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Variance analysis and linear contracts in agencies with distorted performance measures

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ABSTRACT

This paper investigates the role of variance analysis procedures in aligning objectives under the condition of distorted performance measurement. A risk-neutral agency with linear contracts is analyzed, whereby the agent receives post-contract, pre-decision information on his productivity. If the performance measure is informative with respect to the agent's marginal product concerning the principal's objective, variance investigation can alleviate effort misallocation. These results carry over to a participative budgeting situation, but in this case the variance investigation procedures are less demanding.

1. Introduction

Variance investigation has frequently been the subject of management accounting research in terms of both facilitating and influencing decision-making. Such research has often concentrated on incentive effects. From an agency perspective, the literature has mainly focused on the trade-off between risk-sharing and incentives. This viewpoint essentially reduces the agency problem to the question of which contractual agreement can induce a certain desired action at minimal cost. Several results on the use of variance analysis procedures have been derived by applying Holmström's (1979) informativeness criterion. The crucial requirement for useful variance investigation in this context is the provision of *additional* information with respect to the agent's action. If observation of an overall result is not statistically sufficient, there are potential gains from analyzing further details regarding the agent's actions (Baiman and Demski, 1980a).[1]

In the last decade, however, economic agency research has emphasized the misallocation of effort rather than the trade-off between risk and incentives as the central issue in the provision of incentives. Starting with Holmström and Milgrom (1991), a rich literature has analyzed the effects of dysfunctional behavior. This problem may arise whenever an agent's performance indicator does not fully accord with his principal's objective, which can occur for a variety of reasons. On the one hand, the principal could have a noncontractible objective such as the value of a privately traded firm. On the other hand, the objective might be a very risky measure of the agent's performance and result in a high risk premium to be paid. In both cases, the principal may seek alternative performance measures to provide contractual incentives. Such measures, however, may induce effort allocations that do not coincide with those preferred by the principal, particularly if the agent performs a variety of tasks.

Multi-dimensional effort may result for two reasons. First, the agent might work on different tasks and might have to decide not only on the total amount of effort, but also on where to put it. Beside this classical multi-task situation, the agent's productivity might depend on some state

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[1] In practical applications, these benefits of course have to be weighed against the cost of data gathering, and conditional monitoring might become advantageous. See Baiman and Demski (1980b) and Young (1986) for a detailed analysis.

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of nature that he observes before choosing his action, leading to a state-contingent action. If the principal's objective and the performance measure are influenced by the state of nature in different ways, a misallocation problem similar to that under multi-tasking arises from the agent's private information.

From an accounting perspective, an obvious question in both cases is whether management accounting procedures such as variance analysis can help to alleviate the problem. To answer this question, I first analyze how additional input information can best be incorporated into a linear contract. Building on these results, their relation to variance analysis procedures is then studied. It emerges that certain special variances can be naturally interpreted as predictors of the agent's impact on the firm's objective. Consequently, they appear in the agent's compensation function. Distortion of performance measures is therefore another rationale for tying compensation to variances in corporate practice. I derive these results for the case of a privately informed agent, for which the adoption of variance analysis procedures and participative budgeting has a natural interpretation. However, since the general effects of distortion are the same under both multi-tasking and private information, the results on variance investigation would apply to the classical multi-task setting as well, with a slightly different interpretation of variances.

In a broader sense, my aim is to connect two branches of literature: one on distortion in performance measurement, and the other on the use of variance analysis procedures for incentive contracting. In the first respect, the paper is most closely related to Baker (1992) and Feltham and Xie (1994). Like Baker, I consider the combined use of output and input data to improve the congruity of performance measures. In addition, I consider the role of participation in budgeting and discuss the relation of performance measures to accounting data. In this last respect, the paper is more closely related to the work of Feltham and Xie, but they do not consider the use of accounting procedures such as variance analysis. In this regard, I follow Darrough (1988) and Kloock and Schiller (1997). Kloock and Schiller describe different decomposition methods proposed for variance analysis, particularly in the German cost accounting literature. I refer to them when I describe the optimal contract in terms of variance decomposition results. Kloock and Schiller present only verbal arguments on the use of variance investigation for incentive purposes. I find evidence supporting their statements in a quantitative interpretation of the model. Darrough (1988) considers the use of ex post budgets in splitting the efficiency variance in cost accounting. Although Darrough does not explicitly employ an agency model, both her work and mine use the agent's reaction to his pre-decision information. While in Darrough's paper this information is publicly observable ex post, in this paper only its impact on the performance measure can be used for contracting. Accordingly, in Darrough's paper an agent would always choose the first-best input mix, whereas in my model implementation depends on the relation of the principal's objective and the agent's performance measure.

The remainder of the paper is organized as follows. In Section 2, a general model of distorted performance measurement under private information is described. Section 3 studies the impact of additional information and the role of variance analysis procedures. Section 4 considers participative budgeting, and Section 5 draws conclusions and discusses directions for future research.

2. Distorted performance measurement under private information

To introduce the problem of distorted performance measurement under private information, I adapt a model studied by Baker (1992). For this purpose, consider a risk-neutral principal hiring a risk-neutral agent to perform a certain task on his behalf. The agent takes an action $a \in \mathbb{R}^+$, which, along with a random variable $\delta \in [\underline{\delta}, \bar{\delta}] \subset \mathbb{R}^+$, determines the realization of the principal's objective $V(a, \delta) = \delta a$. By choosing a, the agent incurs a private cost $C(a) = a^2/2$. Thus, maximizing the total surplus[2] $V - C$ would require $a = \delta$, equating the marginal product δ and the marginal cost a of the agent's effort. The expected total surplus from this action would be

$$E[V - C]^{PI} = \frac{E[\delta^2]}{2}, \tag{1}$$

where the superscript PI denotes that this is the outcome under perfect information.

To study asymmetric information and performance measure distortion, I assume that δ cannot be observed by the contracting parties. Only the agent receives a signal $\phi \in \mathbb{R}^+$, from which he imperfectly infers the realization of δ. Up to this point, the setting is a linear-quadratic specification of the model studied in the standard agency theory (Harris and Raviv, 1979), for which, owing to the agent's risk-neutrality, a first-best solution could be achieved by selling the business to the agent. The agent would use his information to maximize the conditional expectation of $V - C$, choosing an action $a^{FB} = E[\delta|\phi]$. The expected total surplus from this action would be identical to the first-best solution under symmetric information,[3]

$$E[V - C]^{SI} = E\left[\frac{\delta a^{FB} - (a^{FB})^2}{2}\right] = E_\phi\left[\frac{E[\delta|\phi]^2}{2}\right], \tag{2}$$

where the principal observes ϕ and prescribes a^{FB}. I rule out this trivial case by assuming that V is not the value of the firm as a whole, but only the agent's contribution to the firm's value, which cannot be separated from the remaining assets and sold to the manager. Nevertheless, (2) will serve as a benchmark for the following analysis of the second-best case in which the principal does not observe ϕ and the firm cannot be sold to the agent.

Incentives then have to be provided by tying the agent's compensation to a contractible performance measure P. For instance, if V is the value added to a privately traded firm, P might be some measure of short-term success such as profit or return on investment. Since such short-term perfor-

[2] Since both parties are risk-neutral, maximizing total profit is identical to maximizing total welfare.

