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Information in tournaments under limited liability

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ABSTRACT

The problem of designing tournament contracts under limited liability and alternative performance measures is considered. Under risk neutrality, only the best-performing agent receives an extra premium if the liability constraint becomes binding. Under risk aversion, more than one prize is awarded. In both situations, performance measures can be ranked if their likelihood ratio distribution functions differ by a mean-preserving spread. The latter result is applied to questions of contest design and more general forms of relative performance payment.

1. Introduction

Tournaments and contests are widely used for compensation and incentive purposes.[1] To this end, planners apply a variety of performance indicators. In some cases, the performance measure is identical to the organizer's objective, in others, there seems to be only an indirect relation to what is sought to be procured by the tournament. In all cases, however, the incentive effects of potential performance measures are at least implicitly taken into account.

From an economic perspective, incentive effects of performance measurement have been extensively discussed within a standard agency framework. Starting with the pioneering work of Holmström (1979), numerous papers have analyzed questions of information efficiency under the assumption of optimal contracts in a moral hazard setting. In doing so, they established several criteria for ranking alternative information systems (see, for example, Gjesdal, 1982; Holmström, 1982; Grossman and Hart, 1983; Amershi and Hughes, 1989; Kim, 1995, or Demougin and Fluet, 2001).

Very little work, however, has so far been carried out on information efficiency in a tournament setting in which contracts are exogenously restricted to the order of the agents' performance. Under such a restriction, the criteria derived in the standard setting are not naturally valid. As Holmström (1982, p. 86) states, "if, for administrative reasons, one has restricted attention *a priori* to a limited class of contracts (e.g., linear price functions or instruction-like step functions), then informativeness may not be sufficient for improvements within this class". In general, the same objection applies to the other criteria. Due to

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[1] For example, contests are a predominant instrument in salesforce compensation. Empirical studies found that 75–90% of surveyed firms used sales contests for remuneration of their salespeople (Kalra and Shi, 2001).

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the widespread use of tournaments, it is therefore worthwhile to reason whether the criteria derived in the standard agency setting also apply under the specific restrictions given by a tournament contract.

The classical agency literature discusses the use of information in tournaments mainly with regard to optimal contracts. Adapting his results on sufficient statistics from the standard agency setting, Holmström (1982, Proposition 7) proves that relative performance evaluation will be valuable if and only if the agents' outputs are stochastically dependent. Similarly, Green and Stokey (1983, Proposition 1) show in a more specific setting that individual contracts dominate tournaments whenever the agents' outputs admit only idiosyncratic risk. Conversely, if there is common uncertainty, tournaments will dominate individual contracts when the common shock becomes diffuse (Green and Stokey, 1983, Proposition 2y). Mookherjee (1984, Proposition 4) applies Holmström's (1979) informativeness result to show that a tournament contract will be optimal if an agent's rank in output is statistically sufficient for all available information with respect to his action choice.

In all of these results, informativeness criteria are applied to distinguish between different types of contracts. I return to this important question when applying my general results. First, however, I look for criteria to rank information systems in a setting in which contracts are exogenously restricted to rank orders. To distinguish the analysis from the previous research on *optimal* contracts, I deliberately confine myself to analyzing situations in which the agents' performances are stochastically independent. According to Holmström's (1982) result mentioned above, a tournament contract will not be optimal in this setting, and application of the general informativeness results will thus not be valid. I show that, nonetheless, the main criteria also apply to the tournament setting. Thereafter, I use these criteria to distinguish between different types of relative performance evaluation.

The remainder of this paper is organized as follows. In Section 2, the analytical framework is presented. Section 3 describes the main characteristics of an information system with respect to the principal's optimization problem. Sections 4 and 5 present results on the structure of optimal tournaments and information efficiency as well as an application to the comparison of alternative contract types. Concluding remarks are given in Section 6.

2. Model

Consider a single-period agency setting in which a risk-neutral principal hires a number of agents $i = 1, \ldots, n \quad (n \geq 2)$ to perform identical tasks. The agents who decide to participate provide a productive input $a_i \in A = [\underline{a}, \bar{a}] \subset \mathbb{R}$ not observed by the principal. The resulting outputs $x_i \in X \subseteq \mathbb{R}$ that accrue to the principal are independent, identically distributed (i.i.d.) random variables. Their probability distribution function $F(x_i; a_i)$ is parameterized by the agent's action choice. Increases in a_i are assumed to shift the distribution function to the right in the sense of first-order stochastic dominance. Thus, ceteris paribus, the principal will prefer higher effort levels.

All agents have identical preferences. These can be described by utility functions that are additively separable in monetary income w_i and effort a_i, such as:

$$U_i(w_i, a_i) = u(w_i) - d(a_i),$$

where $u(w^i)$ denotes the agent's utility of monetary income and $d(a_i)$ denotes the agent's disutility of action a_i. I assume that the agents are effort-averse and weakly risk-averse,[2] i.e. $u' > 0$, $u'' \leq 0$, $d' > 0$ and $d'' > 0$.

Before hiring the agents, the principal chooses an information system k from a set $\mathcal{K}$ of feasible information systems. Information system k consists of signals $y_1^k, \ldots, y_n^k \in Y^k = [\underline{y}^k, \bar{y}^k] \subseteq \mathbb{R}$ that become observable to the principal and the agents without cost after the action choices have been taken. The signals y_i^k are i.i.d. random variables with distribution function $G^k(y_i^k; a_i)$ and probability density function $g^k(y_i^k; a_i)$, which depend on the effort of the respective agent. G^k is assumed to be twice differentiable in a_i, and its support is independent of a_i. Signal y_i^k can be regarded as a performance measure for agent i. In the simplest case, performance is measured by output ($y_i = x_i$). More generally, y_i^k may be an index of all available information on the agent's action. I assume that higher effort can be inferred from a higher performance score, i.e. the monotone likelihood ratio property (MLRP) is assumed to hold for any information service k.

Given the information system, the principal designs an ordinal payment scheme that determines the compensation for agent i according to his rank r_i in the order of the observed signals. Let w^j denote the compensation stipulated for the jth-lowest rank j within the order of performance measures. In the case of multiple ranking agents, the respective prize is awarded via randomization.[3] Under these assumptions, the expected utility for agent i from action choices $\mathbf{a} = (a_1, \ldots, a_n)$ can be written as:

$$\mathrm{E}[\mathrm{U}_i(w_i, a_i)|\mathbf{a}] = \sum_{j=1}^{n} u(w^j) p_{ij}^k(\mathbf{a}) - d(a_i),$$

[2] Results are presented separately for risk-neutral and strictly risk-averse agents.

[3] This will occur with zero probability because the density functions g^k have no mass points.

where $p_{ij}^k(\mathbf{a}) = \text{Prob}\{r_i = j|(a_1, \ldots, a_n)\}$ denotes the probability that agent i will achieve rank j in the tournament based on y_i^k.[4] The principal wants all agents to participate. Hence, their expected utilities have to reach a certain reservation level U^{R}, which is assumed to be identical for all agents. Furthermore, I assume that agents are of restricted wealth, and thus compensation has to exceed a liability level $w^{\min}$ for each agent.

3. The principal's problem and properties of information systems

The principal seeks to maximize his expected profit net of wage payments. His problem is to select compensations $\mathbf{w} = (w^1, \ldots, w^n)$ from a set $\mathcal{W}^n \subset \mathbb{R}^n$ of feasible compensations such that the agents choose actions $\hat{a}_i$ that maximize his expected net profit.

