Optimal Risk Taking in an Uneven Tournament Game with Risk Averse Players*

Matthias Kräkel, University of Bonn**

We analyze the optimal choice of risk in a two-stage tournament game between two players that have different concave utility functions. At the first stage, both players simultaneously choose risk. At the second stage, both observe overall risk and simultaneously decide on effort or investment. The results show that those two effects which mainly determine risk taking – an effort effect and a likelihood effect – are strictly interrelated. This finding sharply contrasts with existing results on risk taking in tournament games with symmetric equilibrium efforts where such linkage can never arise. Conditions are derived under which this linkage leads to a reversed likelihood effect so that the favorite (underdog) can increase his winning probability by increasing (decreasing) risk which is impossible in a completely symmetric setting.

Key words: asymmetric equilibria, rank-order tournaments, risk taking.

JEL classification: C72, J3, L1, M5

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** Matthias Kräkel, Department of Economics, BWL II, University of Bonn, Ade-nauerallee 24-42, D-53113 Bonn, Germany, e-mail: m.kraekel@uni-bonn.de, phone: +49-228-739211, fax: +49-228-739210.
1 Introduction

In rank-order tournaments, players compete for given prizes. The best performing player (e.g. the one with the highest output) receives the highest prize, the second best performer gets the second highest prize and so on. Distribution of prizes according to relative performance creates considerable incentives for all contestants since ex ante each player wants to be declared winner of the tournament. There are many examples for rank-order tournaments in practice: sales representatives compete for bonuses which have been fixed in advance (Murphy et al. 2004), workers take part in job-promotion tournaments (Baker et al. 1994), athletes participate in sports contests (Szymanski 2003), lawyers compete in litigation contests (Wärneryd 2000), firms and individuals invest in external or internal rent-seeking contests (Gibbons 2005), managers receive relative performance pay (Gibbons and Murphy 1990), firms spend resources for advertising in winner-take-all markets (Schmalensee 1976), there are research tournaments (Schöttner 2007) and even tournaments in broiler production (Knoeber and Thurman 1994).

Theoretic models which analyze players’ behavior in rank-order tournaments1 typically focus on the effort or investment decision of contestants: The more input a player chooses relative to his opponents the higher will be his probability of winning the tournament. However, in real tournaments

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players can often also decide on the risk of their behavior. For example, before firms choose their advertising expenditures, they can decide on whether introducing a new product (high risk) or not (low risk). In many tournaments, contestants first have the choice between using a standard technique (low risk) or switching to a new technique (high risk); thereafter they decide on effort or, more generally, on input to win the tournament.

This paper addresses such a two-stage tournament game with risk taking at the first stage and effort or input choice at the second stage. We focus on heterogeneous and risk averse workers so that interior pure-strategy equilibria at the effort stage are always asymmetric. This asymmetric outcome has important consequences on those effects which mainly determine risk taking in tournaments: First, the choice of risk influences the equilibrium efforts at the second stage (effort effect). Second, risk also influences the players’ probabilities of winning (likelihood effect). Previous work on risk taking in two-stage tournaments has only considered symmetric equilibria at the effort stage 2. There, the effort effect and the likelihood effect were completely separate. However, this outcome will no longer hold, if players’ utility functions at least slightly differ. Then both effort effect and likelihood effect are strictly interrelated which can be crucial for risk taking. In particular, we can show that in this situation the favorite (underdog), i.e. the player with the higher (lower) probability of winning in equilibrium, may prefer high risk (low risk) in order to maximize his winning probability which is impossible under symmetry. This new effect is labeled reversed likelihood effect since
it works just contrary to the original likelihood effect. We also consider the role of risk aversion. Whereas under mean-variance preferences the more risk averse player always prefers maximum risk this result need not hold for alternative utility functions.

So far there is only a small number of papers which also address the problem of risk taking in two-stage tournaments. Hvide (2002) focuses on the case of homogeneous players. Since equilibrium efforts are always identical in his setting and the winning probability of each player is always one half irrespective of the risk level, basically there is no likelihood effect in the Hvide-model. Since equilibrium efforts are monotonically decreasing in risk, each player chooses maximum risk according to the effort effect so that players exert minimum effort at the second stage of the game. In the setting of Hvide (2002), maximizing risk at the first stage works like an implicit collusion for the effort choices at the second stage.

When introducing heterogeneity between the players, there are two possibilities in principle. Following O’Keeffe et al. (1984) we can differentiate between unfair and uneven tournaments. In an unfair two-person tournament, players choose identical efforts so that again we have a symmetric equilibrium like in the case of homogeneous contestants. However, one of the players has a lead and, hence, a higher probability of winning. In an uneven tournament, only asymmetric equilibria exist since players have different cost-of-effort functions or different preferences of winning (i.e. the subjective tournament prizes or the respective utilities of the players differ). Kräkel and
Sliwka (2004) combine the problem of risk taking with unfair tournaments. In their setting, equilibria at the effort stage are always symmetric like in Hvide (2002). However, now both effort and likelihood effect are important. Whether the players prefer high or low risk in order to reduce effort costs depends on the magnitude of the favorite’s lead. If the lead is small (large) both players are interested in choosing a high (low) risk in order to destroy overall incentives at the second stage according to the effort effect. Concerning the likelihood effect, there is an unambiguous result due to the symmetric equilibrium: The favorite (underdog) maximizes his winning probability by choosing low (high) risk. In this paper, we address risk taking by considering uneven tournaments in which only asymmetric interior equilibria can exist at the second stage. In this setting, it can be shown that the effort effect and the likelihood effect are strictly interrelated, which sharply contrasts with the findings for symmetric equilibria in Hvide (2002) and Kräkel and Sliwka (2004). This linkage between the two effects can lead to a so-called reversed likelihood effect under which the favorite – and not the underdog – prefers high risk in order to maximize his winning probability.