[3] The superscript SI indicates the outcome under symmetric information, and E_ϕ denotes the expectation with respect to ϕ.

mance may at least partly determine the total value added by the manager, it is evident that we can allow that V and P are correlated. I do so by assuming that $P(a, \phi, \nu) = \phi a + \nu$, where the sensitivity[4] of the performance measure is the agent's private information ϕ, and δ and ϕ may be correlated. The noise term $\nu \in \mathbb{R}$ with $\mathrm{E}[\nu] = 0$ is unobservable to both parties, and ensures that a forcing contract cannot be written. Instead, the principal offers a linear payment scheme

$$S = s_0 + sP \tag{3}$$

to the agent.[5] At the time this contract is signed, neither the principal nor the agent has information about the realization of ϕ, and they have common beliefs about its distribution. Before the agent chooses his action, however, ϕ becomes observable to him. From this post-contractual information asymmetry, a chance to improve the agent's decision as well as the problem of distorted performance measurement arises because the agent may use his information for both productive and non-productive purposes.

To see this, first consider a situation in which the agent does not observe ϕ. Ignorant of ϕ, he chooses his action $a^{NI} = s\mathrm{E}[\phi]$ to maximize his expected utility $s_0 + s\mathrm{E}[P] - C = s_0 + s\mathrm{E}[\phi]a - a^2/2$, and the principal can induce an action maximizing the expected total surplus[6] $\mathrm{E}[V - C]$ by setting $s^{NI} = \mathrm{E}[\delta]/\mathrm{E}[\phi]$. The agent chooses $a^{NI} = \mathrm{E}[\delta]$, and an expected total surplus of $\mathrm{E}[V - C]^{NI} = \mathrm{E}[\delta]^2/2$ accrues to the agency.[7]

Since the agent observes ϕ, however, he will choose the action $a(s, \phi) = s\phi$ that maximizes his expected utility $s_0 + s\mathrm{E}[P - C|\phi] = s_0 + s\phi a - a^2/2$. By a comparison of $a(s, \phi) = s\phi$ and $a^{FB} = \mathrm{E}[\delta|\phi]$, it is obvious that the first-best effort under symmetric information can be implemented if and only if $\mathrm{E}[\delta|\phi] = c\phi$ for some $c \in \mathbb{R}$, i.e., if the expected value of δ, given ϕ, is proportionate to ϕ. First best is then achieved by setting $s = c$. In all other cases, the principal obtains a second-best solution by choosing the contract parameters to maximize the expected total surplus $E[V - C|s] = \mathrm{E}[\delta a(s, \phi) - a(s, \phi)^2/2] = s\mathrm{E}[\delta\phi] - \frac{s^2}{2}\mathrm{E}[\phi^2]$. The optimal contract specifies $s^{AI} = \mathrm{E}[\delta\phi]/\mathrm{E}[\phi^2]$ (cf. Baker, 1992), leading to a total surplus

$$\mathrm{E}[V - C]^{AI} = \frac{\mathrm{E}[\delta\phi]^2}{2\mathrm{E}[\phi^2]} \tag{4}$$

under asymmetric information.[8] The outcome $\mathrm{E}[V - C]^{AI}$ under asymmetric information can be compared to the outcome $\mathrm{E}[V - C]^{NI}$ without information. This comparison yields the first result concerning the value of information.

Proposition 1. *In the risk-neutral agency setting with unobservable effort and no communication, the value of information to the agency may be positive or negative.*

Proof. The proof is by construction. Consider a situation with three states of nature $\{1, 2, 3\}$, probabilities $p_1 = p_2 = p_3 = 1/3$, and realisations $(\delta_1, \delta_2, \delta_3) = (1, 2, 3)$ of the agent's productivity. The outcome with a non-informed agent is $\mathrm{E}[V - C]^{NI} = \mathrm{E}[\delta]^2/2 = 2^2/2 = 2$.

Now consider first an informed agent who observes the sensitivity ϕ of his performance measure with realizations $(\phi_1, \phi_2, \phi_3) = (1, 4, 2)$. The agency's expected surplus is

$$\mathrm{E}[V - C]^{AI} = \frac{\mathrm{E}[\delta\phi]^2}{2\mathrm{E}[\phi^2]} = \frac{(15/3)^2}{2(21/3)} = \frac{75}{42} < 2,$$

and the value of information is negative.

If on the other hand $\phi = \delta$, for example, the expected surplus $\mathrm{E}[V - C]^{AI} = \mathrm{E}[\delta^2]/2 = 7/3 > 2$ of the agency is identical to that under perfect information, and the value of information is positive. □

Proposition 1 is a well-known result in the standard agency model (Demski, 1980, p. 97f.; Christensen, 1981, p. 669ff.). Here, it is asserted for two reasons. First, it serves as a benchmark for the case that additional input information is available, which is analyzed in the next section. Second, the proof of Proposition 1 shows that if $\phi = \delta$, the outcome (1) under perfect information can be achieved. This raises the question as to how a weaker relation of the two random variables affects the agency's surplus. Baker (1992) states that it is determined by the correlation of the two variables. Indeed, if (4) is written as

$$\mathrm{E}[V - C]^{AI} = \frac{(\mathrm{E}[\delta]\mathrm{E}[\phi] + \mathrm{Cov}[\delta, \phi])^2}{2\mathrm{E}[\phi^2]}, \tag{5}$$

it is obvious that the surplus is higher if δ and ϕ show a stronger (positive) correlation. However, an inspection of (5) also makes it clear that the performance measure affects the agency's surplus not only by its correlation to the firm's value, but also by its absolute level. In Baker's paper, the latter impact is ruled out by the assumption that, translated to the present model, δ and ϕ have the same expected value. But even under this additional assumption, (5) does not say anything about the extent to which the correlation of δ and ϕ affects the agency's surplus. Baker (1992, p. 605) demonstrates that the outcome under perfect information is obtained if the marginal products of V and the scaled performance measure P are perfectly correlated *and* have the same variance, in which case $\delta = \phi$ would hold in our model.

In all other cases, the effect of correlation crucially depends on the absolute value or the variance of the performance measure. This may be illustrated by the following example.

Example 1. Similar to the example in the proof of Proposition 1, let there be three equally likely states of nature with realizations $(\delta_1, \delta_2, \delta_3) = (1, 2, 3)$ of the agent's productivity. In modification of the example, the sensitiv-

[4] According to Banker and Datar (1989, p. 29), sensitivity "measures the change in the expected value of the signal with changes in the level of effort of the agent". In the present linear representation, the sensitivity of P is ϕ.

[5] In general, the principal could make use of the agent's information by offering menus of contracts. These are analyzed in Section 4. For the time being, assume that ϕ cannot be communicated, and the same contract has to be offered for all realizations of ϕ.

[6] Again, we can focus on the maximization of the total surplus although a complete characterization of the contracting problem would refer to S instead of $C(a)$. But since the agent is risk-neutral, the binding participation constraint $\mathrm{E}[S] - C = U^R$ yields $\mathrm{E}[S] = U^R - C$, from which the principal's net profit becomes $\mathrm{E}[V - S] = \mathrm{E}[V - C] - U^R$.

