

In this paper we compare behavior in all-pay auctions with behavior in static or dynamic wars of attrition. Both institutions are frequently used to model conflicts or fights. While we know a lot about behavior in all-pay auctions much less is known about behaviour in wars of attrition. We use laboratory experiments to investigate whether behavioral anomalies in wars of attrition are similar to those in all-pay auctions.

The laboratory evidence for all-pay auctions documents that human decision makers tend to overbid and make investments that are even higher than the already high and socially wasteful investments in equilibrium. To compare all-pay auctions and wars of attrition we replicate the standard first-price all-pay auction experiment as a benchmark with a single prize and two bidders. As we should expect from other experiments, we find overbidding and stepwise linear bidding functions. We then extend the experimental literature by investigating a war of attrition, which we implement both as a dynamic and as a static bidding process. In contrast to all-pay auctions we find underbidding, in particular in the dynamic implementation of the war of attrition. Furthermore, we do not find stepwise linear bidding functions.

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All-pay auctions and wars of attrition can be interpreted as fights over a prize. Bids are static in the all-pay auction and dynamic in the war of attrition. In an all-pay auction with two players both players make investments (bids). Both players pay their bid and the player with the larger investment wins a prize. In a war of attrition both players incur a cost for each unit of time while they are fighting. Both players pay their investments during the conflict. As soon as one player gives up, the conflict ends and the remaining player earns the prize.

Due to its dynamic character the war of attrition has been used to model a wide range of problems: Arms races (Zimin and Ivanilov 1968), fights between animals (Bishop, Canning, and Smith 1978; Riley 1980), the voluntary provision of public goods (Bliss and Nalebuff 1984; Bilodeau and Slivinski 1996), competition between firms (Ghemawat and Nalebuff 1985; Fudenberg and Tirole 1986; Roth 1996), the settlement of strikes (Kennan and Wilson 1989; Card and Olson 1995), fiscal and political stabilizations (Alesina and Drazen 1991; White 1995), the timing of exploratory oil drilling (Hendricks and Porter 1996), and many other conflicts are all wars of attrition.

Wars of attrition with two players are isomorphic to (static) second-price all-pay auctions. In a second-price all-pay auction with two bidders both bidders submit a bid (these bids are equivalent to the amount of time bidders are willing to hold out during the war of attrition), the bidder with the highest bid wins the prize, and both bidders pay the amount of the second highest bid. Similarly, in the war of attrition the winner does not have to pay more once the loser gives up, i.e., effectively pays the second highest bid.[1]

While there are many experiments on all-pay auctions there are surprisingly few on wars of attrition. Our paper tries to fill this gap. We will show that, although theoretically very similar, behavior is quite different under the two institutions.

We will compare the following situations in our experiment:

- A static version of an all-pay auction.
- A static version of a war of attrition (as an intermediate case).
- A dynamic version of a war of attrition. For the dynamic war of attrition we will also compare the impact of repetition and ranges of the bidding cost.

Table 1 provides an overview of the experimental literature. Experiments with all-pay auctions have been done by Davis and Reilly (1998), Potters, de Vries, and van Winden (1998), Barut, Kovenock, and Noussair (2002), Gneezy and Smorodinsky (2006), and Müller and Schotter (2007). Four of these studies find a significant amount of overbidding while one (Potters, de Vries, and van Winden 1998) finds no significant deviation from equilibrium. A famous variant of an all-pay auction is the dollar auction game (Shubik 1971). Müller and Schotter (2007) observe in experiments with all-pay auctions what they call bifurcation of bidding functions. Compared with the equilibrium prediction bids are too small if bidding is expensive, and too large if bidding is cheap. Bidding functions can be approximated with the help of a stepwise linear function similar to the one in Figure 1. In section 4.4 we will try to replicate Müller and Schotter's results and check whether these stepwise linear functions can be found in a war of attrition, too.

[Figure 1 about here.]

[Table 1 about here.]

Another related auction format is a rent-seeking contest. As in all-pay auctions in rent-seeking contests, too, all players pay their own bid. In contrast to the all-pay auction the highest bidder in the rent-seeking contest does not win the auction with certainty. Instead, the probability of winning is only an increasing function of the bid.[2] Rent-seeking contests have been introduced by Bishop, Canning, and Smith (1978) and Tullock (1980).[3] Possibly the first test of Tullock's model in the laboratory is an experiment by Millner and Pratt (1989). Their, perhaps disappointing, result is that participants make substantially larger bids, i.e., socially wasteful investments, in the rent-seeking contest than they should in equilibrium. Several other experiments with rent-seeking contests followed.[4] Seven of the ten studies of rent-seeking contests find a significant amount of overbidding, two studies (Shogren and Baik 1991; Vogt, Weimann, and Yang 2002) find no significant deviation from equilibrium, and only one (Schmidt, Shupp, and Walker 2004) finds significant underbidding in rent-seeking contests.

A generalized war of attrition, or a volunteer's dilemma, is an nth price all-pay auction with $n-1>1$ prizes.[5] A simple version of such a game has been studied experimentally by Diekmann (1993). Participants decide whether to provide a public good. Each player can choose only between two different bids, a low bid (waiting for somebody else to provide the public good) or a high bid (provision of the public good). Bilodeau, Childs, and Mestelman (2004) study experiments of a sequential version of this game where players are fully informed about each other's bidding costs and find overbidding.

In sum, most of the experiments with variants of all-pay auctions find systematic overbidding, i.e., bids that are higher than the risk neutral equilibrium. Actually, even in regular (winner-only-pays) second-price and first-price auctions overbidding is a common phenomenon, although to a lesser degree.[6] If we extrapolate from all-pay auctions and rent-seeking contests, we should expect some overbidding in wars of attrition, too.

We will proceed as follows: In section 2 we derive the equilibrium bidding functions. In section 3 we discuss the experimental procedure and present our setup. Section 4 presents the results and section 5 concludes.

2 Hypotheses and Equilibrium predictions

2.1 Equilibrium in the war of attrition

In this section we derive the risk neutral Bayesian Nash equilibrium for the war of attrition with two bidders, i and j. As noted above, wars of attrition with two players are isomorphic to second-price all-pay auctions. We will call P the prize from winning the war of attrition. In the auction bidders make bids b by staying for up to b seconds in the auction unless the opponent leaves the auction earlier. If bidder i stays for b seconds in the auction this bidder incurs a cost $b \cdot c_i$, where c_i is private information of bidder i and not known to bidder j. The other bidder knows only that c_i is drawn from a uniform distribution over $[\underline{c}, \overline{c}]$ where $\underline{c}$ is positive and should not be too small compared to $\overline{c}$ to ensure that the participation constraint is fulfilled. We follow a standard approach and assume that the opponent of bidder i, bidder j with cost per second c_j, uses a decreasing bidding function $\beta_e^W(c)$ with inverse $\beta_e^{W(-1)}(\cdot)$. Bidder i does not know c_j, but can calculate the expected utility EU_i, given that i's bidding cost per second is c_i and bidder i bids up to b seconds. If bidders maximize a utility function $u(x)$ the expected utility is

$$\mathrm{EU}_i = \underbrace{\int_{\underline{c}}^{\beta_e^{W(-1)}(b)} u(-b\,c_i)\, f(c_j)\, dc_j}_{\substack{\text{bidder } i \text{ does not win} \\ \text{and pays the own bid } b}} + \underbrace{\int_{\beta_e^{W(-1)}(b)}^{\overline{c}} u\left(P-\beta_e^W(c_j)\,c_i\right) f(c_j)\, dc_j}_{\substack{\text{bidder } i \text{ wins } P \text{ and pays} \\ \text{the second highest bid } \beta_e^W}} . \qquad (1)$$