Due to the principal's risk neutrality, this problem can be split up, first considering the least-cost way of achieving a given action profile and then turning to the question of which actions to implement. For my purpose of comparing performance measures, the first part is the most interesting. Therefore, similar to Kim's (1995) analysis of the standard agency model, I focus on the question of which information system k implements a particular effort profile $\mathbf{a}$ at the lowest cost. In doing so, I restrict the analysis to symmetric Nash equilibria of the tournament game. Consequently, all agents choose the same action $\hat{a}$, and each agent's probability of winning is $1/n$. The principal's cost minimization problem for the symmetric equilibrium $\hat{\mathbf{a}} = (\hat{a}, \ldots, \hat{a})$, given information systems k, is given by:

$$\min_{w^1,\ldots,w^n} \sum_{j=1}^{n} w^j \tag{1}$$

$$\text{such that} \quad \frac{1}{n}\sum_{j=1}^{n} u(w^j) - d(\hat{a}) \geq U^{\text{R}} \tag{2}$$

$$\hat{a} \in \mathit{arg}\max_{a_i} \left\{ \sum_{j=1}^{n} u(w^j) p_{ij}^k(a_i, \hat{\mathbf{a}}_{-i}) - d(a_i) \right\} \tag{3}$$

$$w^j \geq w^{\min} \quad \forall j. \tag{4}$$

The participation constraint (2) guarantees that all agents accept the contract. The Nash-incentive constraint (3) ensures that, given his opponents equilibrium strategies $\hat{\mathbf{a}}_{-i}$, the desired action a_i is in the best interest of agent i. Finally, (4) accounts for the agents' limited liability.

In this optimization problem, information system k is characterized by a vector $\mathbf{p}_i^k = (p_{i1}^k, \ldots, p_{in}^k)$ of ranking probabilities. To compare information systems with regard to their cost of inducing a certain action a, I am interested in properties of these probabilities, which can be written in more detail as[5]:

$$p_{ij}^k(a_i, \hat{\mathbf{a}}_{-i}) = \frac{1}{n} \int_{Y^k} \frac{g^k(y; a_i)}{g^k(y; \hat{a})} g_{j:n}^k(y; \hat{a})\, \mathrm{d}y, \tag{5}$$

where $g_{j:n}^k$ denotes the density of the $(j{:}n)$-order statistic under distribution G^k. For $a_i = \hat{a}$, the integral in (5) is 1, and $p_{ij}^k = 1/n$. Differing from $\hat{a}$, the agent varies his ranking probabilities. The way in which these changes work at $\hat{a}$ mainly determines the incentive effects of information system k. Similar to the standard agency setting, further insight into the quality of a performance measure can be gained under the first-order approach. As in the standard model, it is valid under the additional assumption that $G^k(y^k; a_i)$ is convex in a (convexity of the distribution function condition, CDFC).[6] The incentive constraint (3) can then be replaced by a first-order condition

$$\sum_{j=1}^{n} u(w^j) \frac{\partial}{\partial a_i} p_{ij}^k(\hat{a}_i, \hat{\mathbf{a}}_{-i}) - d'(\hat{a}) = 0 \tag{6}$$

governed by the marginal probabilities

$$\frac{\partial}{\partial a_i} p_{ij}^k(\hat{a}_i, \hat{\mathbf{a}}_{-i}) = \frac{1}{n} \int_{Y^k} \frac{g_a^k(y; \hat{a})}{g^k(y; \hat{a})} g_{j:n}^k(y; \hat{a})\, \mathrm{d}y, \tag{7}$$

where g_a^k denotes the partial derivative of g^k with respect to a. The integral in (7) is the expected value of the score function g_a/g for the $(j{:}n)$-order statistic of performance scores. By MLRP, this function, which for simplicity is often also referred

[4] According to this definition, rank n denotes the highest outcome.
[5] See Green and Stokey, 1983, p. 355.
[6] A proof is in Appendix B.

to as the likelihood ratio, is increasing in y^k (see Milgrom, 1981). Therefore, the agents' action choices â in the symmetric equilibrium are determined by:

$$\frac{1}{n}\sum_{j=1}^{n} u(w^j)E[\mathrm{lr}^{k,\hat{a}}_{j:n}] = d'(\hat{a}), \tag{8}$$

where $\mathrm{lr}^{k,\hat{a}}_{j:n}$ denotes the $(j{:}n)$-order statistic of likelihood ratios derived from g^k at point â. Note that $E[\mathrm{lr}^{k,\hat{a}}_{j-1:n}] < E[\mathrm{lr}^{k,\hat{a}}_{j:n}]$ by MLRP and $\sum_{j=1}^{n} E[\mathrm{lr}^{k,\hat{a}}_{j:n}] = nE[\mathrm{lr}^{k,\hat{a}}] = 0$ by the assumption of non-moving supports.[7] Therefore, the incentive effects of some prizes for lower ranks will be negative, whereas those of the prizes for higher ranks will be positive.

In the following sections, I exploit further properties of moments and distributions of order statistics to compare different information services. Section 4 derives results for the risk-neutral agency, and Section 5 presents the findings for risk-averse agents. Both sections contain applications to questions of contest design and more general forms of relative performance payment.

4. Risk-neutral agents

4.1. Optimal reward structure

If the agents are risk-neutral, the principal's problem under the first-order approach simplifies to:

$$\min_{w^1,\ldots,w^n} \sum_{j=1}^{n} w^j \tag{9}$$

$$\text{such that} \quad \frac{1}{n}\sum_{j=1}^{n} w^j - d(a) \geq U^{\mathrm{R}} \tag{10}$$

$$\frac{1}{n}\sum_{j=1}^{n} w^j E[\mathrm{lr}^{k,\hat{a}}_{j:n}] = d'(a) \tag{11}$$

$$w^j \geq w^{\min} \quad \forall j. \tag{12}$$

Similar to the analysis of Lazear and Rosen (1981) of a tournament with two agents, the first-best solution can be achieved under *any* informative performance measure, as long as the liability constraints (12) are not binding. Starting from equal prizes for all ranks, the principal just has to increase the prize differentials $w^j - w^{j-1}$ for arbitrary ranks j with positive value of $E[\mathrm{lr}^{k,\hat{a}}_{j:n}]$ until the Nash-incentive constraints (11) are fulfilled. By adjustment of w^1, the participation constraint can be fulfilled with equality. Implementation is without additional cost because of the agents' risk neutrality. The resulting total compensation cost is $n(d(\hat{a}) + U^{\mathrm{R}})$.

Under limited liability, however, this procedure is generally not feasible. For low levels of U^{R} and $E[\mathrm{lr}^{k,\hat{a}}_{j:n}]$, the liability constraints will become binding in the optimal solution. As a consequence, it will matter to which ranks the prize differentials are allocated. Due to the agents' risk neutrality and the MLRP, however, the optimal prize structure is apparently simple:

Proposition 1. *If the agents' liability constraint is binding under information system k, the cost-minimizing tournament awards a prize only to the best-performing agent.*

Proof. Suppose the claim does not hold. Let $\mathbf{w} = (w^1, \ldots, w^n)$ denote the respective compensation schedule, with $w^j \geq w^{\min}$ for all j and $w^j > w^{\min}$ for at least one $j \in \{1, \ldots, n-1\}$. I show that this contract can be improved by one of the type described in the proposition.