[7] The superscript *NI* denotes the situation with no information.

[8] The superscript *AI* indicates asymmetric information.

ity of performance measure P is given by $\phi = \delta + h$, where $h \in \mathbb{R}^+$ is a constant which is known to both contracting parties. Obviously, δ and ϕ are perfectly correlated. The expected total surplus $E[V - C]^{AI} = (2h + 14/3)^2/(2h^2 + 8h + 28/3)$, however, depends on the value of h. It is equal to the solution under perfect information if $h = 0$, and approaches the no-information solution for $h \to \infty$.

In the example, δ and ϕ are perfectly correlated and have the same variance. Different to Baker's model, they do not have the same expectation. The gap h between ϕ and δ, however, may completely destroy the positive effect of correlation.[9] Only if both expectations and variances coincide, the solution under perfect information is obtained. Obviously, this only holds if $\delta = \phi$.

From an accounting perspective, the example may well be interpreted in terms of the controllability principle. In its traditional form, this principle demands that a manager should be held responsible only for those measures he can control (cf. Merchant and Van der Stede, 2003, p. 30). Obviously, there is some ambiguity in this definition because in most cases, a variable is not either controlled by the manager or not. Rather, it will be a function of the agent's action and some environmental factor outside his control. Consequently, Antle and Demski (1989) provide a more precise notion of controllability in an agency setting, stating that a manager controls a measure if, conditional to what the principal already knows, the manager's action affects the conditional distribution of that measure (so-called *conditional controllability*).

Referring to this definition, the agent clearly controls P in the example. But since P is no perfect measure of a, conditional controllability would also support the use of any additional information which is capable of filtering out uncertainty, including that of ϕ. As an extreme, observing a would be assessed to be a perfect performance measure because conditional on a, no further information on the agent's action can be provided.

It will be shown in the next section that such purely input-related evaluation in general is not optimal in the present setting. The point is that the agent obtains knowledge of the uncontrollable effects *before* be chooses his action. Since the principal wishes the agent to account for his private information ϕ, the performance measure P is of particular value for incentive purposes, even if the principal can observe the agent's action. The extent to which it is used, however, in the example depends on the impact of h, because h is neither controlled by the agent nor is it predictive of the agent's productivity δ. Thus, to focus the agent's attention on the value-relevant parts of his measured performance, the principal would be interested in filtering out h. In the next section, variance analysis is introduced as a general device for this purpose.

[9] If P were scaled to the level of V, the same would hold for the different variances of δ and ϕ.

3. Use of additional input information and variance analysis

The above example shows that a performance measure may be almost valueless, even if it is perfectly correlated with the principal's objective. The reason for this counterintuitive result is that although the incentive contract could account for any variation in the marginal product δ (this would be done by fixing $s = 1$), the principal will not make use of this opportunity because the absolute level of effort would be too high. Consequently, he will choose a lower level of incentives, which obviously will not fully account for the possible variations of δ.

To adjust the absolute level of effort, the contract has to incorporate additional information related to the agent's input a instead of the output number P. Such information is frequently considered in variance investigation procedures which try to explain deviations between budgeted and realized output numbers by incorporating additional input information. In the present setting, the difference $\Delta P = P^R - P^B$ of the realized value P^R and the budgeted amount P^B of the performance measure could be split into a component $\Delta^\phi P$ due to the variation of ϕ, and a component $\Delta^a P$ due to the deviation of a,

$$\Delta P = P^R - P^B = \underbrace{(P^R - \phi^B a^R)}_{\Delta^\phi P} + \underbrace{(\phi^B a^R - \phi^B a^B)}_{\Delta^a P}, \tag{6}$$

where the superscripts B and R refer to the budgets and realizations, respectively.[10]

To carry out this decomposition, a measure of the agent's input needs to be available. For simplicity, assume that a is observed by the contracting parties and can be used for performance evaluation.[11] Under the linearity assumption, the compensation contract becomes

$$S = s_0 + s_1 a + s_2 P. \tag{7}$$

The agent's action under this contract will be $a(s_1, s_2) = s_1 + s_2\phi$, which enables the principal to control the absolute level of effort. In the optimal contract, he will fix[12]

$$s_1 = E[\delta] - s_2 E[\phi] \quad \text{and} \quad s_2 = \frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}. \tag{8}$$

This allows us to write the agent's compensation in the form

$$S = s_0 + E[\delta]a + \frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}(P - E[\phi]a). \tag{9}$$

Owing to the assumption of a two-piece-rate contract, the optimal compensation can of course be written as a function of some variance. The more interesting question is how (9) relates to the variance decomposition described in (6), and how this procedure corresponds to the controllability principle. To address these issues, we write $a = a^R$ for the

[10] The realized value of ϕ cannot enter the calculation, since ϕ cannot be observed by the principal. Instead, it is implicitly inferred from P by the residual deviation $P^R - \phi^B a^R$, which cannot be explained by the deviation of a.

[11] More generally, we could consider a noisy measure $A = a + \epsilon$ of the agent's input. Owing to the agent's risk-neutrality, however, this would not affect the results of the paper.

[12] See Appendix B.1.

realized action and take $\phi^B = \mathrm{E}[\phi]$ as the budgeted value of ϕ, as suggested in the literature (Booth and Willett, 1997):

$$S = s_0 + \mathrm{E}[\delta]a^R + \frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}(P^R - \phi^B a^R).$$

Thus, the agent is held responsible for the variance due to deviation of ϕ. The controllability principle in its purest form, in contrast, would demand responsibility for the measures that the agent can *control* (cf. Merchant and Van der Stede, 2003, p. 30). At first glance, this is primarily the variance due to deviation $a^R - a^B$ of effort, which can be included in the compensation scheme by expanding the first variable part:

$$\begin{aligned} S &= s_0 + \mathrm{E}[\delta]a^R + (\mathrm{E}[\delta]a^B - \mathrm{E}[\delta]a^B) + \frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}(P^R - \phi^B a^R) \\ &= \underbrace{(s_0 + \mathrm{E}[\delta]a^B)}_{\tilde{s}_0} + \underbrace{\frac{\mathrm{E}[\delta]}{\mathrm{E}[\phi]}}_{\tilde{s}_1}\underbrace{(\phi^B a^R - \phi^B a^B)}_{\Delta^a P} \\ &\quad + \underbrace{\frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}}_{\tilde{s}_2}\underbrace{(P^R - \phi^B a^R)}_{\Delta^\phi P}. \end{aligned} \tag{10}$$

In examining (10), we see that the agent is held responsible for both special variances, $\Delta^a P$ and $\Delta^\phi P$. Does this contradict the controllability principle? The agent obviously controls $\Delta^a P$ because it is the variance assigned to the deviation of a^R and a^B. However, he also controls $\Delta^\phi P$, since it is computed based on the *realized* effort level. The first piece rate, $\tilde{s}_1$, motivates the agent to choose the optimal *average* level of effort, whereas the second part ensures optimization of his effort *profile*. Focusing on the effort profile accounts for a refinement of the controllability principle stating that managers should be held responsible for those numbers they are supposed to *pay attention to* (cf. Merchant and Van der Stede, 2003, p. 464): since the principal wants the agent to care about his pre-decision information ϕ, it is necessary to incorporate a measure of ϕ into the compensation contract. This is done by including the second variance $\Delta^\phi P$. By variance decomposition, the desired effect can be delineated from the basic incentive, which was impossible under the contract (3) based on P. This distinction clarifies the particular value of the variance analysis: by decomposition of variances, the principal is able to fine-tune the compensation contract stipulating different piece rates for the absolute level of effort and its variation.