We will first analyze the risk neutral case. Thus, we can replace $u(x)$ by x, take the first derivative $\partial \mathrm{EU}_i/\partial b$ and substitute $b = \beta_e^W(c_i)$ to obtain the first-order condition

$$-c_i \int_{\underline{c}}^{c_i} f(c_j)dc_j - \frac{P \cdot f(c_i)}{\beta_e^{W\prime}(c_i)} = 0\,. \tag{2}$$

If c_i is uniformly distributed over $[\underline{c}, \overline{c}]$ then (2) can be simplified:

$$\beta_e^{W\prime}(c_i) = -\frac{P}{c_i(c_i - \underline{c})} \tag{3}$$

which, with the restriction $\beta_e^W(\overline{c}) = 0$, yields the candidate equilibrium bidding function:

$$\beta_e^W(c_i) = \frac{P}{\underline{c}} \ln \frac{(\overline{c} - \underline{c})c_i}{(c_i - \underline{c})\overline{c}} \tag{4}$$

To check whether the individual participation constraint is fulfilled we substitute (4) into (1) and obtain

$$\mathrm{EU}_i = P \cdot \frac{\overline{c} - c_i + \ln \frac{c_i}{\overline{c}}}{\overline{c} - \underline{c}}\,. \tag{5}$$

Hence, as long as $\underline{c}$ is not too small the expected utility from participating is positive for all $c_i \in [\underline{c}, \overline{c}]$. This is the case for all combinations of parameters we use in our experiment (see Table 2 below). Hence, the expression given by equation (4) describes an equilibrium bidding function.

From equation (4) we see that a bidder with a very small c_i is ready to make a potentially very high bid. It might even be that $\beta_e^W(c_i) \cdot c_i > P$. Since we are in a war of attrition, a bidder with a very small c_i will pay the bid of the opponent with the larger c_i which is much smaller than $\beta_e^W(c_i)$ most of the time. Therefore, as we see in equation (5), the expected utility, ex ante, is still positive. We should note that, during the bidding process, our bidder does not necessarily want to stop when $b \cdot c_i = P$. During the war of attrition the current bid $b \cdot c_i$ is already a sunk cost at any point in time. Only the marginal increment of the bid and the thereby achieved increase in probability of obtaining P are relevant. Hence, while it might seem strange that bidders make bids potentially larger than P, they will (with our parameters) make a positive profit ex ante, and they should not care about their sunk investment ex interim.

[Figure 2 about here.]

An example of the equilibrium bidding function is shown as the solid line in the left diagram in Figure 2. The expected expenditure[7] is

$$R_e^W = \int_{\underline{c}}^{\overline{c}} \beta_e^W(c_i)\, 2\frac{c_i - \underline{c}}{(\overline{c} - \underline{c})^2}\, dc_i = P\frac{\overline{c} - \underline{c}\left(1 + \ln \frac{\overline{c}}{\underline{c}}\right)}{(\overline{c} - \underline{c})^2} \tag{6}$$

2.2 Equilibrium in the all-pay auction

Similar to the derivation for the war of attrition in equations (1) to (5) we can find the equilibrium bidding function for the all-pay auction. We assume that bidder j with cost c_j uses a decreasing bidding function $\beta_e^A(c)$ with inverse $\beta_e^{A(-1)}(\cdot)$. The expected utility of bidder i with per period cost c_i who bids up to b periods is

$$\mathrm{EU}_i = \underbrace{\int_{\underline{c}}^{\beta_e^{A(-1)}(b)} u(-bc_i)\, f(c_j)\, dc_j}_{\substack{\text{bidder } i \text{ does not win} \\ \text{and pays the own bid } b}} + \underbrace{\int_{\beta_e^{A(-1)}(b)}^{\overline{c}} u(P - bc_i)\, f(c_j)\, dc_j}_{\substack{\text{bidder } i \text{ wins } P \text{ and pays} \\ \text{the own bid } b}} . \tag{7}$$

Again, we will first consider the risk neutral case $u(x) = x$. We take the first derivative $\partial \mathrm{EU}_i / \partial b$ and substitute $b = \beta_e^A(c_i)$ to obtain the first-order condition

$$-c_i \int_{\underline{c}}^{c_i} f(c_j) dc_j - c_i \int_{c_i}^{\overline{c}} f(c_j) dc_j - \frac{f(c_i)}{\beta_e^{A\prime}(c_i)} P = 0 . \tag{8}$$

With c_i distributed uniformly over $[\underline{c}, \overline{c}]$ we can simplify (8) to

$$\beta_e^{A\prime}(c_i) = -\frac{P}{c_i \cdot (\overline{c} - \underline{c})} \tag{9}$$

which, with the restriction $\beta_e^A(\overline{c}) = 0$, yields the candidate equilibrium bidding function:

$$\beta_e^A(c_i) = \frac{P}{\overline{c} - \underline{c}} \ln \frac{\overline{c}}{c_i} . \tag{10}$$

To check whether the individual participation constraint is fulfilled we substitute (10) into (7) and obtain

$$\mathrm{EU}_i = P \cdot \frac{(\overline{c} - c_i)(\overline{c} - \underline{c}) - \ln \frac{\overline{c}}{c_i}}{\overline{c} - \underline{c}} . \tag{11}$$

Again, this expression is positive for all $c_i \in [\underline{c}, \overline{c}]$ as long as $\underline{c}$ is not too small. For our parameters this is always the case; hence, the expression given by equation (10) describes an equilibrium bidding function. An example of the equilibrium bidding function is shown as a dashed line in the left part of Figure 2. As long as $c_i \in (\underline{c}, \overline{c})$ we always have $\beta_e^A(c_i) < \beta_e^W(c_i)$. The expected expenditure of the all-pay auction is

$$R_e^A = \int_{\underline{c}}^{\overline{c}} \frac{\beta_e^A(c_i)}{\overline{c} - \underline{c}} dc_i = P \frac{\overline{c} - \underline{c} \left(1 + \ln \frac{\overline{c}}{\underline{c}}\right)}{(\overline{c} - \underline{c})^2} . \tag{12}$$

Comparing with equation (6) we see that the expected expenditure in the two auction formats is the same, $R_e^W = R_e^A$. Krishna and Morgan (1997) point out that this need not be the case if bidding costs are not distributed independently. They show that generally the expenditure in the war of attrition is larger than or equal to expenditure in the all-pay auction.