To that purpose, consider the wage schedule $\mathbf{v}' = (v^1, \ldots, v^n)$, with

$$v^j = w^{\min} \quad \text{for}\, j = 1, \ldots, n-1$$

and

$$v^n = w^n + \sum_{j=1}^{n-1} (w^j - w^{\min}) \frac{E[\mathrm{lr}^{k,\hat{a}}_{j:n}]}{E[\mathrm{lr}^{k,\hat{a}}_{n:n}]}.$$

[7] For the relation of order statistics, see Arnold et al. (1992, p. 110).

According to (11), the incentive effects of **v** are identical to those of **w**:

$$\sum_{j=1}^{n} v^j E[\mathrm{lr}_{j:n}^{k,\hat{a}}] = \sum_{j=1}^{n-1} w^{\min} E[\mathrm{lr}_{j:n}^{k,\hat{a}}] + E[\mathrm{lr}_{n:n}^{k,\hat{a}}] \left[w^n + \sum_{j=1}^{n-1} (w^j - w^{\min}) \frac{E[\mathrm{lr}_{j:n}^{k,\hat{a}}]}{E[\mathrm{lr}_{n:n}^{k,\hat{a}}]} \right] = \sum_{j=1}^{n} w^j E[\mathrm{lr}_{j:n}^{k,\hat{a}}].$$

The total wage payment, however, is lower under **v**:

$$\sum_{j=1}^{n} v^j = (n-1)w^{\min} + w^n + \sum_{j=1}^{n-1} (w^j - w^{\min}) \frac{E[\mathrm{lr}_{j:n}^{k,\hat{a}}]}{E[\mathrm{lr}_{n:n}^{k,\hat{a}}]} < (n-1)w^{\min} + w^n + \sum_{j=1}^{n-1} (w^j - w^{\min}) = \sum_{j=1}^{n} w^j.$$

The inequality follows from the fact that $E[\mathrm{lr}_{j:n}^{k,\hat{a}}] < E[\mathrm{lr}_{n:n}^{k,\hat{a}}]$ for all $j < n$ by MLRP. □

Essentially, the proof of Proposition 1 shows that the compensation cost can be lowered by shifting compensation from the lower ranks to the highest rank of the tournament. The economics of the result are similar to those in the standard agency setting as derived by Demougin and Fluet (1998). If the agents are risk-neutral, income smoothing only matters with regard to the minimum wage. Incentives, however, are provided at the least cost by rewarding only the result for which the probability is most sensitive to changes in an agent's effort.[8] In the contest setting, due to the MLRP, this is the top rank $r_{n:n}$.

The proposition also complies with results of Moldovanu and Sela (2001), who find that in a symmetric equilibrium of privately informed contestants, a total premium is most effectively allocated to only the winner of the contest.[9] Similar to the moral hazard setting analyzed here, the result is driven by the fact that a single prize provides the strongest incentive for risk-neutral contestants. However, since under private pre-decision information, different types of agents choose different effort levels in equilibrium, the result of Moldovanu and Sela requires linear or concave cost functions for which variations in effort do not have cost-increasing effects. In the present setting, convexity of the cost function is not an issue because all agents choose identical actions.

4.2. Information efficiency

Given the structure of the optimal contract, a comparison of alternative information systems is straightforward. Whenever one of the liability constraints is binding, the optimal reward scheme takes the form $\mathbf{w} = (w^{\min}, \ldots, w^{\min}, w^n)$. In this scheme, w^n has to be chosen to fulfil the agents' Nash-incentive compatibility constraint (11), which takes the form:

$$\frac{1}{n}(w^n - w^{\min}) E[\mathrm{lr}_{n:n}^{k,\hat{a}}] = d'(\hat{a}). \tag{13}$$

According to (13), the necessary wage spread to induce action â is given by:

$$(w^n - w^{\min}) = \frac{nd'(\hat{a})}{E[\mathrm{lr}_{n:n}^{k,\hat{a}}]}.$$

As a consequence, the total compensation cost under information system k in a symmetric equilibrium of n contestants with action choices â can be written as:

$$C_n^k(\hat{a}) = n \cdot \max\left\{ d(\hat{a}) + U^{\mathrm{R}}, w^{\min} + \frac{d'(\hat{a})}{E[\mathrm{lr}_{n:n}^{k,\hat{a}}]} \right\}. \tag{14}$$

By inspection of (14), it is obvious that the cost impact of an information system is solely determined by $E[\mathrm{lr}_{n:n}^{k,a}]$:

Proposition 2. *In the symmetric equilibrium* â *of the tournament under information system* k, *the total cost is lower for higher* $E[\mathrm{lr}_{n:n}^{k,\hat{a}}]$.

Proof. The proof is obvious from (14). C_n^k is decreasing in $E[\mathrm{lr}_{n:n}^{k,\hat{a}}]$. □

Given the prominent role of the likelihood ratio in (14), Proposition 2, when related to the literature on informativeness criteria, provides a direct reference to Kim's (1995) criterion of a mean-preserving spread of likelihood ratio distribution functions. Kim proves that in a standard agency setting with one risk-averse agent, an action â can be induced under a signal y^l at a lower cost than under another signal y^m if the distribution function of the likelihood ratio g_a^l/g^l under signal y^l differs from that under signal y^m by a mean-preserving spread (MPS).[10] Since, due to the assumption of a non-moving support, the

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[8] Related results for signals with a continuous distribution are derived by Innes (1990), Park (1995) and Kim (1997). These papers, however, mainly focus on implementation of the first-best solution, whereas Demougin and Fluet (1998) analyze the structure of the second-best contract. All cited papers differ from the present analysis by considering optimal contracts, whereas Proposition 1 proves that the property applies to a tournament model.

[9] The same result is derived by Glazer and Hassin (1988) in a related framework, but under more restrictive assumptions.

[10] For the definition of a mean-preserving spread, see Rothschild and Stiglitz (1970).

expected value of the likelihood ratio is zero for all information systems, the MPS relation reduces to second-order stochastic dominance. To exploit this property, Kim essentially shows that the compensation cost is a concave function of likelihood ratios, the expectation of which is lower under second-order stochastic dominance.[11] A related convexity argument can be applied here to establish the MPS criterion as a device to rank information systems in the tournament setting.

Proposition 3. *In the symmetric equilibrium* $\hat{a}$ *of the tournament, total compensation cost under information system* y^l *is lower than that under information system* y^m *if the distribution function of the likelihood ratio* $\text{lr}^{l,\hat{a}} = g_a^l/g^l$ *under signal* y^l *differs from that under signal* y^m *by a mean-preserving spread.*