In order to make the refined version of the controllability principle more precise, it is useful to relate my results to the informativeness principle. With regard to the analysis of Antle and Demski (1989), a comparison requires a more comprehensive definition of what information the principal is looking for. Of course, once a has been observed, the performance measure P provides no additional information with respect to the agent's action. It is nevertheless useful for contracting because the principal is not interested in implementing a certain *fixed* effort level, but the effort level that is optimal under the agent's private information ϕ. Therefore, even if the principal observes the agent's action a, he is still interested in inferring whether the action chosen is the one most suitable to maximise the expected total surplus. For this purpose, information on both a and ϕ is needed, which is provided by a and P.

The approach described conforms to the accounting literature on variance investigation as well. In general, the variance decomposition method complies with those proposed in management accounting textbooks, for instance the price and efficiency variances in cost accounting (Horngren et al., 2006, p. 227ff.). More specifically, Kloock and Schiller (1997, p. 317) state that variances computed on a budgeted basis are capable of creating proper *ex ante* incentives. As mentioned above, this ex ante perspective is covered by $\Delta^a P$. From an ex post perspective, variances based on realized amounts are considered advantageous since they provide relevant information for planning purposes. This is fulfilled by the second variance. However, while the conventional argument refers to *future planning periods* in this respect, planning in the present model concerns the agent's action in the *current period*. The realized effort is used to quantify the (expected) benefits arising from deviations in ϕ, motivating the agent to choose the right action.

Once we have shown that variance analysis procedures represent a proper instrument to implement the second-best solution in the present model, we can turn to the question of whether the inclusion of input information resolves the issue raised in Proposition 1, namely that the value of an informed agent may be negative. For this purpose, consider the agent's action $a(s_1, s_2) = s_1 + s_2\phi$ resulting from the contract (7) with variance investigation. Obviously, the optimal action $a^{NI} = \mathrm{E}[\delta]$ of an uninformed agent is readily obtained by choosing $s_1 = \mathrm{E}[\delta]$ and $s_2 = 0$, and the ambiguity of Proposition 1 disappears.

Proposition 2. *In the risk-neutral agency setting with observable effort, the value of information to the agency is non-negative.*

Proof. Obvious from the above considerations. □

My next goal is to derive the conditions under which optimal alignment is achieved. For this purpose, compare the action[13]

$$a^{OE} = \mathrm{E}[\delta] + \frac{\mathrm{Cov}[\delta, \phi]}{\mathrm{Var}[\phi]}(\phi - \mathrm{E}[\phi]) \tag{11}$$

resulting from the second-best contract (9) to the first-best action $a^{FB} = \mathrm{E}[\delta|\phi]$, as given in Section 2. Obviously, the latter can be implemented by a simple linear contract of the form (3)(without observation of a) only if the expected value of δ is proportionate to ϕ. With observation of a, there is an additional degree of freedom in the compensation scheme. Scheme (9) will align the interests of the agent and principal if (11) is the conditional expectation of δ, given ϕ.

Proposition 3. *In the risk-neutral agency setting with observable effort, the first-best solution can be obtained by a linear contract if the conditional expectation of the agent's marginal product δ is a linear function of the sensitivity ϕ to his performance measure.*

[13] The superscript *OE* indicates the situation with observable effort.

Proof. If the expected value of δ, given ϕ, is a linear function of ϕ, there exist $\lambda_1, \lambda_2 \in \mathbb{R}$ such that $\mathrm{E}[\delta|\phi] = \lambda_1 + \lambda_2\phi$ for all realizations of ϕ. Using a linear contract of the form (9), the agent chooses $a(s_1, s_2, \phi) = s_1\mathrm{E}[\phi] + s_2(\phi - \mathrm{E}[\phi])$. Thus, setting $s_2 = \lambda_2$ and $s_1 = \lambda_2 + \lambda_1/\mathrm{E}[\phi]$ yields the first-best action $a^{FB} = \mathrm{E}[\delta|\phi]$. □

If the conditional expectation of δ is a linear function of ϕ, it can be replicated by a linear contract.[14] A sufficient (but not necessary) condition for the linearity requirement is perfect correlation of δ and ϕ, in which case δ itself is a linear function of ϕ by definition. Moreover, under perfect correlation the first-best outcome is identical to the perfect information outcome, where the principal has knowledge of δ.

Corollary 1. *If δ and ϕ are perfectly correlated in the risk-neutral agency setting with observable effort, the solution under perfect information can be obtained by a linear contract.*

Proof. Under perfect correlation, a linear relation $\delta = \lambda_1 + \lambda_2\phi$ holds for some $\lambda_1, \lambda_2 \in \mathbb{R}$ and all realizations of δ and ϕ. Setting $s_2 = \lambda_2$ and $s_1 = \lambda_2 + \lambda_1\mathrm{E}[\phi]$ induces $a = \delta$, which yields the solution (1) under perfect information. □

Corollary 1 is a special case of Proposition 3. Under perfect correlation, δ can be inferred unambiguously from ϕ, and symmetric information is equivalent to perfect information. Thus, it seems obvious that, contrary to the example of Section 2, the correlation of δ and ϕ has a positive impact on the principal's benefit, at least if the conditions of Proposition 1 are fulfilled. To analyze this in general, we denote the correlation of δ and ϕ by ρ. The agent's action (11) under the two-piece-rate contract (9) can then be written as

$$a^{OE} = \mathrm{E}[\delta] + \rho\sqrt{\frac{\mathrm{Var}[\delta]}{\mathrm{Var}[\phi]}}(\phi - \mathrm{E}[\phi]).$$

By substitution of this term in the principal's objective function, the expected total surplus becomes[15]

$$\mathrm{E}[V - C]^{OE} = \frac{1}{2}(\mathrm{E}[\delta]^2 + \rho^2\mathrm{Var}[\delta]). \tag{12}$$

A comparison of (12) and (5) reveals that, contrary to the situation with unobservable effort, the performance measure now affects the agency's surplus only by the correlation of δ and ϕ. The latter therefore perfectly indicates the quality of a performance measure for a linear contract.