There are some other papers that also deal with risk taking in tournaments. In the models by Gaba and Kalra (1999), Hvide and Kristiansen (2003) and Taylor (2003), players can solely decide on risk taking in the tournament; hence there is no effort effect and no possible linkage with the likelihood effect. Other papers analyze risk taking empirically. For example, Becker and Huselid (1992) consider individual behavior in stock-car racing.
Their results show that drivers take more risk if tournament prizes and prize spreads are large. Knoeber and Thurman (1994) find out that more able contestants tend to choose less risky strategies. However, their empirical analysis is not based on a theoretical model that allows effort choices to react on risk. The findings of Brown et al. (1996) and Chevalier and Ellison (1997) point out that presumable losers – contrary to presumable winners – prefer high risks in tournaments between mutual fund managers. Finally, the paper by Grund and Gürtler (2005) on professional soccer confirms the previous findings that leading players or teams (players that lie behind) prefer low-risk (high-risk) behavior. However, neither of the empirical papers addresses the effort effect.

The paper is organized as follows. The next section introduces the model. Section 3 considers the effort stage. Section 4 focuses on risk taking at the first stage of the game; it contains the main results. In particular, the reversed likelihood effect is highlighted in Subsections 4.2 and 4.3. Section 5 concludes.

2 The Basic Model

Two risk averse players $A$ and $B$ participate in a two-stage tournament. Player $i$’s ($i = A, B$) production or performance function is given by

$$q_i = e_i + \varepsilon_i$$  \hspace{1cm} (1)
where $e_i$ denotes investment or effort chosen by player $i$. $\varepsilon_A$ and $\varepsilon_B$ are exogenous noise terms. The density of the composed random variable $\varepsilon_j - \varepsilon_i$ ($i, j = A, B; i \neq j$) is denoted by $g(\cdot)$ and the corresponding cumulative distribution function by $G(\cdot)$ which is assumed to be continuous and twice differentiable. The density $g(\cdot)$ is assumed to be symmetric around its unique mode at zero with variance $\sigma^2 = \sigma_i^2 + \sigma_j^2$. For example, $\varepsilon_i$ and $\varepsilon_j$ may be stochastically independent and normally distributed with $\varepsilon_i \sim N(\mu, \sigma_i^2)$ and $\varepsilon_j \sim N(\mu, \sigma_j^2)$. In that case, the convolution $g(\cdot)$ again is a normal distribution with $\varepsilon_j - \varepsilon_i \sim N(0, \sigma_i^2 + \sigma_j^2)$.

Each player $i$ ($i = A, B$) has a strictly concave utility function $U_i$ which is separable in monetary income, $I_i$, and effort costs, $c(e_i)$:

$$U_i = u_i(I_i) - c(e_i)$$

with $u_i'(I_i) > 0$, $u_i''(I_i) < 0$, $\forall I_i$, and $c'(e_i) > 0$, $c''(e_i) > 0$, $\forall e_i > 0$, and $c(0) = 0$. The utility functions are assumed to be common knowledge.

At the first stage of the tournament game (risk stage), both players $i$ and $j$ simultaneously choose their respective risks measured by the variances $\sigma_i^2$ and $\sigma_j^2$. At the second stage (effort stage), both players observe $\sigma_i^2$ and $\sigma_j^2$, and then simultaneously choose efforts $e_i$ and $e_j$. Effort choices together with the realizations of $\varepsilon_i$ and $\varepsilon_j$ determine $q_i$ and $q_j$ according to (1). If $q_i > q_j$, player $i$ is declared winner of the tournament and receives the monetary

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2 The assumption of a unimodal distribution is not unusual in tournament models; see, e.g., Dixit (1987), Drago et al. (1996), Hvide (2002), Chen (2003).
prize $I_i = w_1$, whereas player $j$’s monetary income is given by the loser prize $I_j = w_2 < w_1$ ($i, j \in \{A, B\}; i \neq j$).

3 Optimal Effort Choices

The subgame-perfect equilibrium of the two-stage game is derived by backward induction. First, the effort stage is considered where the players choose $e_i$ and $e_j$ in order to maximize their expected utilities $EU_i(e_i)$ and $EU_j(e_j)$, respectively, for given values of $\sigma_i^2$ and $\sigma_j^2$. Then we go backwards to the risk stage where the two players anticipate effort choices at stage 2, $e_i^* (\sigma_i^2, \sigma_j^2)$ and $e_j^* (\sigma_i^2, \sigma_j^2)$, and choose their respective variances $\sigma_i^2$ and $\sigma_j^2$.

In stage 2, player $i$ chooses $e_i$ to maximize

$$EU_i(e_i) = u_i(w_1) G(e_i - e_j) + u_i(w_2) [1 - G(e_i - e_j)] - c(e_i)$$

with $G(e_i - e_j) = \text{prob}\{q_i > q_j\} = \text{prob}\{\varepsilon_j - \varepsilon_i < e_i - e_j\}$ denoting $i$’s probability of winning. In analogy, player $j$’s objective function can be written as

$$EU_j(e_j) = u_j(w_1) [1 - G(e_i - e_j)] + u_j(w_2) G(e_i - e_j) - c(e_j).$$

If an interior equilibrium solution in pure-strategies, $(e_i^*, e_j^*)$, exists at the effort stage, it will be characterized by the players’ first-order conditions

$$\Delta u_i g (e_i^* - e_j^*) - c'(e_i^*) = 0$$

$$\Delta u_j g (e_i^* - e_j^*) - c'(e_j^*) = 0$$
with \( \Delta u_k := u_k (w_1) - u_k (w_2) \) \((k = i, j)\) describing player \( k \)'s additional utility from receiving more money in case of winning the tournament. Conditions (5) and (6) show that, in an interior pure-strategy equilibrium, the player with the larger utility spread \( \Delta u_k \) exerts more effort. Of course, if luck becomes extremely large so that effort has not any real impact on the outcome of the tournament, an interior solution will not exist. Especially, if \( \sigma^2 = \infty \) the winner of the tournament is determined by pure luck, and each player’s likelihood of winning is one half irrespective of the effort choices. Hence, in equilibrium both players choose zero efforts to minimize effort costs.