2.3 Risk aversion

In the previous two sections we have derived equilibrium bidding functions under the assumption of risk neutrality. We know from other auction experiments that bidders' behavior can often be well explained by risk aversion. In this section we derive equilibrium bidding functions for risk averse bidders. While we cannot find a closed form solution for general utility functions, it is possible to follow the above steps for specific utility functions. In our context, constant relative risk aversion $u(x) = x^{1-r}$ is not very meaningful since equilibrium payoffs can be positive or negative. Thus, utility is sometimes real and sometimes complex, which is difficult to interpret. Here we consider only the case of constant absolute risk aversion $u(x) = -e^{-r\cdot x}$.

War of attrition With constant absolute risk aversion r the equilibrium bidding function is

$$\beta_e^{WR}(c_i) = \frac{\overline{c}-\underline{c}}{r\underline{c}}\left(1-e^{\frac{rP}{\underline{c}-\overline{c}}}\right)\cdot\ln\frac{(\overline{c}-\underline{c})c_i}{(c_i-\underline{c})\overline{c}} \tag{13}$$

and the expected expenditure is

$$R_e^{WR} = \frac{1-e^{\frac{rP}{\underline{c}-\overline{c}}}}{r\cdot(\overline{c}-\underline{c})}\cdot\left(\overline{c}-\underline{c}\left(1+\ln\frac{\overline{c}}{\underline{c}}\right)\right). \tag{14}$$

In the risk neutral limit, $r \to 0$, these expressions coincide with their risk neutral counterparts given by (4) and (6). For all positive values of r bids and expenditures are decreasing in r and smaller than the risk neutral values.

First-price all-pay auction The equilibrium bidding function is

$$\beta_e^{AR} = \frac{(\overline{c}-\underline{c})\cdot\left(1-e^{\frac{rP}{\underline{c}-\overline{c}}}\right)}{r\left(\overline{c}\cdot e^{\frac{rP}{\underline{c}-\overline{c}}}-\underline{c}\right)}\cdot\ln\frac{\left((\overline{c}-c_i)\cdot e^{\frac{rP}{\underline{c}-\overline{c}}}+c_i-\underline{c}\right)\cdot\overline{c}}{(\overline{c}-\underline{c})\cdot c_i} \tag{15}$$

and the expected expenditure is

$$R_e^{AR} = \frac{\left(\underline{c}\cdot\ln\frac{\overline{c}}{\underline{c}}+rP\right)e^{\frac{rP}{\underline{c}-\overline{c}}}-\underline{c}\cdot\ln\frac{\overline{c}}{\underline{c}}}{r\cdot(e^{\frac{rP}{\underline{c}-\overline{c}}}\overline{c}-\underline{c})}. \tag{16}$$

In the risk neutral limit, $r \to 0$, these expressions coincide with their risk neutral counterparts given by (10) and (12). For positive values of r bids are smaller if c_i is sufficiently large. This is consistent with the observation of Fibich, Gavious, and Sela (2006). In their paper risk is modeled in a different way, as 'weak risk aversion', which is a small perturbation of the risk neutral utility function. They find that with increasing risk their 'low type', in our model the bidder with a high c_i, bids less the more risk averse bidders become. However, the 'high type', in our model the bidder with a low c_i, bids more the larger the degree of risk aversion. The intuition given by Fibich, Gavious, and Sela also applies in our case: Bidders with a high c_i will, most likely, not win the auction and just lose their bid. Thus, making a high bid is risky for them. The more risk averse bidders are, the lower is the equilibrium bid. Bidders with a small c_i have good chances of winning. Making a slightly too small bid can be very risky for them. Thus, the more risk averse bidders become, the higher are the equilibrium bids for bidders with a low c_i. The right diagram in Figure 2 shows bidding functions for different values of risk aversion r. We see that for most values of c_i risk averse bids are below the risk neutral bid. However, if c_i is very small, risk averse bids are higher than the risk neutral bid. The expenditure is decreasing in r and smaller than the risk neutral expenditure. For the parameters we are using R_e^{AR} decreases more slowly than R_e^{WR}, i.e., with risk averse bidders the expected expenditure is larger with all-pay auctions.

3 Experimental setups

We will use Table 1 to discuss our setup.

Institution: Most experiments summarized in Table 1 have been done with all-pay auctions and rent-seeking contests. There is one experiment with a volunteer's dilemma.

In this paper we concentrate on wars of attrition. We use all-pay auctions as a reference point. With our experiment we want to find out whether the seemingly small differences between the two auction formats affect bidding behavior and bidding anomalies.

Another game that is similar to a war of attrition is the sequential volunteer's dilemma studied by Bilodeau, Childs, and Mestelman (2004). In this game players have symmetric information about each other's bidding cost. In the only subgame perfect equilibrium of their game the bidding process ends in the first round with a bid of zero, i.e., in equilibrium one does not observe dynamically increasing bids. Bilodeau, Childs, and Mestelman observe substantially higher bids, i.e., again overbidding. We depart from Bilodeau, Childs, and Mestelman in analyzing a situation with asymmetric information. The mutual bidding cost is revealed only at the end of the game, i.e., players do not know ex ante who is the weakest bidder.

Bidding procedure: Many other auction experiments use a static bidding procedure. In this paper we will compare two procedures: A static and a dynamic procedure. With the static procedure players simply fill in a number on the screen and the computer immediately determines the winner and the gains and losses from the game. With the dynamic procedure participants see their bid and the time like an ascending clock on the screen and can, similar to the procedure in the Dutch auction, decide to stop the clock. Unlike in the Dutch auction, the person who stops the clock is the loser of the auction. With two bidders the static and the dynamic process are isomorphic,[8] but behaviorally there could be a difference. The dynamic procedure applies naturally in the context of the war of attrition. However, if we find behavioral differences between all-pay auctions and wars of attrition it will be interesting to know more about the reasons for these differences: Does behavior differ as the result of a change from a static to a dynamic procedure, or as the result of a change in the strategic situation? Hence, we will study the war of attrition both from dynamic and static perspectives.

Asymmetric information: Most of the early experiments with all-pay auctions and rent-seeking contests use symmetric information of players about all aspects of the game. This symmetry simplifies the experiment considerably. Only some of the more recent experiments (see Table 1) introduce asymmetric information. In our introduction we mentioned a couple of examples of wars of attrition. In these examples the bidding cost, i.e., the strength of the army or the animal or the strength of the R&D department of a firm, the waiting cost during the settlement of a strike, and so on, neither is known perfectly to everybody nor is the same for all contestants. More technically, the rather abstract case of perfect symmetry rules out equilibria in pure strategies. To avoid mixed equilibria as a benchmark solution and to be closer to the applications we have in mind we will assume that bidding costs are private information and are uniformly distributed over some interval $[\underline{c}, \overline{c}]$.