Proposition 4. *In the risk-neutral agency setting with observable effort, the agency's surplus is increasing in the absolute level of the correlation ρ between the agent's marginal product δ and the sensitivity ϕ of his performance measure.*

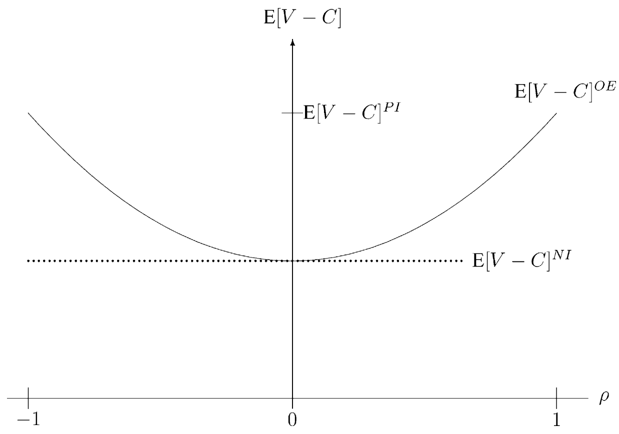

Fig. 1. The agency's surplus as a function of the correlation ρ of δ and ϕ.

Proof. Obvious from Eq. (12). □

Proposition 4 condenses the insight that in the risk-neutral agency with distorted performance measurement, dealing with the agent's private information is merely a matter of inferring his marginal product from the sensitivity of the performance measure applied. Observing the agent's effort enables the principal to use a linear regression, where the correlation coefficient indicates the quality of the inference.

A graphical illustration of this relation is provided in Fig. 1.

For perfect positive or negative correlation, the expected surplus $\mathrm{E}[V - C]^{PI}$ under perfect information is obtained. For smaller values of correlation, the agency's surplus decreases and reaches the level $\mathrm{E}[V - C]^{NI}$ without information when δ and ϕ are uncorrelated. Under the conditions of Proposition 3, $\mathrm{E}[V - C]^{OE}$ is also the first best solution under symmetric information. If not, the principal would benefit from observing ϕ.

4. Participative budgeting

Proposition 3 defines conditions under which the first-best outcome under symmetric information can be achieved. An obvious question is whether any improvement can be obtained when these conditions are not met. The most frequently discussed way to achieve such improvements is budget participation. In economic terms, this corresponds to offering a menu of contracts to the agent.

In the present model, the first-best solution under symmetric information can easily be achieved by participative budgeting if the agent's action a is observable. The principal just has to prescribe the first-best effort level as a budget $a^B = a(\hat{\phi}) = \mathrm{E}[\delta|\hat{\phi}]$ for any reported productivity $\hat{\phi}$ of ϕ, thereby compensating the agent for the resulting cost of effort $(a^B)^2/2$. This eliminates the moral hazard problem, and budget participation becomes merely a matter of eliciting the agent's private information. Since the agent obtains private information only after he has signed the contract, truthful reporting can be achieved without an additional cost. To that purpose, the principal can apply a mechanism

[14] We could look for distributions that meet this condition and allow for the first-best solution. This is obviously the case for all distributions meeting the linear conditional expectation. A well-known example is the class of elliptical distributions, of which the normal distribution is a special case. A larger class is the Pearson family (see Wei et al., 1999 for details). Beyond this, discrete distributions that fulfil the requirement can also be constructed.

[15] See Appendix B.2.

according to Osband and Reichelstein (1985), in which deviations of budgeted and realized performance are punished by a convex incentive scheme[16]

$$S(P^B, P) = l(P^B) + l'(P^B)[P - P^B], \tag{13}$$

where $P^B = \phi^B a^B$ with $\phi^B = \hat{\phi}$ and $a^B = a(\hat{\phi}) = \mathrm{E}[\delta|\hat{\phi}]$ denotes the budgeted performance, and $l(\cdot)$ is an arbitrary convex function.[17] Since l is a convex function, the penalty $l'(P^B)[P^B - P]$ for an overreport of ϕ always outweighs the benefits $l(P^B)$ from that misreport. In turn, underreporting is also disadvantageous because the benefits $l'(P^B)[P - P^B]$ from an underreport of ϕ are outbalanced by the loss $l(P^B)$ from that report. By the comparison of budgeted and reported performance numbers, the structure of variance analysis again is brought into the contract. But contrary to the situation without communication, variance analysis is used to discipline the agent in reporting instead of making indirect use of the agent's private information, as was done in the preceding analysis.

If a cannot be fixed in the contract, the problems of moral hazard and private information cannot be separated, and a more subtle contract has to be used. Again, the agent will not earn an informational rent because he obtains private information only after contracting. But since a is not observable, the contract has to provide incentives for both truth-telling *and* the desired first-best effort. Therefore, unlike in a situation with observable effort, it is not clear whether the first-best solution can be obtained by menu of linear contracts. In general, these contracts will take the form

$$S^0 = s_0^0(\hat{\phi}) + s_1^0(\hat{\phi})P, \tag{14}$$

where both the fixed payment s_0^0 and the share parameter s_1^0 are based on the agent's report $\hat{\phi}$ of ϕ.

To obtain the first-best solution by this menu of contracts, two requirements have to be met. First, it has to be in the agent's best interest to truthfully report his private information. This requirement arises from the revelation principle which states that the analysis of optimal contracts can focus on truth-inducing mechanisms without loss of generality.[18] Second, given that the agent reports $\hat{\phi} = \phi$, his rational choice of effort has to be the first-best action $a^{SI} = \mathrm{E}[\delta|\phi]$ under symmetric information. This solves the moral hazard problem.

Since from the analysis of Section 2 we know that the agent chooses $a(s, \phi) = s_1\phi$, the second requirement is met by setting

$$s_1^0(\hat{\phi}) = \frac{\mathrm{E}[\delta|\hat{\phi}]}{\hat{\phi}}. \tag{15}$$

Taking this share parameter as given, a revelation contract for the truthful report of ϕ can be derived by the standard techniques of mechanism design. It is given by the compensation contract

$$S^0(\hat{\phi}) = \underbrace{\underline{s} + \int_{\underline{\phi}}^{\hat{\phi}} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi - \frac{\mathrm{E}[\delta|\hat{\phi}]^2}{2}}_{s_0^0(\hat{\phi})} + \underbrace{\frac{\mathrm{E}[\delta|\hat{\phi}]}{\hat{\phi}}}_{s_1^0(\hat{\phi})}P. \tag{16}$$

In (16), the base level $\underline{s}$ of compensation is chosen to make the agent accept the contract. The remaining terms of s_0^0 guarantee that it is in the agent's best interest to report his private information ϕ. To see this, consider the agent's expected utility $\mathrm{E}[U^A] = s_0^0(\hat{\phi}) + s_1^0(\hat{\phi})\mathrm{E}[P] - C(a(s_1^0))$ from the contract (14). After substitution for the agent's action $a(s_1^0) = s_1^0\phi$, for his cost of effort $C(a(s_1^0)) = a(s_1^0)^2/2$, for the expected performance $\mathrm{E}[P] = \phi a(s_1^0)$ and for the contract parameters s_0^0 and s_1^0 of (16), the agent's expected utility is

$$\begin{aligned}\mathrm{E}[U^A] &= \underline{s} + \int_{\underline{\phi}}^{\phi} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi - \frac{\mathrm{E}[\delta|\phi]^2}{2} \\ &\quad + \frac{\mathrm{E}[\delta|\hat{\phi}]^2}{\hat{\phi}^2}\phi^2 - \frac{1}{2}\frac{\mathrm{E}[\delta|\hat{\phi}]^2}{\hat{\phi}^2}\phi^2 \\ &= \underline{s} + \int_{\underline{\phi}}^{\phi} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi - \frac{\mathrm{E}[\delta|\phi]^2}{2} + \frac{1}{2}\frac{\mathrm{E}[\delta|\hat{\phi}]^2}{\hat{\phi}^2}\phi^2.\end{aligned}$$