As it is known in the tournament literature, pure-strategy equilibria will only exist if there is sufficient noise in the tournament and the players’ cost function \( c (\cdot) \) is sufficiently steep (Lazear and Rosen 1981, 845). In particular, strict concavity of the players’ objective functions and, hence, existence of pure-strategy equilibria is assured if

\[
\sup_{\sigma^2, \Delta e} \Delta u_k \cdot |g' (\Delta e)| < \inf_{e > 0} c'' (e) \tag{7}
\]

with \( \Delta e := e_i - e_j \). Let \( \sigma^2_{\min} \) denote the respective value of \( \sigma^2 \) at the left-hand side of (7), i.e. \( \sigma^2_{\min} \) characterizes the minimum amount of risk necessary for assuring the existence of a pure-strategy equilibrium at the effort stage of the tournament game. We obtain the following results:

**Proposition 1** If \( c' (0) = 0 \) and \( \sigma^2_{\min} \leq \sigma^2 < \infty \), there will exist a pure-strategy equilibrium at the effort stage, \((e^*_i, e^*_j)\), being described by (5) and

\[3\] For a similar condition see Schöttner (2007).
Moreover, \( \text{sign} \left( \frac{\partial e^*_i}{\partial \sigma^2} \right) = \text{sign} \left( \frac{\partial e^*_j}{\partial \sigma^2} \right) \).

**Proof.** The first part of the proposition has already been proved. To see that \( \text{sign} \left( \frac{\partial e^*_i}{\partial \sigma^2} \right) = \text{sign} \left( \frac{\partial e^*_j}{\partial \sigma^2} \right) \) we have to implicitly differentiate the system of equations (5) and (6):

\[
\begin{align*}
\frac{\partial e^*_i}{\partial \sigma^2} &= \frac{1}{|J|} \Delta u_i \frac{\partial g \left( e^*_i - e^*_j \right)}{\partial \sigma^2} \cdot c'' \left( e^*_j \right) \quad (8) \\
\frac{\partial e^*_j}{\partial \sigma^2} &= \frac{1}{|J|} \Delta u_j \frac{\partial g \left( e^*_i - e^*_j \right)}{\partial \sigma^2} \cdot c'' \left( e^*_i \right) \quad (9)
\end{align*}
\]

with

\[ |J| := \text{EU}''_i \left( e_i \right) \text{EU}''_j \left( e_j \right) + \Delta u_i \Delta u_j \left[ g' \left( e^*_i - e^*_j \right) \right]^2 > 0 \]

denoting the Jacobian determinant. Comparing (8) and (9) yields

\[ \text{sign} \left( \frac{\partial e^*_i}{\partial \sigma^2} \right) = \text{sign} \left( \frac{\partial e^*_j}{\partial \sigma^2} \right) = \text{sign} \left( \frac{\partial g \left( e^*_i - e^*_j \right)}{\partial \sigma^2} \right) \].

(10)

Proposition 1 shows that marginally increasing risk makes both players either increase or decrease efforts. From the first-order conditions (5) and (6), we know that if one player chooses more (less) effort, the other player will react in the same way (i.e. \( c' \left( e^*_i \right) / \Delta u_i = c' \left( e^*_j \right) / \Delta u_j \)). However, it is interesting to see in which situations both players increase or decrease their efforts. Increasing risk means that the probability mass under the density \( g \left( \cdot \right) \) is shifted from the middle towards the tails of the distribution.

If the effort difference \( |e^*_i - e^*_j| \) is located near the mode, \( \partial g \left( e^*_i - e^*_j \right) / \partial \sigma^2 \) will be negative and the two players reduce their equilibrium efforts. If,
on the contrary, $|e^*_i - e^*_j|$ is large and, hence, lies at one of the tails, then
\[ \frac{\partial g(e^*_i - e^*_j)}{\partial \sigma^2} \] will be positive so that marginally increasing risk makes both players choose higher efforts in equilibrium. As an example consider $\varepsilon_j - \varepsilon_i \sim N(0, \sigma^2)$ which gives
\[ \frac{\partial g(e^*_i - e^*_j)}{\partial \sigma} = \exp \left\{ -\frac{(e^*_i - e^*_j)^2}{2\sigma^2} \right\} \cdot \frac{\left( \frac{(e^*_i - e^*_j)^2}{\sigma^2} \right) - 1}{\sigma^2 \sqrt{2\pi}}. \]

Thus, the derivative is positive (negative) if $|e^*_i - e^*_j| > (<) \sigma$. Note that the effort difference $e^*_i - e^*_j$ positively depends on the difference of the players’ utility spreads, $\Delta u_i - \Delta u_j$, which yields the following corollary:

**Corollary 1** If $|\Delta u_i - \Delta u_j|$ is sufficiently large (small), an increase in risk $\sigma^2$ will result in larger (smaller) equilibrium efforts $(e^*_i, e^*_j)$.

The intuition for this result is the following: If the players are very heterogeneous in the sense that their utility spreads $\Delta u_i$ and $\Delta u_j$ differ significantly, then competition is highly uneven resulting in low effort levels of both players – the player with the very low utility spread (the underdog) is hardly motivated and exerts very little effort so that the other player (the favorite) also chooses a very low effort level as best response. However, increasing risk in this situation brings the poorly motivated underdog back into the race, since higher uncertainty works against the uneven competition. This effect makes both contestants exert higher effort levels. Consider now the opposite case. If there is only a small degree of heterogeneity (i.e. $|\Delta u_i - \Delta u_j|$ is small) in the tournament, competition will be rather even and efforts rather
high. Increasing risk in such situation destroys incentives – the outcome of the tournament is rather determined by luck than by effort choice – so that equilibrium efforts fall.

4 Optimal Risk Taking

We differentiate between two situations. In the first scenario, each player can freely choose any variance which is equal or greater than $\sigma^2_{\text{min}}/2$. In the second scenario, each player can either choose low risk $\hat{\sigma}^2_L$ or high risk $\hat{\sigma}^2_H$ with $\hat{\sigma}^2_L < \hat{\sigma}^2_H$ and $2\hat{\sigma}^2_L \geq \sigma^2_{\text{min}}$.