Most of the treatments we did were based on intervals with $\underline{c}/\overline{c} = 1/2$ with $\overline{c}$ between 3.6 and 5.2.[9] After normalizing the bids we did not find any significant differences between these intervals. Hence, we decided to pool treatments with $\underline{c}/\overline{c} = 1/2$ but different $\overline{c}$.

Number of contestants and prizes: Most of the experiments that we summarise in Table 1 have two bidders. A setup with two bidders is particularly attractive for a comparison of a dynamic game with a static game, since with two bidders the static and the dynamic game are isomorphic. We will, hence, have two bidders and one prize in our experiment.

In this context we should mention Anderson, Goeree, and Holt (1998), who develop a model of boundedly rational bidders which relates overbidding to the number of bidders. The more bidders there are the more overbidding we should observe in this model. With two bidders we have the smallest possible number of bidders; thus we should not expect too much overbidding. Since we are not interested in the absolute level of overbidding but in a comparison of bidding behavior in different auction formats, the number of bidders is not essential for us.

Repetitions: The Bayesian Nash equilibria, which we present in sections 2.1, 2.2, and 2.3, are the equilibria of a game that is played once or, by backward induction, equilibria of a game that is played finitely many times. We will study both situations: random matching of players after each round and random matching of players only after a fixed number of repetitions. While the one-shot game constitutes a simple benchmark, we find the treatment with repetitions more interesting. Players who have fought over the allocation of a resource in the past will do this again. Regardless of whether we interpret the war of attrition as an arms race, as the provision of public goods, the competition between firms, the settlement of strikes, or as political stabilization—wars of attrition tend to repeat. Hence, most of our experiments are based on the finitely repeated war of attrition. The repeated situation is not only closer to the conflict we actually want to model, but studying repeated wars of attrition also has a technical advantage in the laboratory since the waiting time for other participants is considerably reduced.[10] Nevertheless, we include a treatment with random rematching of players after each round in sections 4.2 and 4.3.

Implementation in the lab: All treatments of the experiment were implemented with the software z-Tree (Fischbacher 2007) and carried out at the experimental laboratory of the SFB 504 at the University of Mannheim.

[Table 2 about here.]

In our experiment we want to study the impact of the number of repetitions, the role of static versus dynamic bidding, and the difference between a war of attrition and an all-pay auction. Our baseline treatment is a (dynamic) war of attrition with six repetitions and cost intervals with $2\underline{c} = \overline{c}$. The other four treatments are variants of the baseline treatment. Table 2 lists the different treatments.[11] In the remainder of this section we describe how the treatments were implemented.

In the treatments with dynamic bidding groups of typically 10 to 14 participants read instructions,[12] answer computerised control questions to check whether they understand the experiment, and are matched randomly in pairs to bid for a prize. During the bidding process participants see information similar to the one shown in Figure 3.[13]

[Figure 3 about here.]

The number of seconds bid and the actual cost of that bid are updated every second. As soon as one bidder stops bidding the other is declared winner of the auction.

In the static treatment, the screen look looks like the one in Figure 4.

[Figure 4 about here.]

In contrast to the dynamic procedure (Figure 3) players make decisions before the actual bidding begins. They fix either the maximum cost or the maximum number of seconds they are willing to bid. The interface in the experiment automatically converts cost into seconds and vice versa. Once players have made their choices the actual bidding process completes instantaneously.[14]

In both the dynamic and the static treatment participants get feedback similar to the one shown in Figure 5.

[Figure 5 about here.]

They are asked to copy this information manually to a table on their desk. Some of the feedback information, such as the other bidder's cost, might not always be available to bidders in a natural context. However, during our pilot experiments we saw that presenting the information in a symmetric way helps participants to understand the nature of the game.

Depending on the treatment participants are either rematched every round or every six rounds to a new, randomly selected partner. This procedure is repeated until the 24th round. At the end of the experiment participants complete a questionnaire, and receive their earnings from the experiment in sealed envelopes. Sessions last for about 75 minutes. The average payoff per subject was 13.05 with a standard deviation of 2.53.

4 Results

4.1 Bidding

Equations (4) and (10) describe equilibrium bids β_e^W and β_e^A in the war of attrition and all-pay auction, respectively. The higher the individual cost c_i the sooner a participant will give up. The last column of Table 2 shows the relative frequency of bids b that are larger than equilibrium bids β_e for the different treatments. We can already see that overbidding is more frequent in the all-pay auction and in the static version of the war of attrition while in all dynamic versions of the war of attrition the majority of bids are smaller than equilibrium bids.

Figure 6 shows scatterplots of bids in the war of attrition and in the all-pay auction.

[Figure 6 about here.]

Let us first have a look at the left graph in Figure 6. Each dot corresponds to one bid in the dynamic war of attrition. We superimpose contour lines of a kernel density estimate. The solid line indicates the equilibrium bid (equation (4)). Generally, bidders with a large cost tend to make smaller bids which is in line with the equilibrium prediction. Many bids for the smaller values of c in the war of attrition concentrate around P/c_i (dashed line). These bidders leave the auction when they have exactly spent the value of the prize. Bids also seem to follow different patterns for small and for large values of c. We will come back to this point in section 4.4 when we talk about bifurcated bidding functions. In section 4.4 we will also discuss why the graphs in Figure 6 look so differently.

Meanwhile, we will provide a more formal comparison of the treatments in the next sections.

4.2 Comparison with equilibrium bids

We estimate the following equation:

$$b_{i,t} = \beta_{e,i} \cdot (1 + \beta_{\text{war}} d_{\text{war}} + \beta_{\text{stat}} d_{\text{stat}} + \beta_{\text{rnd}} d_{\text{rnd}} + \beta_{\text{allp}} d_{\text{allp}}) + v_i + u_{i,t} \tag{17}$$

where $\beta_{e,i}$ is the equilibrium bid from equation (4) or (10) and $b_{i,t}$ is the actual bid. Since we do not observe the winner's bid in the dynamic war of attrition we use an interval regression (see Tobin 1958; Amemiya 1973, 1984; Quing and Pierce 1994). d_{war} is a dummy that is one in the dynamic war of attrition with repeated interaction, d_{stat} is one in the static war of attrition, d_{rnd} is one in the war of attrition in which players are randomly rematched after each period, and d_{allp} is one in the all-pay auction. v_i is a random effect for each participant; $u_{i,t}$ is the noise term for each individual choice. If bidders follow the equilibrium bidding function then we should estimate $\beta_{\text{war}} = \beta_{\text{stat}} = \beta_{\text{rnd}} = \beta_{\text{allp}} = 0$.[15] Table 3 presents the estimation results.[16]

[Table 3 about here.]

In the estimation of equation (17) we observe the following:

- In the all-pay auction there is a significant amount of overbidding (about 69%). This is what we should expect from previous experiments.
- In all other situations, the dynamic war of attrition, the static war of attrition, and the war of attrition with random rematching we find underbidding (between 20% and 42% depending on the treatment). In particular, underbidding in the war of attrition is always significant.

We also ran a regression with an additional dummy which was one if the participant won the auction in the previous period and zero otherwise. The coefficient of such a dummy is small (-0.045) and not significant (p-value of 0.113). Estimates of the other coefficients (and their p-values) do not change substantially; we therefore do not provide an extra table for this estimation.