Differentiation with respect to $\hat{\phi}$ yields the first-order condition for the agent's report,

$$\begin{aligned}\frac{\partial \mathrm{E}[U^A]}{\partial \hat{\phi}} &= \frac{\mathrm{E}[\delta|\hat{\phi}]^2}{\hat{\phi}} - \mathrm{E}[\delta|\phi]^2 \frac{\partial \mathrm{E}[\delta|\phi]}{\partial \hat{\phi}} \\ &\quad + \frac{\hat{\phi}^2 \mathrm{E}[\delta|\hat{\phi}]((\partial \mathrm{E}[\delta|\hat{\phi}])/\partial\hat{\phi}) - \hat{\phi}\mathrm{E}[\delta|\hat{\phi}]^2}{\hat{\phi}^4}\phi^2 = 0,\end{aligned}$$

which after some rearrangement can be simplified to

$$(\hat{\phi}^2 - \phi^2)\mathrm{E}[\delta|\phi] = \hat{\phi}(\hat{\phi}^2 - \phi^2)\frac{\partial \mathrm{E}[\delta|\phi]}{\partial \hat{\phi}}. \tag{17}$$

The first-order condition (17) is fulfilled if $\hat{\phi} = \phi$ or if $\mathrm{E}[\delta|\phi] = \hat{\phi}\partial \mathrm{E}[\delta|\phi]/\partial\hat{\phi}$. The first case identifies truthful reporting. The second case describes a situation in which the first-best allocation can be achieved without communication because $\mathrm{E}[\delta|\phi]$ is proportionate to ϕ, and the share parameter (15) that implements the first-best action is identical for all types of agents. Analytically, the revelation term in (16) vanishes, and the contract (16) is identical to the contract (3). It was already stated in Section 2 that the first-best effort can be implemented by this type of contract if $\mathrm{E}[\delta|\phi]$ is proportionate to ϕ.

16 Another well-known mechanism is that proposed by Weitzman (1976). Unlike the scheme of Osband and Reichelstein, however, the Weitzman scheme would reveal a quantile of the distribution of P instead of its expectation.

17 Since P is distorted by white noise, the agent in this case has to be compensated for the expected punishment under truthful reporting. Due to the agent's risk-neutrality, such sanctions are free of cost to the principal.

18 See Dasgupta et al. (1979) or Myerson (1979).

Like in the previous analysis of Section 3, the contract (16) is well interpreted as a variance analysis procedure:

$$S^0(\hat{\phi}) = \underline{s} + \int_{\underline{\phi}}^{\phi} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi - \frac{\mathrm{E}[\delta|\phi]^2}{2} + \frac{\mathrm{E}[\delta|\hat{\phi}]}{\hat{\phi}}P$$

$$= \underline{s} + \underbrace{\int_{\underline{\phi}}^{\hat{\phi}} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi}_{\text{information rent}} + \underbrace{\frac{\mathrm{E}[\delta|\hat{\phi}]^2}{2}}_{\text{effort cost}} + \frac{\mathrm{E}[\delta|\hat{\phi}]}{\hat{\phi}}(P - \mathrm{E}[\delta|\hat{\phi}]\hat{\phi}).$$

Again, the last term exhibits a variance structure which can further be clarified by fixing budgets $\phi^B = \hat{\phi}$ and $a^B = \mathrm{E}[\delta|\hat{\phi}]$:

$$S^0(\phi^B) = \underline{s} + \underbrace{\int_{\underline{\phi}}^{\phi^B} \frac{\mathrm{E}[\delta|\varphi]^2}{\varphi}\,\mathrm{d}\varphi}_{\text{information rent}} + \underbrace{\frac{(a^B)^2}{2}}_{\text{effort compensation}} + \underbrace{s_1(\phi^B)[P - a^B\phi^B]}_{\text{variance term}}. \tag{18}$$

The budget ϕ^B is set by the agent. From this budget, the required (or budgeted) effort $a^B = \mathrm{E}[\delta|\phi^B]$ is fixed in a prespecified way. Under truthful reporting, the expected value $\mathrm{E}[P] = \phi a$ of the performance measure is identical to its budget $\phi^B a^B$, and the variance term in (18) has an expected value of zero. Consequently, the agent in expectation will be compensated for his disutility of effort, and he will receive a rent for his private information. The base salary $s_0^0(\underline{\phi}) = \underline{s}$ for the least profitable type $\underline{\phi}$ is fixed to provide the agent's reservation utility.

The structure of the compensation contract (18) shows that, in contrast to the setting with observable effort and no communication, the agent is made responsible for the whole deviation $P - a^B\phi^B = P - P^B$ of realized and budgeted performance, without any distinction in the magnitude of incentives, as applied in (10). Different rates for different special variances are not necessary under participative budgeting because the agent's private information is revealed by his choice among the offered contracts, and his effort $a(s_1^0) = s_1^0(\phi)\phi$ directly follows from his report. Since productivity ϕ and effort a are tied to each other in this budgeting procedure, both the average level of effort and its distinction according to the marginal product can be controlled in a single step. Once the agent has committed to a certain budget, the total variance suffices to ensure that he will not differ from the corresponding action. Again, this is well in line with the controllability principle: since the agent observes ϕ and eventually fixes the budget $P^B = \phi^B a^B$ by announcing ϕ^B, he fully controls the deviation $P - P^B$, except for the additive noise. Since the latter cannot be factored out, rewarding $P - P^B$ is the best the principal can do if he does not observe a.

Obviously, (17) is only a necessary first-order condition for the solution to the manager's reporting problem. A sufficient condition can be derived from the general mechanism design problem (Salanie, 1997, p. 26ff.; Fudenberg and Tirole, 1991, p. 257ff.), and is stated in the following proposition.

Proposition 5. *In the risk-neutral agency setting without observable effort, the first-best action can be implemented by a menu of linear contracts if and only if* $\mathrm{E}[\delta|\phi]/\phi$ *is a nondecreasing function of* ϕ.

Proof. See Appendix A. □

Proposition 5 is an application of a familiar result in mechanism design stating that if the agent's marginal utility from his private information is monotonically increasing (or decreasing) in the allocation resulting from his report $\hat{\phi}$ (the so-called single-crossing or sorting condition[19]), any nondecreasing (nonincreasing) allocation is implementable (Guesnerie and Laffont, 1984, Theorem 2). In the present model, this sorting condition condenses to the requirement of the share parameter $s_1(\cdot)$ being a nondecreasing function in the agent's report ϕ. That is, not only should a higher effort be optimal for higher levels of ϕ, but it also needs to be implemented by a higher share of P.[20] The agent must be willing to pay more (i.e., to accept higher cuts in his base salary $s_0(\phi)$) for an increase in his share s_1 if he observes higher values of ϕ. Then, different types of agents can be separated by offering larger variable payments for higher-sensitivity products and making the agent pay for this privilege.