Proof. [12] Let $L^{k,\hat{a}}$ denote the distribution function of the likelihood ratio $\text{lr}^{k,\hat{a}}$, $k = l, m$. If $L^{l,\hat{a}}$ differs from $L^{m,\hat{a}}$ by a MPS, it is said to be *larger than* $L^{m,\hat{a}}$ *in the convex order*, which means that

$$E_{L^{l,\hat{a}}}[\phi] \geq E_{L^{m,\hat{a}}}[\phi]$$

for any convex function $\phi : \mathbb{R} \to \mathbb{R}$, provided the expectation exists.[13] The same holds for the joint distributions

$$M^{l,\hat{a}}(z_1, \ldots, z_n) = \prod_{i=1}^{n} L^{l,\hat{a}}(z_i) \quad \text{and} \quad M^{m,\hat{a}}(z_1, \ldots, z_n) = \prod_{i=1}^{n} L^{m,\hat{a}}(z_i)$$

of i.i.d. random variables $z_i \in \mathbb{R}$, which are distributed according to L_a^l and L_a^m, respectively. The expectation of any convex function $\psi : \mathbb{R}^n \to \mathbb{R}$ is higher under distribution $M^{l,\hat{a}}$ (this follows from Theorem 5.A.3. in Shaked and Shanthikumar (1994)). The relation also applies to the convex function $\psi(z_1, \ldots, z_n) = \max_{i=1,\ldots,n} z_i$. Thus,

$$E[\text{lr}_{n:n}^{l,\hat{a}}] = E_{M^{l,\hat{a}}}\{\max_{i=1,\ldots,n} z_i\} \geq E_{M^{m,\hat{a}}}\{\max_{i=1,\ldots,n} z_i\} = E[\text{lr}_{n:n}^{m,\hat{a}}], \tag{15}$$

which establishes the proposed relation due to Proposition 2. □

The proof of Proposition 3 makes use of the fact that the well-known consequences of second-order stochastic dominance between univariate distributions extend to the product distribution of i.i.d. random variables. By the convexity of the maximum operator, it is thus obvious that an MPS relation yields a unique order of highest-order statistics. Therefore, as in the standard agency setting, information systems can be compared by the distributions of their likelihood ratios. Similarly, the MPS property only provides a local criterion for a specific action $\hat{a}$. To make general predictions for arbitrary levels of a, the relation must hold for *all* $a \in (\underline{a}, \bar{a}]$. This, again, has been proven by Kim (1995, Proposition 4) to follow from the criterion of Blackwell informativeness (the opposite is not true). From this, the following conclusion is obvious.

Corollary 1. *In any symmetric equilibrium of the tournament, the total compensation cost under information system* y^l *is lower than that under information system* y^m *if* y^l *is Blackwell-sufficient for* y^m *with respect to* a.

Proof. The claim follows from Proposition 3 by the relation of Blackwell sufficiency and the MPS criterion. □

Although a greater number of information systems will be comparable by the MPS criterion, Blackwell sufficiency is useful for at least two reasons. First, it is a global criterion that does not focus on a particular effort level $\hat{a}$. Therefore, information systems can be compared by the criterion without specifying which action is sought to be induced. Second, and even more importantly for the application, the criterion refers to the signal distributions instead of the distributions of likelihood ratios. Usually, this will make its use much easier. In the following subsection, I apply Corollary 1 to rank different types of relative performance payment.

4.3. Application

4.3.1. Alternative forms of relative performance evaluation

Perhaps the most important property of tournament contracts is the fact that the total compensation paid to all n agents is constant. Malcomson (1984) uses this property to propose tournaments as a general device to overcome the unverifiability problem, i.e. tournaments can be used for compensation even if the applied performance measures are not verifiable and the principal could misreport these measures in order to cut wages. Obviously, this is impossible under a tournament contract as long as contracts and payments are observable.

Tournaments, however, are not the only compensation form to fulfil the desired property of a constant total wage payment. In particular, Japanese firms make extensive use of a special kind of relative performance payment in which a constant bonus

[11] The result can also be carried forward to a standard agency model with a risk-neutral agent who is of limited wealth.

[12] A related proof for risk-averse agents can be found in Budde and Gaffke (1999).

[13] See Shaked and Shanthikumar (1994, p. 55), for a definition of convex orders, and Scarsini (1994, p. 357), for the (explicit) relation of convex orders and mean-preserving spreads.

W is distributed to workers of a group according to their relative outputs. Agent i's wage in accordance with outputs $x_1, \ldots, x_n$ is given by

$$w_i = w_0 + \frac{x_i}{\sum_{j=1}^{n} x_j} W. \tag{16}$$

Due to its similarity to a tournament, this type of compensation has also been referred to as a *J-type tournament* after its Japanese origin as opposed to *U-type tournaments* of the form described in Section 2, which are predominantly applied in the US (Kräkel, 2003).

By inspection of (16), a similarity to the tournament type derived in Proposition 1 becomes obvious: (16) has the same structure as the expected wage payment under a winner-takes-all tournament, where $x_i/\sum_{j=1}^{n} x_j$ is agent i's winning probability. With regard to the general question of compensation cost, the two types of compensation contracts can therefore be compared by application of the criteria derived in the previous section:

Proposition 4. *In the symmetric equilibrium* $\hat{a}$ *of the tournament game, total compensation cost in a U-type tournament is lower than that in a J-type tournament.*

Proof. Consider performance measures $y_i \in [-\infty, 0]$ with cumulative distribution function $G(y_i|x_i) = \exp(x_i y_i)$ and probability density function $g(y_i|x_i) = x_i \exp(x_i y_i)$ parameterized by the output x_i. Suppose that the signals y_i are used in a U-type tournament of the form derived in Proposition 1, and only the best-performing agent receives a prize. Given $\mathbf{x} = (x_1, \ldots, x_n)$, agent i's probability of winning this prize is

$$\text{Prob}\{y_i = \max\{y_j\}_{j=1,\ldots,n}|\mathbf{x}\} = \int_{-\infty}^{0} g(y|x_i) \prod_{\substack{j=1 \\ j \neq i}}^{n} G(y|x_j)\,\mathrm{d}y = \int_{-\infty}^{0} x_i \exp(x_i y) \prod_{\substack{j=1 \\ j \neq i}}^{n} \exp(x_j y)\,\mathrm{d}y$$

$$= \int_{-\infty}^{0} x_i \exp\left(y \sum_{i=1}^{n} x_j\right) \mathrm{d}y = \frac{x_i}{\sum_{j=1}^{n} x_j}.$$

Taking into account the stochastic nature of the outputs x_j, agent j's (ex ante) expected utility is given by

$$\text{EU}_i = \int_X \left[w^{\min} + \frac{x_i}{\sum_{j=1}^{n} x_j} (w^n - w^{\min}) \right] \prod_{j=1}^{n} f(x_j; a_j)\,\mathrm{d}x_1 \ldots \mathrm{d}x_n - d(a_i).$$

This is identical to his utility in a J-type tournament in which the shared bonus W is equal to the winner's bonus $w^n - w^{\min}$ and the base salary w_0 is given by $w^{\min}$. Therefore, a comparison of compensation cost in a U-type to that in a J-type tournament is equivalent to a comparison of the costs in U-type tournaments under performance measures x_i and y_i.

Given the previous results, however, the latter is straightforward. Since y_i depends on a_i only via x_i, its probability density function, given a_i, can be written as

$$g(y_i|a_i) = \int_X g(y_i|x_i) f(x_i|a_i)\,\mathrm{d}x_i.$$

Since the function $g(y_i|x_i)$ meets the requirements of a Markov kernel, x_i is Blackwell sufficient for y_i. From this, the claim immediately follows by Corollary 1. □

The proof of Proposition 4 makes use of the fact that the bonus portion in (16) is identical to a contest success function. This contest success function, in turn, is known in a two-player contest to be identical to the winning probability under exponentially distributed outputs (see Hirshleifer and Riley, 1992, p. 380). The proof generalizes this property to an n-player tournament and shows that risk-neutral agents assess a J-type tournament equal to a U-type tournament with an additional randomization based on outputs. This randomization, however, weakens the incentives of the contest, leading to a higher compensation cost.