To see whether players systematically change their behavior during the experiment and perhaps converge to the Bayesian Nash equilibrium we divide the periods in the experiment into four parts of equal size, the first, the second, the third, and the last quarter of the experiment. We estimate equation (17) separately for each quarter. Results are shown in Figure 7.

[Figure 7 about here.]

We see that for all auction formats bids decrease over time in a similar way. The distance between the war of attrition and the all-pay auction remains similar during the course of the experiment.

To summarise this subsection: While the experimental literature on all-pay auctions and rent-seeking contests typically finds overbidding our results document underbidding in the war of attrition.[17]

4.3 Comparison of expenditures

Based on the equilibrium expenditures given by equations (6) and (12) and for our distribution of bidding cost we should expect that the expenditures are the same under the war of attrition and the all-pay auction. For other distributions of the bidding cost this need not be the case. One can show that in equilibrium expected expenditures in the war of attrition are never smaller than in the all-pay auction; they can only be larger (see Krishna and Morgan 1997).

In the experiment we have seen that bids are smaller than equilibrium bids in the war of attrition and larger in the all-pay auction. What can we now say about expenditures? Average overbidding in the all-pay auction does not necessarily imply higher expenditures, at least not if the amount of overbidding depends on the bidding cost. Similarly, average underbidding in the war of attrition does not need to imply lower than equilibrium expenditures.

To compare expenditures in the experiment we estimate the following random effects model

$$R_{i,t} = R^e_{i,t} \cdot \left(1 + \beta_{\text{war}} d_{\text{war}} + \beta_{\text{stat}} d_{\text{stat}} + \beta_{\text{rnd}} d_{\text{rnd}} + \beta_{\text{allp}} d_{\text{allp}}\right) + v_i + v_s + u_{i,t} \qquad (18)$$

where $R_{i,t}$ is the actual expenditure obtained in the auction and $R^e_{i,t}$ the ex-post expected expenditure in equilibrium. d_{war}, d_{stat}, d_{rnd}, d_{allp}, v_i, and $u_{i,t}$ have the same interpretation as in equation (17). v_s is a random effect for each session. If all bidders follow the Bayesian Nash equilibrium bidding function we should have $\beta_{\text{war}} = \beta_{\text{allp}} = \beta_{\text{rnd}} = \beta_{\text{stat}} = 0$. If the coefficients are negative the expenditure is smaller than in equilibrium. If the coefficients are positive then expenditure is larger than in equilibrium. Results of the estimation are shown in Table 4.[18]

[Table 4 about here.]

We observe the following:

- For all variants of the war of attrition that we studied (dynamic or static bidding, random or repeated matching) the estimated coefficients are significantly negative, i.e., the expenditure in the war of attrition is smaller than the expenditure in equilibrium.
- Expenditures in the all-pay auction are larger than in the war of attrition[19] but significantly smaller than equilibrium expenditures.

Thus, while expenditures in the war of attrition and the all-pay auction are still socially wasteful in our experiment their level is lower than predicted by equilibrium. This might look surprising since we found bids in the all-pay auction to be larger than equilibrium. The next section might offer an answer to this puzzle.

4.4 Bifurcations in all-pay auctions and in wars of attrition

Let us have another look at Figure 6. The three scatterplots show bids for the dynamic war of attrition, the static war of attrition, and the all-pay auction. The figure suggests that, in particular for the all-pay auction, bids can be divided into two distinct groups: bids for large values of c and bids for small values of c. The latter are high (and do not depend much on the specific value of c), the former are low (and, again, do not depend much on c).

In an experiment with all-pay auctions Müller and Schotter (2007) observe a similar phenomenon and call it bifurcation of bidding functions. If bidding is expensive, bidders underbid. If bidding is cheap, bidders overbid (see Figure 1. We want to find out whether this is a stable pattern that repeats in our experiments with wars of attrition, i.e., whether individuals with low costs fight longer than predicted by equilibrium and those with high cost stop fighting earlier. The left diagram in Figure 6 already suggests that this phenomenon is less pronounced in the war of attrition. However, Figure 6 only shows the aggregate distribution. In this section we will look at individual bidding functions.

We follow the procedure suggested by Müller and Schotter and estimate the following switching regression:

$$b_i(c) = \begin{cases} \beta_{e,i}c + \beta_{0,i} + u & \text{if } c \leq \hat{c}_i \\ \gamma_{e,i}c + \gamma_{0,i} + u & \text{if } c > \hat{c}_i \end{cases}. \tag{19}$$

We use an OLS regression for the static war of attrition and the all-pay auction. For the dynamic war of attrition we have to use an interval regression (Tobin 1958; Amemiya 1973, 1984).[20] For each individual i we estimate coefficients $\hat{\beta}_{e,i}, \hat{\beta}_{0,i}, \hat{\gamma}_{e,i}, \hat{\gamma}_{0,i}$. We choose the position of each individual step $\hat{c}_i$ such that the likelihood of the regression is maximized.[21]

[Figure 8 about here.]

Figure 8 shows several examples of individual bids and estimated individual bidding functions. The nine diagrams on the left show examples for bidders who can best be approximated with a descending step in the bidding function; the nine on the right are examples for bidders who can best be approximated with an ascending step in the bidding function.

Müller and Schotter test their approach by comparing the residual sum of squares of the switching regression model (19) with the residual sum of squares of an alternative model (which contains the equilibrium bidding function as a special case). We did the same with our data[22] and come to the same conclusion: the switching regression model has a better fit.

We should keep in mind that the switching regression model permits all kinds of stepwise functions. Figure 8 suggests that in the war of attrition not all participants use bidding functions that are flat for small bidding costs, then decrease sharply, and then remain flat for high bidding costs. We have to ask ourselves which examples are more representative of bidding in the war of attrition: the ones in the left part of Figure 8 or the ones in the right part? To answer this question we will look at the step size, i.e., the difference

$$\Delta_i = \hat{\beta}_{e,i}\hat{c}_i + \hat{\beta}_{0,i} - (\hat{\gamma}_{e,i}\hat{c}_i + \hat{\gamma}_{0,i}). \tag{20}$$

[Figure 9 about here.]

This difference is positive if participants have a decreasing step in their bidding function like in the left part of Figure 8. Figure 9 shows the distribution of the step size. Let us first look at the dotted line which shows the distribution of the step size Δ_i for the all-pay auction. As we should expect from Müller and Schotter, more than 50% (actually 59.1%) have an estimated positive stepsize.

[Table 5 about here.]

For the different treatments we test against $\Delta_i = 0$ and report results in Table 5. For the all-pay auction and the static war of attrition we do not find Δ_i to be significantly different from zero. Δ_i is actually significantly negative in the dynamic war of attrition where only 36.6% of participants seem to use a positive stepsize.

How can it be that in our experiment most participants seem to use a negative stepsize in the dynamic war of attrition treatment? To understand this better let us compare the equilibrium bidding functions in the war of attrition and the all-pay auction.