The intuition behind this result is straightforward. If the share parameter $s_1^0(\hat{\phi}) = \mathrm{E}[\delta|\hat{\phi}]/\hat{\phi}$ that induces the first-best action were decreasing on an interval $\Phi_{12} = [\phi_1, \phi_2]$, ceteris paribus it would be beneficial for all types $\phi \in \Phi_{12}$ to report the lowest level ϕ_1 in order to receive the highest variable payment. To preclude this misreporting, $s_0^0(\hat{\phi})$ had to be increasing in ϕ such that higher types are kept from understating ϕ. However, given the sorting condition CS^+ this increase makes it beneficial to report the highest level ϕ_2 in order to receive the highest fixed payment s_0^0. Only if s_1^0 is increasing in ϕ, both overstating and understating of ϕ can be precluded.

By comparison of the contract (18) to the incentive scheme (13) with observable effort, the benefit from additional input information under participative budgeting becomes obvious: although both schemes exhibit a similar structure, S^0 is limited because, contrary to (13), in (18) it does not suffice to punish deviations of realized and budgeted performance by an arbitrary increasing function $l'(\cdot)$. The respective share parameter s_1 also has to induce the first-best action, which under observable effort can be enforced by the contract. With unobservable effort, the contract has to serve two purposes, which—as Proposition 5 shows—can only be brought in line under certain conditions. Thus, at least with regard to the first-best allocation, the observation of a will be of value whenever the requirement of Proposition 5 is not fulfilled.

The finding that budget participation may be of value seems to contrast the results of classical models on communication in agencies. In these models, a well-known result

[19] Graphically, the sorting condition states that the slope of agent's indifference curves in the allocation s_1 and the transfer s_0 is monotonic in the agent's type. The indifference curves of two different types therefore intersect only once, and agents can be sorted by their choice of contracts.

[20] This limitation arises because the sorting condition is always fulfilled in its positive form CS^+.

is that under o ptimal contracting, using a menu of contracts is of no value compared to a single contract if (a) the agent attains full information in the sense that his action and private information unambiguously determine the outcome or (b) only additive noise disturbs the outcome (cf. Melumad and Reichelstein, 1989, or Caillaud et al., 1992). In the present model, the conditions of (b) are met, but contract design is restricted because contracts are linear and can only be written on P, but not on V. The analysis shows that under this restriction communication may be useful: with observable effort, communication is of value whenever the requirements of Proposition 3 are not met. Without this input information, Proposition 5 shows that communication is of value if the first-best action is implemented in a separating equilibrium. This highlights the fact that performance measure distortion substantially limits the use of participative budgeting: if budgets refer to V instead of P, implementing the first-best solution would of course not be an issue. The restriction to linear contracts alone would not warrant the use of participative budgeting. Communication is only of value under distorted performance measurement.

5. Conclusion

This paper has analyzed the role of variance investigation procedures in mitigating the problem of effort misallocation in an agency setting with distorted performance measurement. It was shown that variance analysis improves the optimal linear contract in most cases for which the performance measure is not in line with the principal's objective. By application of variance investigation, the quality of a particular performance measure for a privately informed agent can be quantified using the correlation of its marginal product and that of the principal's objective. Special variances have a natural interpretation in this setting: they quantify the expected deviation from the principal's objective, given a performance measure deviation. Using deviations instead of total amounts, the effects of measured performance can be translated into value effects. Thus, the objectives of the principal and agent can be aligned via compensation.

If the agent can communicate his private information, budget participation changes the role of variance analysis. While under top-down budgeting the agent is held responsible for the two variances to different extents, under participation he bears responsibility for the total deviation of realized and budgeted performance. Such complete stewardship, however, is feasible only under certain conditions. Otherwise, additional information on the agent's input may be used to achieve complete alignment.

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Appendix A. Proof of Proposition 5

A general insight in mechanism design is that a non-monotonic allocation cannot be implemented (see Salanie, 1997, p. 36). Monotonic allocations can be implemented if certain regularity conditions are met. I analyze which of these conditions are satisfied in the present setting. To that purpose, consider the agent's expected utility

$$V(\hat{\phi}, \phi) \equiv \mathrm{E}U(\hat{\phi}, \phi, a) = s_0^0(\hat{\phi}) + s_1^0(\hat{\phi})\phi a(s_1^0(\hat{\phi}), \phi) - \frac{a(s_1^0(\hat{\phi}), \phi)^2}{2}$$

from a contract (14). Taking into account the agent's action choice $a(s_1^0(\hat{\phi}, \phi)) = s_1^0(\hat{\phi}, \phi)\phi$, this utility becomes

$$V(\hat{\phi}, \phi) = s_0^0(\hat{\phi}) + \frac{s_1^0(\hat{\phi})^2 \phi^2}{2}.$$

Differentiation with respect to $\hat{\phi}$ yields the first-order condition

$$\left.\frac{\partial V(\hat{\phi}, \phi)}{\partial \hat{\phi}}\right|_{\hat{\phi}} = \phi = \left.\frac{\mathrm{d}s_0^0(\hat{\phi})}{\mathrm{d}\hat{\phi}}\right|_{\hat{\phi}=\phi} + \frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\phi^2}{2}\right)\left.\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi)\right|_{\hat{\phi}=\phi} = 0 \tag{A.1}$$

of the agent's reporting problem. Eq. (A.1) defines a necessary condition for the base wage s_0^0 to be capable of inducing a truthful report of ϕ. Building the differential

$$\left.\frac{\partial^2 V(\hat{\phi}, \phi)}{\partial \hat{\phi}^2}\right|_{\hat{\phi}=\phi} + \left.\frac{\partial^2 V(\hat{\phi}, \phi)}{\partial \hat{\phi}\partial \phi}\right|_{\hat{\phi}=\phi} = 0$$

allows to rewrite the local second-order condition

$$\left.\frac{\partial^2 V(\hat{\phi}, \phi)}{\partial \hat{\phi}^2}\right|_{\hat{\phi}=\phi} \leq 0$$

of a maximum as

$$\left.\frac{\partial^2 V(\hat{\phi}, \phi)}{\partial \hat{\phi}\partial \phi}\right|_{\hat{\phi}} = \phi = \frac{\partial^2}{\partial s_1^0 \partial \phi}\left(\frac{s_1(\hat{\phi})^2\phi^2}{2}\right)\left.\frac{\mathrm{d}s_1}{\mathrm{d}\phi}(\phi)\right|_{\hat{\phi}} = \phi = (2s_1(\phi)\phi)\frac{\mathrm{d}s_1}{\mathrm{d}\hat{\phi}}(\phi) \geq 0, \tag{A.2}$$

meaning that the sensitivity ϕ affects the agent's marginal utility from an increase in s_1 in a systematic way (Salanie, 1997, p. 30). The so-called sorting condition requires that the cross-derivative in (A.2) is positive (SC^+) or negative (SC^-) for all ϕ. In the present model, substitution of the first-best share parameter $s_1^0(\hat{\phi}) = \mathrm{E}[\delta|\hat{\phi}]/\hat{\phi}$ in (A.2) yields

$$(2s_1^0(\phi)\phi)\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi) = 2\mathrm{E}[\delta|\phi]\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{\mathrm{E}[\delta|\phi]}{\phi}\right) \geq 0 \tag{A.3}$$

as a local second-order condition for the first-best action to be implementable. Since by assumption the marginal product δ of the agent's action is positive, the same holds for its expectation $\mathrm{E}[\delta|\phi]$, which is identical to the cross derivative in (A.2). Thus, the sorting condition is always fulfilled in its positive form CS^+. Consequently, only an allocation $s_1(\phi)$ that is nondecreasing in ϕ can be implemented because condition (A.3) condenses to

$$\frac{\mathrm{d}}{\mathrm{d}\phi}s_1(\phi) = \frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{\mathrm{E}[\delta|\phi]}{\phi}\right) \geq 0, \tag{A.4}$$

the desired allocation s_1 being nondecreasing in the agent's information ϕ.