4.3.2. Contest design

Moldovanu and Sela (2006) analyze the question of whether a contest should be split into several sub-contests in a situation of private pre-decision information. They prove that for linear or convex cost functions, the grand contest generates a higher expected output than any contest divided into subgroups of equal size (Moldovanu and Sela, 2006, Theorem 1). Adapting this question to the present moral hazard situation, I find the following result.

Proposition 5. *The total compensation cost to induce a certain action* $\hat{a}$ *in a symmetric equilibrium of risk-neutral contestants is lower under a grand contest of n agents than under any split contest of subgroups with* $n_1 \in \{2, \ldots n-2\}$ *and* $n_2 = n - n_1$ *agents.*

Proof. The average compensation cost per agent in each of the subgroups is:

$$c_{n_i}(\hat{a}) = \frac{C_{n_i}(\hat{a})}{n_i} = \max\left\{d(\hat{a}) + U^R, w^{\min} + \frac{d'(\hat{a})}{E[\mathrm{lr}_{n_i:n_i}^{k,\hat{a}}]}\right\}, \quad i = 1, 2. \tag{17}$$

Since $E[\mathrm{lr}_{n_i:n_i}^{k,\hat{a}}] < E[\mathrm{lr}_{n:n}^{k,\hat{a}}]$ for $n_i < n$, the average compensation cost is higher in each subgroup, from which the claim follows by the fact that $n_1 + n_2 = n$. □

The result is derived from the fact that the average cost (17) is decreasing in the number of contestants.[14] Due to the agents' risk neutrality, the fact that each agent's probability of winning is smaller in the grand contest does not result in an additional cost. Due to the MLRP, however, compensation reacts most sensitively to changes in the agents' efforts if they compete in a grand contest.

5. Risk-averse agents

5.1. Optimal reward structure

If the competing agents are risk-averse, the proposed extreme prize schedule in which only the best-performing agent receives an extra payment will no longer be optimal. This can be illustrated by the following counter-example.

Example. Consider a group of $n = 3$ risk-averse agents competing in a contest with prize structure $\mathbf{w} = (w^1, w^2, w^3)$. Prizes are allocated according to signals $y_i \in \mathbb{R}^+$, which follow the same family of probability distributions described by cumulative distribution functions $G(y_i|a_i) = 1 - \exp(y_i/a_i)$. Thus, the agents' performance measures are exponentially distributed with mean a_i. Furthermore, let the agents' preferences be described by identical utility functions $U_i(w_i, a_i) = \sqrt{w_i} - a_i^2$, and let their reservation utilities be $U^R = 0$. Prizes have to be non-negative. Assuming a symmetric equilibrium of the contest game, the principal wants to implement an equilibrium effort $\hat{a} = 1$ for all agents. His cost minimization problem described in (1)–(4) then becomes:

$$\min_{w^1, w^2, w^3} w^1 + w^2 + w^3 \tag{18}$$

$$\text{such that} \quad \frac{1}{3}\sqrt{w^1} + \frac{1}{3}\sqrt{w^2} + \frac{1}{3}\sqrt{w^3} - 1 \geq 0 \tag{19}$$

$$-\frac{2}{3}\sqrt{w^1} - \frac{1}{6}\sqrt{w^2} + \frac{5}{6}\sqrt{w^3} = 2 \tag{20}$$

$$w^j \geq 0 \quad j = 1, 2, 3. \tag{21}$$

The coefficients in (20) are the expected values of the likelihood ratio order statistics under the exponential distribution with mean 1. The cost-minimizing prize structure is given by $w^1 = 0$, $w^2 = (6/7)^2$ and $w^3 = (18/7)^2$. Obviously, it assigns positive prizes to more than just the top ranking position.

Similar to the situation analyzed in Proposition 1, the agents' liability constraint is binding in the example. However, the contract proposed there would impose too much risk on the agents. Therefore, incentives also have to be provided by w^2. This is less effective than solely rewarding the best-performing agent, but under risk aversion it is also less costly.

5.2. Information efficiency

Given the counter-example, the ranking criteria derived in the previous section cannot be directly translated to the model with risk-averse agents because they build on the extreme contract of Proposition 1. Under a more general prize structure, the compensation will depend not only on the value of $E\left[\mathrm{lr}_{n:n}^{k,\hat{a}}\right]$, as in (14), but in general on the expectations of *all* likelihood ratio order statistics. However, since $\sum_{j=1}^{n} E[\mathrm{lr}_{j:n}^{k,\hat{a}}] = nE[\mathrm{lr}^{k,\hat{a}}] = 0$ for all k, the relation of order statistics used in Propositions 2 and 3 cannot hold for *all* ranks. Nevertheless, if the distribution function of the likelihood ratio $lr^{l,\hat{a}}$ under signal y^l differs from that under signal y^m by an MPS, the same should hold for the likelihood ratio distribution functions of the ranks

[14] This is in line with Proposition 2 in Moldovanu and Sela (2006).

achieved in a contest under these measures. Intuitively, this leads to prizes that are less dispersed, which in turn yield a lower compensation cost for risk-averse agents.

To prove this intuition, I first give a condition of less dispersed prizes under which the total compensation cost is reduced (Lemma 1). Subsequently, I prove that this condition is fulfilled under the MPS criterion (Proposition 6).

Lemma 1. *Let* $\mathbf{w} = (w^1, \ldots, w^n)$ *and* $\mathbf{v} = (v^1, \ldots, v^n)$ *be incentive-compatible prize schedules fulfilling restrictions (2), (4)and (8) in the symmetric equilibrium of the tournament. If all utility spreads resulting from these prizes under a concave utility function u are higher under* $\mathbf{w}$, *then the total compensation cost is less under* $\mathbf{v}$, *i.e.*:

$$u(w^j) - u(w^{j-1}) \geq u(v^j) - u(v^{j-1}) \quad \text{for} j = 2, \ldots, n \tag{22}$$

$$\Rightarrow \sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j. \tag{23}$$

Proof. The proof is in Appendix A. □

The lemma intuitively follows from the agents' risk aversion and limited liability. Under an optimal prize structure, either the agents' participation constraint or their liability constraint will be binding. If the participation constraint is binding under both schedules, the higher utility spreads under schedule **w** produce an MPS relation of the distribution functions of utilities. The claim then follows from the agents' risk aversion. If, on the other hand, the liability constraint is binding, the higher utility spreads under **w** result in prizes that are higher for each rank. In this case, the claim is even more obvious.