[Figure 10 about here.]

Figure 10 shows examples for the equilibrium bidding functions in the war of attrition (equation (4)) and the all-pay auction (equation (10)) as solid lines. Since $\lim_{c\to\underline{c}}\beta_e^W(c)=\infty$ the function is not easy to approximate with a stepwise linear function. The dotted line shows the result of an OLS regression of a random sample of cost values. We see that, since the left part of the function is very steep, we obtain a negative stepsize. The equilibrium bidding function for the all-pay auction is much easier to fit with a stepwise linear function (the dotted line is very close to the equilibrium bidding function). Any step of a stepwise linear approximation will be small.

5 Concluding remarks

In this series of experiments we address a couple of hypotheses, some related to theoretical studies of the war of attrition, some to other experiments.

We start from the observation that in equilibrium expected expenditures in the war of attrition are never smaller than in the all-pay auction; they can only be larger. In our experiment we study a situation where equilibrium expenditures are the same in both cases. In line with previous experiments we find overbidding in all-pay auctions. However, we see that overbidding does not carry over to wars of attrition. As a result expenditures are actually smaller in wars of attrition than in all-pay auctions.

We find that two effects work hand-in-hand: First, moving from a first-price format (as in the all-pay auction) to a second-price format (as in the static version of the war of attrition) reduces expenditures. In equilibrium expenditures should not change. Second, moving from a static format (as in the static version of the war of attrition) to a dynamic format (as in the proper war of attrition) lowers expenditures even more. Again, in equilibrium expenditures should not change.

These results are interesting for the designer of a contest. In a situation where we want to maximize expenditures (as, e.g., in an athletic competition where the bids of the contestants (their training) provide a positive externality) the all-pay format is behaviorally more attractive and the designer might want to make the competition resemble an all-pay auction. This can be done be keeping the number of remaining contestants and their expenditures private and by excluding dynamic adjustments of bids. E.g., athletic competitions are often held within a time interval which is so short that contestants cannot dynamically adjust their bids (i.e. their training) during the competition.

In a situation where high expenditures are socially wasteful (as in strikes, fiscal stabilizations, arms races, patent races, etc.) a dynamic war of attrition can be preferable for behavioral reasons. The designer of the contest should ban commitments and ease the flow of information between fighting parties to make the contest resemble a war of attrition. E.g. requiring unions (by law or by convention) to publicly announce increases in their strike funds might lead to shorter strikes.

In line with previous findings of Müller and Schotter (2007) we find support for stepwise linear bidding functions in the all-pay auction. For bids in all-pay auctions it matters mainly whether costs are high or low. With low costs bids are high (and are not much affected by the exact level of the costs), with high costs bids are low (and are, again, not much affected by the exact level of the costs). Also in the war of attrition bidders have a tendency to follow two different regimes, one for low and one for high costs. However, the very simple approximation, which worked quite well for the all-pay auction, is not a good approximation for equilibrium bids in the war of attrition and, hence, is not used by many bidders.

A designer of a contest who wants to maximize expenditures and who takes into account stepwise linear bidding functions might want to give bidders with high costs extra encouragement, knowing that otherwise these bidders make inefficiently low bids. If expenditures are socially wasteful then it might be sensible to educate bidders with low bidding cost.

While our study answers some questions it opens a couple of new ones. A game theorist might be glad to hear that overbidding in competitive situations seems to be restricted to a specific class of competitive and rather static situations. We still have to find out why behavior is closer to equilibrium predictions when competition becomes more dynamic.

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Notes

[1]With more than two players the two formats are no longer isomorphic, though, weakly equivalent. In the experiment we present below we will concentrate on the situation with two players.

[2]In Krishna and Morgan (1997) only those all-pay auctions in which the highest bidder obtains the prize with certainty are called *all-pay auctions*.

[3]Following Tullock all-pay auctions can be seen as a special case of rent-seeking contests.

[4]Shogren and Baik (1991), Davis and Reilly (1998), Potters, de Vries, and van Winden (1998), Vogt, Weimann, and Yang (2002), Anderson and Stafford (2003), Schmidt, Shupp, Swope, and Cardigan (2004), Schmidt, Shupp, and Walker (2004), Öncüler and Croson (2005), Abbink, Brandts, Herrmann, and Orzen (2007).

[5]See Bulow and Klemperer (1999) for a derivation of the equilibrium of such a game.

[6] See Cox, Smith, and Walker (1988); Kagel, Harstad, and Levin (1987); Kagel and Levin (1993).

[7]Many auction theorists call the sum of all bids *revenue* and use the letter R to denote revenue. Since in many applications of wars of attrition there is no seller and no revenue we use the term *expenditure* in this paper.

[8]Note that, as in the Dutch auction, bidders in a two-player war of attrition do not learn anything unexpected until the auction ends. If ex ante a bidder finds it optimal to stay in the auction until a bid of b, no relevant new information is gained ex interim.

[9]A list with the parameters for each session can be found in http://www.kirchkamp.de/pdf/supplementWoa.pdf.

[10]Since bids can have a large variance some wars of attrition will end quickly while others last for a long time. If players are rematched in each period, most players will have to wait most of the time. Otherwise these different bids average out, which speeds up the experiment considerably.

[11]A list with the parameters for each session can be found in http://www.kirchkamp.de/pdf/supplementWoa.pdf.

[12]The instructions to the experiment can be found in http://www.kirchkamp.de/pdf/supplementWoa.pdf.

[13]For technical reasons the top right corner of the screen in the experimental interface showed a "remaining time" slowly counting down. We set this time to an exorbitantly high value, so it never became binding in any of our experiments. In the unlikely event of remaining time actually counting down to zero the text would change to "please make your decision now" but bidders would still be able to increase their bids. This, however, never happened in any of our experiments.

[14]As long as we look at bidders who make a positive bid participants spend on average about the same amount of time with the dynamic and the static decision screen (33.41 seconds in the dynamic treatment and 33.08 seconds in the static treatment).

[15] The ratio between risk averse equilibrium bids and risk neutral bids is $(1-e^{-200r})/200r$ for the war of attrition and $(200r+\ln 2-e^{200r}\ln 2)/(200\cdot(2-e^{200r})\cdot r\cdot(1-\ln 2))$ for the all-pay auction. Hence, when we compare the all-pay auction with the war of attrition we can't simply compare levels of these ratios (or levels of estimated coefficients). We can still compare bids in the experiment with risk neutral bids. If we observe a significant amount of underbidding (compared with risk neutral bids) in the war of attrition and

a significant amount of overbidding in the all-pay auction no degree of risk aversion can be consistent with both observations.

[16]A model with random effects for each session of the experiment yields very similar coefficients and *p*-values. A model with fixed effects for participants cannot be used here since participants are fully nested in treatments.