To show that (A.4) is also sufficient for s_1 to be implementable, similar arguments as in Salanie (1997, p. 31) can be used to show that ϕ is also the global maximizer to the agent's reporting problem. To that purpose, consider the agent's marginal expected utility

$$\frac{\partial V(\hat{\phi},\phi)}{\partial\hat{\phi}} = \frac{\mathrm{d}s_0^0(\hat{\phi})}{\mathrm{d}\hat{\phi}} + \frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\phi^2}{2}\right)\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi)$$

of his report. Since s_0^0 is chosen as to satisfy the first-order condition (17) for all ϕ, this is identical to

$$\begin{aligned}\frac{\partial V(\hat{\phi},\phi)}{\partial\hat{\phi}} &= -\frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\hat{\phi}^2}{2}\right)\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi) \\ &\quad+\frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\phi^2}{2}\right)\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi) \\ &= \left[-\frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\hat{\phi}^2}{2}\right)+\frac{\partial}{\partial s_1^0}\left(\frac{s_1^0(\hat{\phi})^2\phi^2}{2}\right)\right]\frac{\mathrm{d}s_1^0}{\mathrm{d}\hat{\phi}}(\phi).\end{aligned}$$

The sign of the term in brackets is identical to that of

$$\frac{\partial^2}{\partial s_1^0\partial\phi}\left(\frac{s_1(\hat{\phi})^2\breve{\phi}^2}{2}\right)(\phi-\hat{\phi}) \tag{A.5}$$

for some $\breve{\phi}$ between ϕ and $\hat{\phi}$. By the sorting condition CS^+, the cross derivative in (A.5) is positive. Since s_1^0 is nondecreasing, the sign of $\partial V(\hat{\phi},\phi)/\partial\hat{\phi}$ is therefore identical to that of $\phi-\hat{\phi}$. Consequently, the agent's utility is increasing in $\hat{\phi}$ for $\hat{\phi}<\phi$ and decreasing in $\hat{\phi}$ for $\hat{\phi}>\phi$.

Appendix B. Further computations

B.1. Contract parameters (8)

Based on P and a, a linear contract is of the form $S = s_0 + s_1 a + s_2 P$, resulting in an action choice $a(\phi) = s_1 + s_2\phi$. Substituting this into the principal's objective of maximizing total surplus

$$\mathrm{E}[V-S] = \mathrm{E}\left[\delta a(\phi) - \frac{a(\phi)^2}{2}\right]$$

yields the optimization problem

$$\begin{aligned}&\max_{s_1,s_2}\mathrm{E}[V-S] \\ &\quad = s_1\mathrm{E}[\delta] + s_2\mathrm{E}[\delta\phi] - \frac{s_1^2}{2} - s_1 s_2\mathrm{E}[\phi] - \frac{s_2^2}{2}\mathrm{E}[\phi^2].\end{aligned}$$

From the first-order conditions

$$\frac{\partial\mathrm{E}[V-S]}{\partial s_1} = \mathrm{E}[\delta] - s_1 - s_2\mathrm{E}[\phi] = 0$$

and

$$\frac{\partial\mathrm{E}[V-S]}{\partial s_2} = \mathrm{E}[\delta\phi] - s_1\mathrm{E}[\phi] - s_2\mathrm{E}[\phi^2] = 0$$

I derive

$$s_1 = \mathrm{E}[\delta] - s_2\mathrm{E}[\phi] \text{ and } s_2 = \frac{\mathrm{E}[\delta\phi]-\mathrm{E}[\delta]\mathrm{E}[\phi]}{\mathrm{E}[\phi^2]-\mathrm{E}[\phi]^2} = \frac{\mathrm{Cov}[\delta\phi]}{\mathrm{Var}[\phi]}$$

for the optimal values of s_1 and s_2 which coincide with the values in (9).

B.2. Expected surplus (12) from a contract with two piece rates

The agent's incentive constraint yields

$$a^{OE} = s_1 + s_2\phi = \mathrm{E}[\delta] + s_2(\phi - \mathrm{E}[\phi])$$

where I use the fact that $s_1 = \mathrm{E}[\delta] - s_2\mathrm{E}[\phi]$. Substituting this into the gross profit yields

$$\begin{aligned}V - C &= \delta a^{OE} - \frac{(a^*)^2}{2} = \delta[s_1 + s_2\phi] + \frac{[s_1+s_2\phi]^2}{2} \\ &= \delta[\mathrm{E}[\delta] + s_2(\phi-\mathrm{E}[\phi])] - \frac{[\mathrm{E}[\delta]+s_2(\phi-\mathrm{E}[\phi])]^2}{2}.\end{aligned}$$

Computation of expectations gives

$$\begin{aligned}\mathrm{E}[V-C]^{OE} &= \mathrm{E}[\delta]^2 + s_2\mathrm{E}[\delta\phi - \delta\mathrm{E}[\phi]] - \frac{1}{2}(\mathrm{E}[\delta]^2 \\ &\quad + 2s_2\mathrm{E}[\delta]\mathrm{E}[\phi-\mathrm{E}[\phi]] + s_2^2\mathrm{E}[\phi-\mathrm{E}[\phi]]^2) \\ &= \frac{1}{2}\mathrm{E}[\delta]^2 + s_2\mathrm{Cov}[\delta,\phi] - \frac{1}{2}s_2^2\mathrm{Var}[\phi].\end{aligned}$$

Substitution of $s_2 = (\mathrm{Cov}[\delta,\phi]/\mathrm{Var}[\phi])$yields

$$\begin{aligned}&\mathrm{E}[V-C]^{OE} \\ &\quad = \frac{1}{2}\mathrm{E}[\delta]^2 + \frac{\mathrm{Cov}[\delta,\phi]}{\mathrm{Var}[\phi]}\mathrm{Cov}[\delta,\phi] - \frac{1}{2}\frac{\mathrm{Cov}[\delta,\phi]^2}{\mathrm{Var}[\phi]^2}\mathrm{Var}[\phi] \\ &\quad = \frac{1}{2}\left(\mathrm{E}[\delta]^2 + \frac{\mathrm{Cov}[\delta,\phi]^2}{\mathrm{Var}[\phi]}\right) = \frac{1}{2}(\mathrm{E}[\delta]^2 + \rho^2\mathrm{Var}[\delta]).\end{aligned}$$

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