4.1 The Continuous Case

In this subsection, player $i$ ($j$) chooses $\sigma^2_i$ ($\sigma^2_j$) at the first stage of the game with $\sigma^2_i, \sigma^2_j \geq \sigma^2_{\text{min}}/2$ anticipating that, at the second stage, efforts are chosen according to (5) and (6) in case of an interior solution, or that $e^*_i = e^*_j = 0$ in case of $\sigma^2 = \sigma^2_i + \sigma^2_j = \infty$. We obtain the following result:

**Proposition 2** At the first stage, only corner or semi-corner solutions exist in which the player with the lower utility spread $\Delta u_k$, $k \in \{i, j\}$, chooses $\sigma^2_k = \infty$ as weakly dominant strategy.

**Proof.** The non-existence of interior solutions can be shown by contradiction. In case of an interior solution at the second stage, the players’
objection functions at the first stage can be written as

\[
EU_i (\sigma_i^2) = u_i (w_2) + \Delta u_i \cdot G \left( e_i^* - e_j^*; \sigma_i^2 + \sigma_j^2 \right) - c (e_i^*)
\]

(11)

\[
EU_j (\sigma_j^2) = u_j (w_2) + \Delta u_j \cdot G \left( - [e_i^* - e_j^*]; \sigma_i^2 + \sigma_j^2 \right) - c (e_j^*)
\]

(12)

with \( G (\cdot; \sigma_i^2 + \sigma_j^2) \equiv G (\cdot) \), and \( e_i^* = e_i^* (\sigma_i^2 + \sigma_j^2) \) and \( e_j^* = e_j^* (\sigma_i^2 + \sigma_j^2) \) being described by (5) and (6). The first-order conditions yield

\[
\Delta u_i \left( g (\Delta e^*) \left( \frac{\partial e_i^*}{\partial \sigma_i^2} - \frac{\partial e_j^*}{\partial \sigma_j^2} \right) + \frac{\partial G (\Delta e^*)}{\partial \sigma_i^2} \right) = c' (e_i^*) \frac{\partial e_i^*}{\partial \sigma_i^2}
\]

(13)

\[
\Delta u_j \left( -g (-\Delta e^*) \left( \frac{\partial e_i^*}{\partial \sigma_i^2} - \frac{\partial e_j^*}{\partial \sigma_j^2} \right) + \frac{\partial G (-\Delta e^*)}{\partial \sigma_i^2} \right) = c' (e_j^*) \frac{\partial e_j^*}{\partial \sigma_j^2}
\]

(14)

with \( \Delta e^* := e_i^* - e_j^* \). Since \( g (\Delta e^*) = g (-\Delta e^*) \) (due to symmetry) and because of (5) and (6) (hence, the envelope theorem applies) we have that

\[
g (\Delta e^*) \frac{\partial e_j^*}{\partial \sigma_i^2} = \frac{\partial G (\Delta e^*)}{\partial \sigma_i^2}
\]

(15)

and

\[
g (\Delta e^*) \frac{\partial e_i^*}{\partial \sigma_j^2} = \frac{\partial G (-\Delta e^*)}{\partial \sigma_j^2}.
\]

(16)

However, since \( \frac{\partial \sigma_i^2}{\partial \sigma_i^2} = \frac{\partial \sigma_j^2}{\partial \sigma_j^2} \) so that \( \text{sign} \left( \frac{\partial e_i^*}{\partial \sigma_i^2} \right) = \text{sign} \left( \frac{\partial e_j^*}{\partial \sigma_j^2} \right) \) according to Proposition 1, and since \( \text{sign} \left( \frac{\partial G (\Delta e^*)}{\partial \sigma_i^2} \right) \neq \text{sign} \left( \frac{\partial G (-\Delta e^*)}{\partial \sigma_j^2} \right) = \text{sign} \left( \frac{\partial}{\partial \sigma_j^2} (1 - G (\Delta e^*)) \right) \), equations (15) and (16) cannot hold at the same time.

Hence, at least one player chooses either \( \sigma^2_{\text{min}} / 2 \) or \( \infty \) in equilibrium. We can show that the player with the smaller utility spread always weakly prefers \( \infty \). Let, w.l.o.g., \( \Delta u_i > \Delta u_j \). If \( i \) chooses \( \sigma_i^2 = \infty \), player \( j \) will be indifferent between all possible values of \( \sigma_j^2 \) since \( \sigma^2 = \sigma_i^2 + \sigma_j^2 \). If \( i \) chooses \( \sigma_i^2 < \infty \),

\[4\text{Note that } G (-x) = 1 - G (x) \text{ because of the symmetry of the convolution.}\]
player \( j \) will either have

\[
EU_j \left( \sigma_j^2 < \infty \right) = u_j(w_2) + \Delta u_j \cdot G(-\Delta e^*) - c(e_j^*)
\]

or

\[
EU_j \left( \sigma_j^2 = \infty \right) = u_j(w_2) + \frac{\Delta u_j}{2} - c(0)
\]

with \( \Delta e^* = e_i^* - e_j^* > 0 \) and \((e_i^*, e_j^*)\) being described by (5) and (6). Since \( G(-\Delta e^*) < 1/2 \), we have \( EU_j \left( \sigma_j^2 = \infty \right) > EU_j \left( \sigma_j^2 < \infty \right) \). □

The intuition for the result of Proposition 2 is the following: The underdog, i.e. the player with the lower utility spread, has always an incentive to completely erase competition at the second stage by choosing infinitely large risk. Note that risk taking has two effects. First, it influences a player’s effort choice and, hence, his effort costs (effort effect). Second, it determines the distribution \( G(\cdot) \) and therefore a player’s likelihood of winning (likelihood effect). For the underdog, both effects work into the same direction: By choosing infinite risk, effort incentives and effort costs are minimized. In addition, the underdog would exert less effort than the other player in case of an interior solution at the second stage and, therefore, would have a winning probability strictly smaller than one half. Infinitely high risk counterbalances the players’ winning probabilities so that each one wins with probability one half.