[17]Among experimental papers on all-pay auctions and rent-seeking contests only Schmidt, Shupp, and Walker (2004) find significant underbidding in three different types of rent-seeking contests (a random single prize, a random multiple prize and a deterministic proportionate prize contest). The results of Schmidt, Shupp, and Walker (2004) are hard to compare to ours since their setup differs from ours along many dimensions, e.g. the type of game, its static character, the number of contestants. Additionally, in Schmidt, Shupp, and Walker (2004) each participant played the three different contests but each contest was played one-shot, each participant had the same bidding costs and there was no asymmetric information concerning these costs.

[18]Outliers have been eliminated using Hadi's method (Hadi 1992, 1994).

[19]A two-sided (parametric bootstrap) test of $\beta_{\text{war}} = \beta_{\text{allp}}$ yields a $p = 0.00167$

[20]Instead of using OLS we can use an interval regression for the static war of attrition and the all-pay auction, too. We did this and we did not find any significant difference.

[21]For the interval regression we do not obtain convergence of the estimator for two of our 282 participants. To check overall convergence we did all our estimates twice, once with at most five iterations, and once with at most 25 iterations. The results are practically the same (we have looked at distributions of estimated coefficients like those presented in Figure 9 and found that they are visually indistinguishable for five and for 25 iterations) so that we have no reason to believe that a larger number of iterations might change the results.

[22]As Müller and Schotter we use sums of squares for the static auctions. Additionally, we use likelihood ratios for all three treatments.

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	institution	bidding procedure	asymmetric information about	number of contestants	number of prizes	repetitions in a group	periods	result
Millner and Pratt (1989)	rent-seeking	static	—	2	1	1	20	overbidding
Shogren and Baik (1991)	rent-seeking, framed in expected payoffs	static	—	2	1	32	32	close to equilibrium
Davis and Reilly (1998)	rent-seeking / first-price	static	—	5	1	1	15	overbidding
Potters, de Vries, and van Winden (1998)	rent-seeking / first-price	static	—	2	1	1	30	overbidding in the rent-seeking case
Vogt, Weimann, and Yang (2002)	rent-seeking	sequential	—	2	1	1	1	efficient equilibrium selected
Barut, Kovenock, and Noussair (2002)	first-price	static	valuation	6	2 or 4	1	20 or 50	overbidding
Anderson and Stafford (2003)	rent-seeking with entry-fee	static	cost, no. of contestants	1…10	1	1	1	overbidding
Schmidt, Shupp, Swope, and Cardigan (2004)	rent-seeking	static	—	2	1	1	5	overbidding
Schmidt, Shupp, and Walker (2004)	rent-seeking	static	—	4	1,3,∞	1	1	underbidding
Bilodeau, Childs, and Mestelman (2004)	volunteer's dilemma	dynamic	—	3	1	1	24–32	overbidding
Öncüler and Croson (2005)	risky rent-seeking	static	—	2,4	1	1	1	overbidding
Gneezy and Smorodinsky (2006)	first-price	static	—	4,8,12	1	10	10	overbidding
Abbink, Brandts, Herrmann, and Orzen (2007)	rent-seeking	static	—	2	1	20	20	overbidding
Müller and Schotter (2007)	first-price	static	cost	4	1,2	1,50	50	overbidding, bifurcation
our experiment	second-price / first-price	dynamic / static	cost	2	1	1,6	24	underbidding in the sequential format, no bifurcation

Table 1 Comparison with other experiments

	$\underline{c}/\overline{c}$	repetitions	static	firstprice	participants	sessions	$\beta_e < b$
1	0.1	6	0	0	34	3	0.2586
2	0.5	1	0	0	36	3	0.2130
3	0.5	6	0	0	118	10	0.2113
4	0.5	6	1	0	50	3	0.4192
5	0.5	6	1	1	44	3	0.4555

The last column shows the relative fraction of bids b that are larger than the equilibrium bid β_e.

Table 2 Treatments

	β	σ	z	$P_{>t}$	95%conf. interval
β_{war}	$-.4245$	.02079	-20.415	0.000	$-.46526, -.38375$
β_{stat}	$-.25721$	.03892	-6.609	0.000	$-.33349, -.18093$
β_{rnd}	$-.19658$	.04671	-4.209	0.000	$-.28813, -.10504$
β_{allp}	.68562	.10535	6.508	0.000	.47913, .89211

The estimation includes only experiments with $\bar{c}/\underline{c} = 2$.

Table 3 Interval regression random effects estimation of equation (17)

	β	σ	t	p value	95% conf	interval
β_{war}	-0.299	0.0247	-12.1	0.0000	-0.348	-0.251
β_{stat}	-0.226	0.0227	-9.97	0.0000	-0.271	-0.182
β_{rnd}	-0.297	0.0259	-11.5	0.0000	-0.348	-0.247
β_{allp}	-0.167	0.0344	-4.86	0.0000	-0.234	-0.0995

Random effects regression with a random effect for subject and one for matching group. The estimation includes only experiments with $\overline{c}/\underline{c} = 2$.

Table 4 Expenditure: Results of estimating equation (18)

	$\overline{\Delta_i}$	t	$P_{>\lvert t\rvert}$	P_{bin}
dynamic war of attrition (interval regression)	-17.09	-4.198	0.001	0.077
static war of attrition (OLS)	-.0043	-0.001	0.999	1.000
static all-pay auction (OLS)	2.474	1.521	0.268	1.000

t-statistics are derived using the Huber-White method to correct for correlated responses from cluster samples. P_{bin} are p values of a binomial test.

Table 5 Two-sided tests against $\Delta_i = 0$

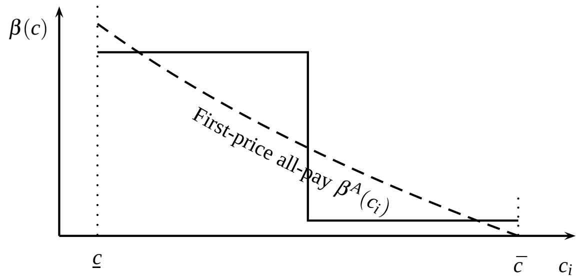
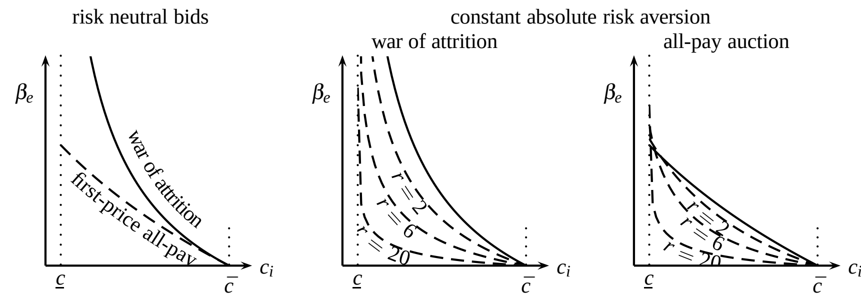


Figure 1 A bifurcated bidding function (solid) and an equilibrium bidding function (dashed)



The left diagram shows risk neutral equilibrium bidding functions for the war of attrition (solid line) and all-pay auction (dashed lines). The diagram in the middle shows equilibrium bidding functions for the war of attrition for bidders with constant absolute risk aversion and risk aversion parameters $r = 2$, $r = 6$, and $r = 20$. The diagram on the right shows similar equilibrium bidding functions for the all-pay auction.