The lemma can be used to compare different information structures. For this purpose, it is convenient to write the agents' expected utility in a way that refers to utility spreads:

$$E[U_i(w_i, a_i)|\mathbf{a}] = u(w^1) + \sum_{j=2}^{n} [u(w^j) - u(w^{j-1})] P_{ij}^k(\mathbf{a}) - d(a_i). \tag{24}$$

The term $P_{ij}^k(\mathbf{a}) = \text{Prob}\{r_i \geq j\}$ denotes the probability that agent i achieves *at least* rank j in the tournament under information system y^k. Given his opponents' effort $\hat{a}$ in the symmetric equilibrium, this probability is given by:

$$P_{ij}^k(a_i, \hat{\mathbf{a}}_{-i}) = \int_{Y^k} (1 - G^k(y^k; a_i)) g_{j-1:n-1}^k(y^k; \hat{a}) \, \mathrm{d}y^k. \tag{25}$$

After substitution of (25) in (24), the agent's first-order condition becomes:

$$\frac{\partial}{\partial a_i} \text{EU}_i(\hat{a}_i, \hat{\mathbf{a}}_{-i}) = \sum_{j=2}^{n} ([u(w^j) - u(w^{j-1})] \int_{Y^k} -G_a^k(y^k; a_i) g_{j-1:n-1}^k(y^k; \hat{a}) \, \mathrm{d}y^k) - d'(a_i). \tag{26}$$

This expression can be used to prove that the MPS criterion also applies to the setting with risk-averse agents. For this purpose, I make use of a finding by Demougin and Fluet (2001) who prove that Kim's MPS criterion is equivalent to their so-called integral condition, which is defined for the transformed signals $z^l = G^l(y^l; \hat{a})$ and $z^m = G^m(y^m; \hat{a})$. Due to the assumption of non-moving supports, z^k is as informative as y^k, since G^k is strictly monotonic. Thus, an optimal contract can be based on z^k, as well as on y^k. I denote the cumulative distribution functions of these signals by $H^l(z^l, a)$ and $H^m(z^m, a)$, given a. The integral condition is fulfilled if:

$$-H_a^l(z|\hat{a}) \geq -H_a^m(z|\hat{a}) \quad \forall z \in [0, 1]. \tag{27}$$

Condition (27) is identical to the fact that the distribution function of $\text{lr}^{l,\hat{a}}$ differs from that of $\text{lr}^{m,\hat{a}}$ by a mean-preserving spread (see Demougin and Fluet, 2001, Proposition 3). The main advantage of the criterion is that, in contrast to the MPS relation, it allows for a simple and intuitive comparison of information structures in the standard agency setting (see Demougin and Fluet, 2001, Proposition 1). Similarly, the criterion can be applied in the contest setting to prove the following result.

Proposition 6. *In the symmetric equilibrium* $\hat{a}$ *of the tournament of risk-averse agents, the total compensation cost under information system* y^l *is lower than that under information system* y^m *if the distribution function of the likelihood ratio* $\text{lr}^{l,\hat{a}}$ *under signal* y^l *differs from that under signal* y^m *by a mean-preserving spread.*

Proof. Let $\mathbf{w} = (w^1, \ldots, w^n)$ denote the optimal prize structure under information system y^m or z^m, respectively,[15] fulfilling the agents' incentive compatibility constraint

$$\frac{\partial}{\partial a_i} \text{EU}_i(\mathbf{a}) = \sum_{j=2}^{n} ([u(w^j) - u(w^{j-1})] \int_0^1 -H_a^m(z^m; a_i) h_{j-1:n-1}^m(z^m; \hat{a}) \, \mathrm{d}z^m) - d'(a_i). \tag{28}$$

[15] Optimal prizes are identical under y^m and z^m because of the monotonicity of the distribution function.

By comparing this to the incentive constraint

$$\frac{\partial}{\partial a_i}\mathrm{EU}_i(\mathbf{a}) = \sum_{j=2}^{n}([u(v^j)-u(v^{j-1})]\int_0^1 -H_a^l(z^l;a_i)h_{j-1:n-1}^l(z^l;\hat{a})\,\mathrm{d}z^l) - d'(a_i) \tag{29}$$

under information system z^l derived from y^l and the respective prize schedule $\mathbf{v}=(v^1,\ldots,v^n)$, the integral condition can be applied: since z^l and z^m are values of the cumulative distribution functions, they follow a uniform distribution on [0,1]. Thus, $h_{j-1:n-1}^l = h_{j-1:n-1}^m$ for all j. From this, the integral in (28) is smaller than that in (29) for each j, provided that (27) is fulfilled. Therefore, there exists a prize schedule $\mathbf{v}$ such that (29) is fulfilled and $u(w^j)-u(w^{j-1}) \geq u(v^j)-u(v^{j-1})$ for $j=2,\ldots,n$. The claim then follows from Lemma 1. □

The intuition of the result is readily carried forward from the arguments in Demougin and Fluet (2001). Relating the integral condition to their previous findings on bonus-type contracts in the risk-neutral agency (Demougin and Fluet, 1998), they argue that under risk aversion, a signal is preferred in an optimal contract if it is also preferred under any bonus contract (Demougin and Fluet 2001, p. 490). The latter is obviously fulfilled under the integral condition.

I also make use of this fact and show that if a signal is preferred under any bonus contract, it is also preferred in a tournament. From a single agent's perspective, a tournament in this regard can best be described as a series of bonus contracts with randomized aspiration levels. These levels are given by the performances of the agent's rivals in the tournament. If a signal is more sensitive with respect to the agent's action for any possible value of these levels, it is also more sensitive in expected terms.

5.3. *Application*

Similar to the analysis of a tournament with risk-neutral agents, the information efficiency results can be applied to compare different types of tournaments. In doing so, I again refer to the analysis of Moldovanu and Sela (2006) of contest architecture. My aim is to reinforce their result on the efficiency of the grand contest in the moral hazard setting analyzed here. Different to the proof in Section 4, however, I cannot simply compare total compensation cost as in (14), because now the compensation cost depends on the agent's risk attitude. To derive the desired result, I therefore first prove that the average compensation cost is decreasing in the number of agents (Proposition 7), and then turn to the question of whether to split the contest or not (Corollary 2).

Proposition 7. *The average compensation cost to induce a certain action* $\hat{a}$ *in a symmetric equilibrium of risk-averse agents is decreasing in the number of contestants.*

Proof. In the symmetric equilibrium of n risk-neutral contestants, each player's compensation is based on his rank $r_{in} \in \{1,\ldots,n\}$, and all ranks are equally likely. In what follows, it is shown that the likelihood ratio distribution function of the rank $r_{in}^{k,\hat{a}}$ in a tournament of n contestants, based on y^k to implement $\hat{a}$, is an MPS of that of $r_{i,n-1}^{\hat{a},k}$, the rank in a contest of $n-1$ participants. The claim then follows from Kim's (1995) results in the standard agency setting.

From (7) and the fact that $p_{ij}^{k,n}(\hat{a})=1/n$, the likelihood ratio $(\partial/\partial a_i)p_{ij}^{k,n}(\hat{\mathbf{a}})/p_{ij}^{k,n}(\hat{\mathbf{a}})$ is given by $E[\mathrm{lr}_{j:n}^{k,\hat{a}}]$. This allows to make use of the triangle rule in order statistics (Arnold et al., 1992, Theorem 5.3.1), which states that expectations of order statistics are related as follows:

$$jE[\mathrm{lr}_{j+1:n}^{k,\hat{a}}] + (n-j)E[\mathrm{lr}_{j:n}^{k,\hat{a}}] = nE[\mathrm{lr}_{j:n-1}^{k,\hat{a}}]. \tag{30}$$

The identity can be exploited to construct the likelihood ratio distribution function of $r_{i,n}^{k,\hat{a}}$ from that of $r_{i,n-1}^{k,\hat{a}}$ by a sequence of mean-preserving spreads s_j, $j=1,\ldots,n-1$, where s_j is defined as follows:

$$s_j = \begin{cases} \dfrac{n-j}{n}\dfrac{1}{n-1} & \text{for } E[\mathrm{lr}_{j:n}^{k,\hat{a}}] \\ -\dfrac{1}{n-1} & \text{for } E[\mathrm{lr}_{j:n-1}^{k,\hat{a}}] \\ \dfrac{j}{n}\dfrac{1}{n-1} & \text{for } E[\mathrm{lr}_{j+1:n}^{k,\hat{a}}]. \end{cases}$$