**Example 1: Mean-variance preferences**

To analyze how the players’ degree of risk aversion may influence their risk taking, let the utility function \( u_i(I_i) \) in (2) be further specified. We assume that both players have mean-variance preferences and hence a quadratic util-
ity function
\[ u_k(I_k) = I_k - r_k I_k^2 \quad (k = i, j) \] (17)

with \( r_k > 0 \) indicating player \( k \)'s degree of risk aversion and \( r_k < 1/(2I_k), \forall I_k \), which guarantees a non-decreasing utility function. By using (17), the expected utilities of the two players at the second stage for given variances \( \sigma_i^2 \) and \( \sigma_j^2 \) can be written as

\[
EU_i(e_i) = E[I_i] - r_i E[I_i^2] - c(e_i) \\
= w_2 - r_i w_2^2 + \left[ \Delta w - r_i \left( w_i^2 - w_2^2 \right) \right] G(\Delta e) - c(e_i)
\]
\[
EU_j(e_j) = w_2 - r_j w_2^2 + \left[ \Delta w - r_j \left( w_i^2 - w_2^2 \right) \right] G(-\Delta e) - c(e_j)
\]

with \( \Delta e = e_i - e_j \) and \( \Delta w = w_1 - w_2 \). The first-order conditions for an interior solution at stage 2 are

\[
\left[ \Delta w - r_i \left( w_i^2 - w_2^2 \right) \right] g(\Delta e) = c'(e_i) \quad (18)
\]
\[
\left[ \Delta w - r_j \left( w_i^2 - w_2^2 \right) \right] g(\Delta e) = c'(e_j). \quad (19)
\]

By redefining \( \Delta u_k := [\Delta w - r_k (w_i^2 - w_2^2)] \) and \( u_k(w_2) := w_2 - r_k w_2^2 \) in the previous results, particularly in Propositions 1 and 2, we immediately get the following corollary:

**Corollary 2** If the two players have mean-variance preferences according to (17), the player with the higher degree of risk aversion \( r_k, k \in \{i, j\} \), will choose maximum risk \( \sigma_k^2 = \infty \) as weakly dominant strategy at stage 1.

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6Recall that \( g(\Delta e) = g(-\Delta e) \) due to symmetry. Note that the terms in brackets are strictly positive because of the technical assumption \( r_k < 1/(2I_k), \forall I_k \), above.
At first sight, it sounds counterintuitive that the more risk averse player unambiguously prefers to maximize risk. However, note that chosen risk is not identical with the players’ income risks. Inspection of (18) and (19) gives the intuition for the finding of Corollary 2: The more risk averse player has the lower marginal return of winning, $[\Delta w - r_k (w^2_1 - w^2_2)] g (\Delta e) (k \in \{i, j\})$, and therefore lower incentives at the second stage.\(^7\) Thus, he is the underdog who chooses maximum risk in order to minimize effort costs (effort effect) and to maximize his likelihood of winning (likelihood effect). However, note that the result of Corollary 2 need not hold for all possible utility functions as the player with the lower utility spread $\Delta u_k, k \in \{i, j\}$, is not necessarily the more risk averse individual.

**Example 2: Normally distributed noise and finite risk**

Let $\varepsilon_j - \varepsilon_i \sim N(0, \sigma^2_i + \sigma^2_j)$. To show that the underdog’s preference for maximum risk crucially depends on the possibility of choosing infinitely high risk, in this example we assume that risk choice is limited to $\sigma^2_{\text{max}}/2$ for each player so that total maximum risk $\sigma^2 = \sigma^2_i + \sigma^2_j = \sigma^2_{\text{max}}$ still leads to an interior solution at the effort stage.\(^8\) Hence, at stage 1, the players have to choose $\sigma^2_i$ and $\sigma^2_j$ from the interval $\left[\frac{\sigma^2_{\text{min}}}{2}, \frac{\sigma^2_{\text{max}}}{2}\right]$. To allow for explicit equilibrium solutions, let effort costs be exponential $c(e_k) = \exp \{c \cdot e_k\} -

\(^7\)Note that player $i$’s expected risk costs $E [r_i I^2_i] = r_i (w^2_1 + (w^2_1 - w^2_2) G (\Delta e))$ monotonically increase in $i$’s effort level. Analogous considerations hold for player $j$.

\(^8\)The calculations in the appendix show that equilibrium efforts will only be positive for risk being not too large.
1 with \( c > 0 \) (\( k = i, j \)).9 W.l.o.g., let player \( i \) be the favorite and \( j \) the underdog, i.e. \( \Delta u_i > \Delta u_j \). Simple computations (see Appendix) show that, in stage 1, we have

\[
\frac{\partial EU_i}{\partial \sigma^2_i} = \frac{\Delta u_i}{2c\sigma^3} \left( 1 - \frac{(\Delta e^*)^2}{\sigma^2} - c \cdot \Delta e^* \right) \phi \left( \frac{\Delta e^*}{\sigma} \right) \tag{20}
\]

\[
\frac{\partial EU_j}{\partial \sigma^2_j} = \frac{\Delta u_j}{2c\sigma^3} \left( 1 - \frac{(\Delta e^*)^2}{\sigma^2} + c \cdot \Delta e^* \right) \phi \left( \frac{\Delta e^*}{\sigma} \right) \tag{21}
\]

with \( \phi (\cdot) \) denoting the density of the standardized normal distribution and \( \Delta e^* = \frac{1}{c} \ln \left( \frac{\Delta u_i}{\Delta u_j} \right) = \text{const} \). Comparing (20) and (21) immediately shows that both derivatives cannot be zero at the same time. In both equations, the expression in parentheses is monotonically increasing in \( \sigma^2 \). Hence, if stationary points exist for \( \sigma^2_i, \sigma^2_j \in \left[ \frac{\sigma^2_{\min}}{2}, \frac{\sigma^2_{\max}}{2} \right] \), they will correspond to (local) minima so that we have strict corner solutions in the given setting. Interestingly, depending on the parameter values there may be an equilibrium in which both players prefer minimum risk: If, for example, \( c \cdot \Delta e^* > 1 \Leftrightarrow \ln \left( \frac{\Delta u_i}{\Delta u_j} \right) > 1 \), the favorite’s objective function \( EU_i (\sigma^2_i) \) is monotonically decreasing so that \( i \) chooses \( \sigma^2_{\min} / 2 \) as a dominant strategy. Note that (21) has a unique root

\[
\hat{\sigma}^2_j = \frac{\Delta e^*}{1 + c \cdot \Delta e^*} - \sigma^2_i.
\]