Figure 2 Equilibrium bidding functions

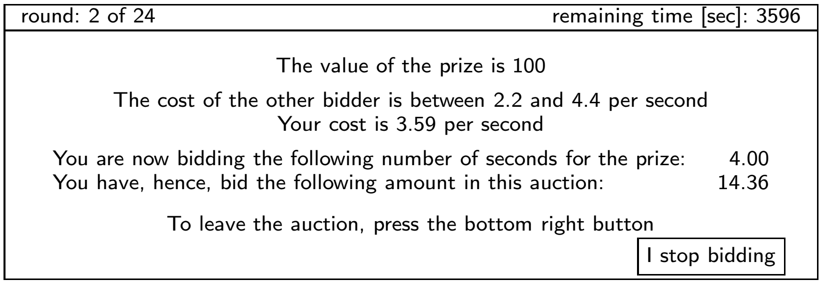


Figure 3 The bidding interface in dynamic treatment

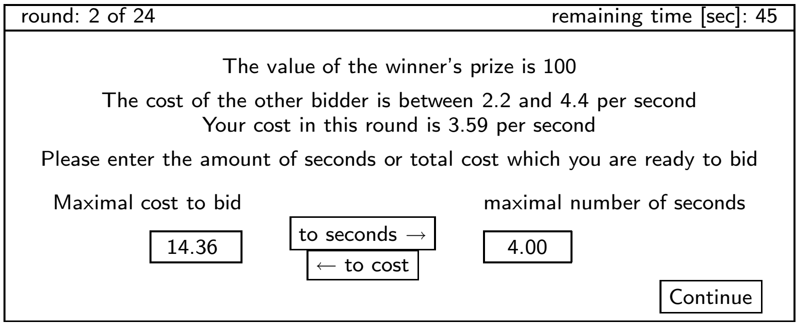
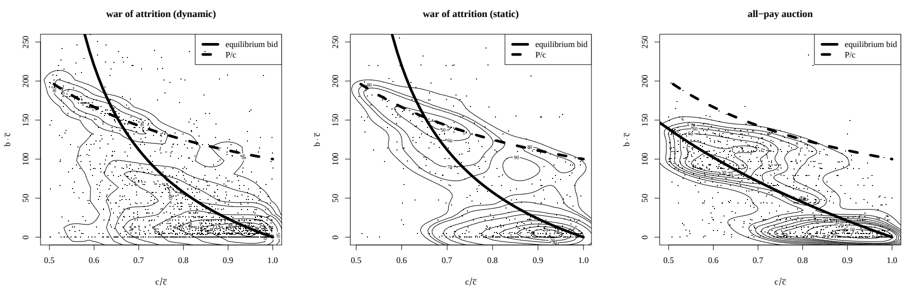


Figure 4 The bidding interface in the static treatment

round: 2 of 24							remaining time [sec]: 17
The other bidder has won the auction							
auction	your cost per second	other bidder's cost per second	length of the auction in seconds	your cost (total)	other bidder's cost (total)	your profit in this auction	new balance of your account
2	3.59	2.91	4	14.36	11.64	-14.36	1876.54

Figure 5 Feedback given at the end of a round



The figure shows bids and contour lines of a kernel density estimate from all treatments where $\bar{c} = 2\underline{c}$. Out of the 1788 individual bids in the left graph 64 are out of the range of the graph. Out of the 600 individual bids in the center graph 9 are out of the range of the graph. The graph for the all-pay auction includes the bids of the winners. We use R's `kde` procedure with default parameters for the kernel density estimate.

Figure 6 Scatterplots of bids

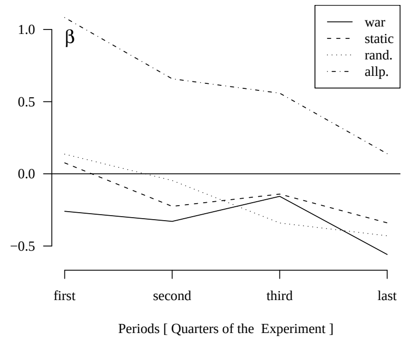
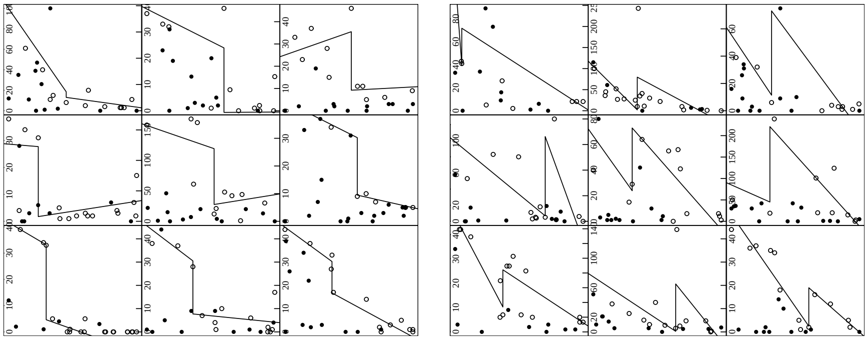


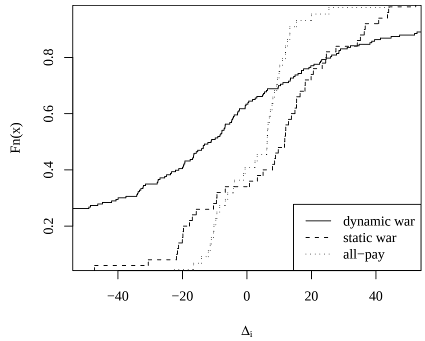
Figure 7 Estimation of equation (17) for different quarters of the experiment



— estimated stepwise bidding function, ○ loss, ● win

The nine diagrams on the left are examples for bidders who can best be approximated with a descending step in the bidding function, the nine on the right are examples for bidders who can best be approximated with an ascending step in the bidding function.

Figure 8 Stepwise approximation of bidding functions for 18 participants in the war of attrition



The graph shows the cumulative distribution of the step size Δ_i. For the static treatments Δ_i is given for an OLS approach, for the dynamic treatment Δ_i is shown for the interval regression.

Figure 9 Bifurcation, the distribution of Δ_i

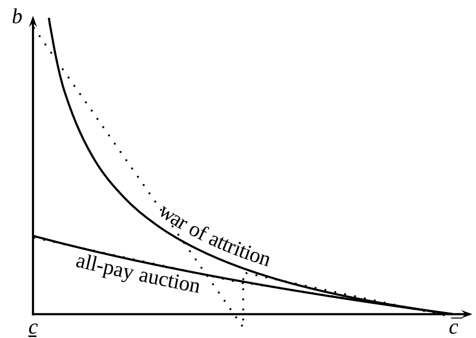


Figure 10 Approximating equilibrium bidding functions with stepwise linear functions