Thus, s_j distributes the probability mass $1/(n-1)$ of $E[\mathrm{lr}_{j:n-1}^{k,\hat{a}}]$ to $E[\mathrm{lr}_{j:n}^{k,\hat{a}}]$ and $E[\mathrm{lr}_{j+1:n}^{k,\hat{a}}]$. It is a spread (which defers probability mass to the tails of a distribution) because $E[\mathrm{lr}_{j:n}^{k,\hat{a}}] \leq E[\mathrm{lr}_{j:n-1}^{k,\hat{a}}] \leq E[\mathrm{lr}_{j+1:n}^{k,\hat{a}}]$, and it is mean-preserving because

$$E[s_j] = \frac{j}{n}\frac{1}{n-1}E[\mathrm{lr}_{j:n}^{k,\hat{a}}] - \frac{1}{n-1}E[\mathrm{lr}_{j:n-1}^{k,\hat{a}}] + \frac{n-j}{n}\frac{1}{n-1}E[\mathrm{lr}_{j+1:n}^{k,\hat{a}}] = \frac{1}{n}\frac{1}{n-1}(jE[\mathrm{lr}_{j:n}^{k,\hat{a}}] - nE[\mathrm{lr}_{j:n-1}^{k,\hat{a}}] + (n-j)E[\mathrm{lr}_{j+1:n}^{k,\hat{a}}]) = 0$$

by the triangle rule (30). The resulting probabilities

$$p\left(\frac{(\partial/\partial a_i)p_{ij}^{k,n}(\hat{\mathbf{a}})}{p_{ij}^{k,n}(\hat{\mathbf{a}})}\right)=\begin{cases}\frac{n-1}{n}\frac{1}{n-1}=\frac{1}{n} & \text{for } j=1\\ \frac{i}{n}\frac{1}{n-1}+\frac{n-(i+1)}{n}\frac{1}{n-1}=\frac{1}{n} & \text{for } j=1,\ldots,n-1\\ \frac{n-1}{n}\frac{1}{n-1}=\frac{1}{n} & \text{for } j=n\end{cases}$$

are those in the contest of n agents. □

The proof of Proposition 7 makes use of the fact that the principal's optimization problem (1)–(4) is similar to that of a standard single-agent model in which the agent's performance is measured by his rank among $n-1$ agents choosing the equilibrium action â. The proof shows that this signal becomes more informative in the sense of the MPS criterion when the number n of competitors increases. At first glance, this seems counterintuitive because each of the contestants adds noise to the performance measure. At the same time, however, the number of ranks increases, thereby enriching the principal's opportunities to calibrate the contract. As Malcomson (1986) shows, for an infinite number of competitors, this results in the equivalence of a rank-order contract and a piece-rate contract. The main contribution of Proposition 7 is therefore to prove the monotonicity of agency costs in the number of agents.

The result can directly be applied to answer the initial question.

Corollary 2. *The total compensation cost to induce a certain action* â *in a symmetric equilibrium of risk-averse contestants is lower under a grand contest of n players than under any split contest of subgroups with* $n_1 \in \{2, \ldots n-2\}$ *and* $n_2 = n - n_1$ *players.*

Proof. The claim is obvious because the average compensation cost is higher in both sub-contests compared to the grand contest, which follows from Proposition 7. □

The reasoning behind Corollary 2 is similar to that of the preceding Proposition 7. Although the grand contest determines an agent's compensation based on the noisiest information, it dominates all other architectures because it allows for the most precise stipulation of prizes. Since, in general, the tradeoff of these two effects is not obvious, the main contribution of the two results is to prove that the latter effect always dominates the former. At the same time, the difference from a model without exogenous restriction to a rank-order tournament is highlighted. Without the restriction, each agent would receive a payment based only on his individual performance, because outputs are assumed to be independent. Since any contract based on y_i^k can be written, any information on another agent's output only adds noise to the compensation. In that sense, the result contrasts with Holmström's (1979) informativeness result.

6. Conclusion

This paper analyzed whether the informativeness criteria derived for information systems in a standard agency setting of moral hazard, in which the principal chooses an optimal contract in the second-best solution, also apply to a tournament setting in which the contract is exogenously restricted to be rank-dependent. As a main result, it was shown that Kim's (1995) MPS criterion was capable of ranking performance measures in the symmetric equilibrium of the tournament game. As a consequence, Blackwell sufficiency also applies. The key feature connecting the standard model and the tournament setting is that the MPS relation of likelihood ratios carries forward from the original signals to the ranks in the contest. Only from this, does the result from second-best contracts also hold in the constrained model.

Various applications of the result are possible. I used it to compare different types of contracts. The key idea is to replace the comparison of contracts by that of different information systems using the same type of contract. While the present paper focussed on the comparison of specific contracts, the procedure could also be applied to more general questions of contract design. In particular, it may be used to identify conditions under which tournaments are optimal agreements with regard to a special class of contracts. One such class could be given by contracts that distribute a constant sum of payments among a group of agents. This class is of particular interest with respect to unverifiable or subjective performance information, as mentioned in Section 4.3. Therefore, the furnished results may represent a device to prove the optimality of tournaments as a solution to the so-called *unverifiability problem*.

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Appendix A. Proofs

Proof of Lemma 1. The proof analyzes the possible cases regarding the agents' liability constraints.

1. The agents' liability constraint is not binding under **v** and **w**.

In this case, the participation constraint is binding and $E[u(w)] = E[u(v)]$. Let F^w and F^v denote the cumulative distribution functions of one agent's utilities resulting from **w** and **v** in the symmetric equilibrium. From the relation of utility spreads (22), it follows that:

(a) $u(w^n) \geq u(v^n)$ (obvious).
(b) $u(w^1) \leq u(v^1)$ (obvious).
(c) The distribution functions F^v and F^w only cross once, i.e. $\exists\hat{u}$ such that

$$F^w(u)\begin{cases} \leq F^v(u) & \forall u < \hat{u} \\ \geq F^v(u) & \forall u > \hat{u}, \end{cases}$$

because the jumps in the cumulative distribution functions are $p_{ij} = 1/n$ for each rank j.

Since $E[u(w)] = E[u(v)]$, it must hold that

$$\int_{-\infty}^{U} F^w(u)\,\mathrm{d}u = \int_{-\infty}^{U} F^v(u)\,\mathrm{d}u$$

for all U such that $F^v(U) = F^v(U) = 1$. From this and 1a–1c above, it follows that

$$\int_{-\infty}^{U} F^w(u)\,\mathrm{d}u \geq \int_{-\infty}^{U} F^v(u)\,\mathrm{d}u \tag{31}$$

for all $U \in \mathbb{R}$. Therefore, F^w and F^v differ by a mean-preserving spread (cf. Rothschild and Stiglitz, 1970, p. 230 ff.), and the expectation of each convex function is lower under F^v. Since the agent's risk aversion implies that the inverse utility function is convex, the expected compensation of a single agent (and thus the total compensation of all agents) is lower under **v**.

2. The agents' liability constraint is binding under both **w** and **v**.

Then, $v^1 = w^1$. From this and (22), it follows that $w^j \geq v^j \quad \forall j$, and therefore $\sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j$.