If \( \hat{\sigma}^2_j > \sigma^2_{\max} / 2 \) for \( \sigma^2_i = \sigma^2_{\min} / 2 \), then underdog \( j \) will also choose minimum risk \( \sigma^2_j = \sigma^2_{\min} / 2 \).10

---

9 Exponential cost functions are also used elsewhere in the literature. See, for example, Tadelis (2002), Kräkel and Sliwka (2004).

10 A necessary condition for this outcome is that \( \frac{\Delta e^*}{\Delta u_j} - \frac{\sigma^2_{\min}}{\Delta u_j} > \frac{\sigma^2_{\min}}{\Delta u_j} \Leftrightarrow \frac{\Delta e^*}{\Delta u_j} > \sigma^2_{\min} \)

\[
\sigma^2_{\min} = \frac{\Delta u_i \exp \left( -\frac{1}{2} \right) - \Delta u_k \exp \left( -\frac{1}{2} \right) \left( 1 + \ln \left( \frac{\Delta u_i}{\Delta u_j} \right) \right)}{\sqrt{\pi}^2} > 0 \text{ which can be satisfied.}
To sum up, the discussion has shown that the finding of Hvide (2002) for homogeneous players that contestants prefer maximum risk does not in general apply to the case of heterogeneous players. The last example has shown that, on the contrary, both favorite and underdog may choose minimum risk in equilibrium. However, the most important topic – the interrelation of the effort effect and the likelihood effect – should be highlighted in more details in the next subsection.

4.2 A Reversed Likelihood Effect

Basically, two effects influence a player’s risk taking in tournaments. First, the choice of risk determines the players’ incentives at the effort stage (effort effect). If initially competition is rather uneven (i.e. $\Delta u_i$ and $\Delta u_j$ substantially differ) increasing risk brings the underdog back into the race so that overall incentives are restored. However, given a rather even competition increasing risk decreases both players’ incentives in equilibrium. The working of the effort effect has been highlighted in Section 3. Second, there is also a likelihood effect as choosing risk influences both players’ likelihoods of winning the tournament. Since an increase of player $i$’s probability of winning automatically reduces $j$’s probability and vice versa we can concentrate on one of the players. In this subsection, we will focus on player $i$’s likelihood of winning. Let, w.l.o.g., again $\Delta u_i > \Delta u_j$ so that we can speak of player $i$ as the favorite and player $j$ as the underdog. From the discussion so far we know that there is a double impact of overall risk $\sigma^2 = \sigma^2_i + \sigma^2_j$ on $i$’s
winning probability or, in other words, that effort effect and likelihood effect are interrelated: $\sigma^2$ influences both the shape of the cumulative distribution function $G(\cdot)$ and the position $\Delta e^* = e^*_i - e^*_j$ at which the winning probability is computed, namely the spread between the equilibrium efforts. Therefore, $i$’s winning probability can be written as $G(\Delta e^*(\sigma^2); \sigma^2)$.

Interestingly, in the two papers which have focused on symmetric equilibria at the effort stage, Hvide (2002) and Kräkel and Sliwka (2004), an interrelationship between effort and likelihood effect does not exist. In the Hvide paper, we have $G(0; \sigma^2) = \frac{1}{2}, \forall \sigma^2$. Since $i$’s winning probability does not depend on risk taking there is a zero likelihood effect. In the Kräkel-Sliwka paper, $i$’s winning probability is given by $G(\Delta a; \sigma^2) > \frac{1}{2}$ with $\Delta a = a_i - a_j = \text{const} > 0$ denoting the exogenously given ability difference between the favorite and the underdog. In that setting, risk taking only influences the shape of $G(\cdot)$. Since increasing risk makes $G(\cdot)$ "rotate clockwise"\textsuperscript{11} in the point $(0, 1/2)$ and we have $\Delta a > 0$, higher risk unambiguously decreases the favorite’s winning probability. Hence, considering the likelihood effect, the favorite always prefers less risk to more risk.

In this subsection, we will show that in the asymmetric setting considered in this paper where players’ utility functions at least slightly differ the interrelationship between the effort effect and the likelihood effect can re-

\textsuperscript{11}More precisely but, perhaps, less illustratively, the cumulative distribution function flattens at both sides since probability mass is shifted from the region around the mean to the tails.
result in a *reversed likelihood effect*. This means that a higher risk $\sigma^2$ leads to an *increased* winning probability of favorite $i$ which is impossible in the symmetric settings of Hvide and Kräkel-Sliwka.\textsuperscript{12}

**Definition** There is a reversed likelihood effect if $\frac{dG(\Delta e^*(\sigma^2);\sigma^2)}{d\sigma^2} > 0$, that is if

$$g(\Delta e^*(\sigma^2);\sigma^2) \cdot \Delta e''^*(\sigma^2) + \frac{\partial G(\Delta e;\sigma^2)}{\partial \sigma^2}\bigg|_{\Delta e = \Delta e^*(\sigma^2)} > 0. \quad (22)$$

The left-hand side of the inequality emphasizes the double impact of risk taking on $i$’s winning probability. The second expression characterizes the influence of increased risk on the shape of the cumulative distribution function at a certain position $\Delta e = \Delta e^*(\sigma^2)$. This effect is typically negative as we already know from Kräkel and Sliwka (2004) since probability mass is shifted from the mean to the tails of the distribution so that $G(\cdot)$ "rotates clockwise". However, the first expression at the left-hand side, which is always zero in the symmetric setting of Hvide and Kräkel-Sliwka because of $\Delta e''^*(\sigma^2) = 0$, may be positive. In particular, since $g(\Delta e^*(\sigma^2);\sigma^2) > 0$ the first expression will be positive if and only if $\Delta e''^*(\sigma^2) > 0$. In the following paragraph, we show that this will happen if the two players are sufficiently heterogeneous, that is if $\Delta u_i - \Delta u_j$ is sufficiently large. Altogether, if the first expression in (22) is positive and dominates the influence of the second expression, the overall effect will be positive.

\textsuperscript{12}$\Delta e''^*(\sigma^2)$ denotes the derivative of the equilibrium effort spread with respect to risk, $d\Delta e^*(\sigma^2)/d\sigma^2$. 
We can use a concrete probability distribution and a familiar class of cost functions to show that, in a realistic setting, the signs of the two expressions can indeed be as postulated before:

**Proposition 3** Let risk taking be limited so that an interior solution exists at the effort stage. Furthermore, let noise be normally distributed (i.e. \( \varepsilon_j - \varepsilon_i \sim N(0, \sigma^2) \)) and the players’ cost functions \( c(e_k) \) \((k = i, j)\) belong to the family of polynomial cost functions \( c(e_k) = \frac{\gamma}{\delta+1} e_k^{\delta+1} \) with \( \gamma > 0 \) and \( \delta \geq 1 \). There will be a reversed likelihood effect if and only if

\[
\Delta e''(\sigma^2) > \frac{\Delta e^*(\sigma^2)}{2\sigma^2}. \tag{23}
\]

If \( \Delta u_i - \Delta u_j \) is sufficiently large, \( \Delta e''(\sigma^2) \) will be positive.

**Proof.** Favorite \( i \)'s probability of winning is given by

\[
G(\Delta e^*(\sigma^2) ; \sigma^2) = \int_{-\infty}^{\Delta e^*(\sigma^2)} \frac{1}{\sqrt{\sigma^2 \sqrt{2\pi}}} \exp \left\{ -\frac{y^2}{2\sigma^2} \right\} dy.
\]

Differentiation with respect to \( \sigma^2 \) (applying Leibniz’s formula) gives

\[
\begin{aligned}
\frac{\Delta e''(\sigma^2)}{\Delta e^*(\sigma^2)} &\cdot \frac{\Delta e^*(\sigma^2)}{\Delta e^*(\sigma^2)} \\
+ &\int_{-\infty}^{-\infty} \left\{ -\frac{1}{2(\sigma^2)^{3/2} \sqrt{2\pi}} \exp \left\{ -\frac{y^2}{2\sigma^2} \right\} + \frac{1}{\sqrt{\sigma^2} \sqrt{2\pi} 2\sigma^4} \exp \left\{ -\frac{y^2}{2\sigma^2} \right\} \right\} dy \\
= &\frac{\Delta e''(\sigma^2)}{\Delta e^*(\sigma^2)} \cdot \frac{\Delta e^*(\sigma^2)}{\Delta e^*(\sigma^2)} \\
+ &\int_{-\infty}^{-\infty} \left\{ -\frac{1}{2\sigma^2} g(y, \sigma^2) + \left[ g'(y; \sigma^2) \left( -\frac{y}{2\sigma^2} \right) \right] \right\} dy.
\end{aligned}
\]
Integrating by parts the expression in square brackets yields

\[
\begin{align*}
\Delta e''(\sigma^2) & \cdot g(\Delta e^*(\sigma^2);\sigma^2) \\
- \frac{1}{2\sigma^2} G(\Delta e^*(\sigma^2);\sigma^2) & - \frac{\Delta e^*(\sigma^2)}{2\sigma^2} g(\Delta e^*(\sigma^2);\sigma^2) + \frac{1}{2\sigma^2} G(\Delta e^*(\sigma^2);\sigma^2) \\
= \Delta e''(\sigma^2) & \cdot g(\Delta e^*(\sigma^2);\sigma^2) - \frac{\Delta e^*(\sigma^2)}{2\sigma^2} g(\Delta e^*(\sigma^2);\sigma^2)
\end{align*}
\]

which is positive if

\[
\Delta e''(\sigma^2) > \frac{\Delta e^*(\sigma^2)}{2\sigma^2}.
\]

The polynomial cost function and \(e_i^*(\frac{\sigma^2}{\sigma}) = e_j^*(\frac{\sigma^2}{\sigma})\) (see (5) and (6)) lead to

\[
\Delta e^*(\sigma^2) = \left(1 - \left(\frac{\Delta u_j}{\Delta u_i}\right)^\frac{1}{\sigma^2} \right) e_i^*(\sigma^2).
\]

Hence, \(\text{sign}(\Delta e''(\sigma^2)) = \text{sign}\left(\frac{\partial e^*_i}{\partial \sigma^2}\right)\). As we know from Corollary 1, this sign is positive if \(\Delta u_i - \Delta u_j\) is sufficiently large.

Note that the reversed likelihood effect has important implications. In the symmetric setting, the favorite would never prefer to choose high risk from a likelihood perspective since he does not want to jeopardize his beneficial position in the competition. However, if, in an asymmetric setting, the reversed likelihood effect works this argument will not hold any longer. Now increasing risk may raise favorite \(i\)'s winning probability. For the underdog \(j\), a reversed likelihood effect means that \(\frac{\sigma^2}{\sigma} [1 - G(\Delta e^*(\sigma^2);\sigma^2)] < 0\), i.e. decreasing risk increases his probability of winning the tournament, contrary to the symmetric setting. Note that, particularly in the context of Proposition 3, the underdog has strong incentives to reduce risk. First of all, reducing risk leads to an increased winning probability according to the reversed
likelihood effect. Second, under the assumptions of Proposition 3 we have 
\( \text{sign}(\Delta e^*(\sigma^2)) = \text{sign}\left(\frac{\partial e^*}{\partial \sigma^2}\right) \) = \( \text{sign}\left(\frac{\partial e^*}{\partial \sigma^2}\right) > 0 \) (see Proposition 1 for the last equality). Hence, reducing risk would also lead to decreased effort and, therefore, decreased effort costs for the underdog. In other words, now the reversed likelihood effect and the effort effect work into the same direction for the underdog and make choosing low risk rather attractive for him. Again, this outcome is impossible in the symmetric setting.