3. The agents' liability constraint is binding under **w** and not binding under **v**.

In this case, the participation constraints will be binding under **v**, but not necessarily under **w**, and $E[u(w)] \geq E[u(v)]$. From 1 above, it immediately follows that $\sum_{j=1}^{n} w^j \geq \sum_{j=1}^{n} v^j$.

4. The agents' liability constraint is binding under **v** and not binding under **w**.

In this case, the participation constraint is binding under **w** and $w^1 \geq v^1$. From this and (22), it follows that $E[u(v)] \leq E[u(w)] = U^{\mathrm{R}}$, a contradiction. □

Appendix B. Validity of the first-order approach

This section serves to prove that similar to the standard agency model, the first-order approach to the principal's problem in the present tournament model is valid if the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) apply.

The first-order approach here consists of substituting the agent's incentive compatibility constraint (3) in the principal's optimization problem (1)–(4) by the first-order condition (6).

As in the standard agency model, proving the validity of this approach mainly consists of showing that in the optimal solution to the problem, the agents' expected utility is a concave function of his effort, and therefore its maximum can be characterized by a first-order condition.

To that purpose, I follow the arguments of Rogerson (1985) who proves the validity of the first-order approach in the standard agency, and refer to (1)–(4) as the *unrelaxed problem*. I then introduce the *relaxed program* (1), (2), (6) and (4), as well as the *doubly relaxed program* (1), (2), (32) and (4), where the first-order condition (6) of the relaxed program is substituted by the inequality

$$\sum_{j=1}^{n} u(w^j)\frac{\partial}{\partial a_i} p_{ij}^{k}(\hat{\mathbf{a}}) - d'(\hat{a}) \geq 0. \tag{32}$$

The doubly relaxed problem mainly serves as a technical device to proof that higher ranks will be paid higher. This is shown in the following lemma:

Lemma 2. *If $\mathbf{w} = (w^1, \ldots w^n)$ is a solution to the doubly relaxed program and the MLRP holds, then $w^j \geq w^{j-1}$ for all $j = 2, \ldots, n$.*

Proof. The Lagrangian of the doubly relaxed program is

$$\mathcal{L} = \sum_{j=1}^{n} w^j + \lambda \left[\frac{1}{n} \sum_{j=1}^{n} u(w^j) - d(\hat{a}) - U^{\mathrm{R}} \right] + \mu \left[\sum_{j=1}^{n} u(w^j) \frac{\partial}{\partial a_i} p_{ij}^k(\hat{\mathbf{a}}) - d'(\hat{a}) \right] + \sum_{j=1}^{n} \eta_j \left[w^j - w^{\min} \right],$$

where, due to the fact that the agent's incentive compatibility constraint is replaced by an inequality, *all* multipliers are nonnegative. The first-order condition with respect to w^j is

$$\frac{\partial \mathcal{L}}{\partial w^j} = -1 + \frac{\lambda}{n} u'(w^j) + \mu \frac{\partial}{\partial a_i} p_{ij}^k(\hat{\mathbf{a}}) u'(w^j) + \eta_j = 0.$$

Now suppose that the claim does not hold and $w^j < w^{j-1}$ for some j. Consequently, $w^{j-1} > w^{\min}$ and therefore $\eta_{j-1} = 0$. Thus, it must hold that

$$\frac{\lambda}{n} u'(w^{j-1}) + \mu \frac{\partial}{\partial a_i} p_{i,j-1}^k(\hat{\mathbf{a}}) u'(w^{j-1}) \geq \frac{\lambda}{n} u'(w^j) + \mu \frac{\partial}{\partial a_i} p_{ij}^k(\hat{\mathbf{a}}) u'(w^j)$$

because $\eta_j \geq 0$. Since from $w^j < w^{j-1}$ it follows that $u'(w^j) > u'(w^{j-1})$, this means that $(\partial/\partial a_i) p_{i,j-1}^k(\hat{\mathbf{a}}) > (\partial/\partial a_i) p_{ij}^k(\hat{\mathbf{a}})$ must hold. This, however, contradicts the monotone likelihood ratio property because $(\partial/\partial a_i) p_{ij}^k(\hat{\mathbf{a}}) = E[\mathrm{lr}_{j:n}^{k,\hat{\mathrm{a}}}]$ and $E[\mathrm{lr}_{j-1:n}^{k,\hat{\mathrm{a}}}] \leq E[\mathrm{lr}_{j:n}^{k,\hat{\mathrm{a}}}]$. □

Provided that an agent's compensation is increasing in his rank, it is straight-forward to show that his expected utility is a concave function of his effort:

Lemma 3. *If **w** is a solution to the doubly relaxed program and the MLRP and the CDFC hold, then an agent's expected utility under **w** is a concave function of his effort.*

Proof. Consider the agent's expected utility as expressed in (24). From Lemma 2 above, the terms in brackets are all non-negative. Since $d(\cdot)$ is a convex function, it therefore suffices to show that all probabilities $P_{ij}^k(a_i, \mathbf{a}_{-i})$ are concave in a_i.

This becomes clear by inspection of (25). The term in braces is concave by the CDFC. Since $g_{j-1:n-1}^k(y^k; \hat{a})$ is non-negative and independent of a_i, the same holds for the integral. Hence, $P_{ij}^k(a_i, \mathbf{a}_{-i})$ and EU_i are concave in a_i. □

In a last step, it must be shown that analyzing the doubly relaxed program instead of the relaxed program does not change the results.

Lemma 4. *If the MLRP holds, constraint (32) is binding in the solution to the doubly-relaxed program.*

Proof. Suppose the claim does not hold. Then $\mu = 0$ and therefore $-1 + (\lambda/n) u'(w^j) + \eta_j = 0$ for all j. If none of the liability constraints (4) is binding, this implies that $\eta_j = 0$ and therefore $1/u'(w^j) = \lambda$ and $w^j = \text{const.}$ for all j. If some liability constraint is binding, from Lemma 2 it follows that this must be the case for w^1. Thus, $\eta_1 > 0$ and therefore $-1 + (\lambda/n) u'(w^1) < 0$. By Lemma 2, $w^j \geq w^1$ for $j = 2, \ldots, n$. By the concavity of $u(\cdot)$, this implies that $u'(w^j) \leq u'(w^1)$ for $j = 2, \ldots, n$. Therefore, $-1 + (\lambda/n) u'(w^j) < 0$ for all j and $\eta_j > 0$ for all j. Hence, $w^j = w^{\min}$ for all j.

In both cases, the agents receive flat wages and $w^j - w^{j-1} = 0$ for $j = 2, \ldots, n$. Substituting this in the agent's incentive constraint as expressed in (26) yields

$$\frac{\partial}{\partial a_i} \mathrm{EU}_i(\mathbf{a}) = -d'(a_i) < 0,$$

which contradicts the initial assumption that $(\partial/\partial a_i) \mathrm{EU}_i(\mathbf{a}) > 0$. □

The three lemmata in conjunction imply that the first-order approach is valid.

Proposition 8. *Let **w** be a solution to the relaxed program and let the MLRP and the CDFC hold. Then **w** is a solution to the unrelaxed program.*

Proof. By Lemma 4, the solution to the relaxed program is identical to the solution to the doubly relaxed program. Since by Lemma 3 the agent's utility is concave in his action under the optimal reward scheme **w**, a_i is a global maximizer and the solution to the doubly relaxed program is a also a solution to the unrelaxed program. □

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