For the remainder of this paper, two assumptions will be modified which do not seem to be very realistic. First, we skip the assumption of Subsection 4.1 that players can choose infinite risk. Second, now we assume that each player can only choose between two discrete risk levels – low risk and high risk. Both modifications are in line with the examples of Section 1 (e.g. new product introduction, stock-car racing). Before we will give a complete solution of the game with discrete risk choice in the next subsection, we first characterize the reversed likelihood effect in the discrete risk model:

**Proposition 4** Consider two levels of overall risk \( \sigma^2 \), namely \( \sigma^2_H \) and \( \sigma^2_L \) with \( \sigma^2_H > \sigma^2_L \). Favorite i’s winning probability will be larger under high risk \( \sigma^2_H \) than under low risk \( \sigma^2_L \) if and only if

\[
\Delta e^* (\sigma^2_H) > \frac{\sigma_H}{\sigma_L} \Delta e^* (\sigma^2_L).
\]

**Proof.** The favorite’s likelihood of winning will be larger under the higher risk if and only if

\[
G \left( \Delta e^* (\sigma^2_H) ; \sigma^2_H \right) > G \left( \Delta e^* (\sigma^2_L) ; \sigma^2_L \right).
\] (24)
Recall that $G(\cdot)$ describes the cdf of $\varepsilon_j - \varepsilon_i$ with mean zero and variance $\sigma^2$. Let $\tilde{G}(\cdot)$ be the cdf of the corresponding standardized random variable $\Delta \tilde{\varepsilon} := \frac{\varepsilon_j - \varepsilon_i}{\sigma}$ with mean 0 and variance 1. Hence, condition (24) is equivalent to

$$\tilde{G}\left(\frac{\Delta e^*(\sigma_H^2)}{\sigma_H}; 1\right) > \tilde{G}\left(\frac{\Delta e^*(\sigma_L^2)}{\sigma_L}; 1\right)$$

which simplifies to $\frac{\Delta e^*(\sigma_H^2)}{\sigma_H} > \frac{\Delta e^*(\sigma_L^2)}{\sigma_L}$ as $\tilde{G}(\cdot)$ is a monotonically increasing function. $\blacksquare$

To see that the inequality of Proposition 4 can indeed be satisfied we use the following example:

**Example 3: Normally distributed noise and heterogeneous exponential costs**

Let $\varepsilon_j - \varepsilon_i \sim N(0, \sigma^2)$. Consider again two alternative levels of overall risk, $\sigma_H^2$ and $\sigma_L^2$ with $\sigma_H^2 > \sigma_L^2$. Let $\sigma_L^2 \geq \sigma_{\min}^2$ so that total risk $\sigma^2 = \sigma_i^2 + \sigma_j^2$ again leads to an interior solution at the effort stage. In order to derive explicit solutions for the equilibrium efforts, let effort costs be exponential $c(c_k) = \exp\{c_k \cdot e_k\} - 1$ with $c_k > 0$ and $c_i \neq c_j$ ($k = i, j$).\(^{13}\) Let $\Delta u_i > \Delta u_j$ and $c_i < c_j$ so that again player $i$ is the favorite and $j$ the underdog. In this

\(^{13}\)Hence, we slightly modify the cost function of example 2. With identical exponential cost functions, we would have $\Delta e^* = const$ so that effort effect and likelihood effect would be completely separate as in the symmetric setting.
setting, the condition of Proposition 4 becomes

\[
c_i c_j (\sigma_H - \sigma_L) > \sqrt{2 (c_j - c_i) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma_H \sqrt{2 \pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma_H \sqrt{2 \pi}} \right) \right)} + \sigma_H^2 c_i^2 c_j^2
\]

which is satisfied for many feasible parameter constellations, e.g. for \( c_i = 20 \), \( c_j = 22 \), \( \Delta u_i = 1.1 \), \( \Delta u_j = 1 \), \( \sigma_L = 2 \) and \( \sigma_H = 2.2 \).

The reversed likelihood effect is illustrated by Figure 1.

[Figure 1]

Increasing risk from \( \sigma_L^2 \) to \( \sigma_H^2 \) leads to an effort difference \( \Delta e^* (\sigma_H^2) \) that is sufficiently large so that the favorite’s winning probability increases by increased risk despite the fact that the cdf "rotates clockwise". From the figure as well as from the condition of Proposition 4 it becomes clear that \( \Delta e^* (\sigma_H^2) > \Delta e^* (\sigma_L^2) \) is a necessary condition for the revised likelihood effect to work. The left-hand side of the distribution in Figure 1 shows that the underdog benefits from the reversed likelihood effect by both an increased winning probability and a decreased effort difference if risk is reduced from \( \sigma_H^2 \) to \( \sigma_L^2 \).

14 The calculations can be requested from the author.

15 Note that the existence condition for interior solutions at the effort stage, (7), simplifies in the given example to \( \frac{\Delta u_i}{\sqrt{2 \pi \sigma_L^2}} \exp \left( -\frac{1}{2} \right) < c_i^2 \) which is clearly satisfied by the numerical example.
Finally, we can have a short look at the equilibrium candidates in the risk stage of the two-stage game with discrete risk choice in which either player can choose a low or a high risk level. Consider, for example, a possible low-risk equilibrium in which both players prefer the low risk level. Let \( \sigma^2_L \) denote the corresponding overall risk and \( \sigma^2_H > \sigma^2_L \) the higher overall risk if one of the players deviates to the high risk level. Furthermore, let the assumptions of Proposition 3 be satisfied: normally distributed noise, polynomial cost function (which — together with Proposition 1 — ensures that \( \text{sign} \left( \frac{\partial e^*_i}{\partial \sigma^2} \right) = \text{sign} \left( \frac{\partial e^*_j}{\partial \sigma^2} \right) = \text{sign} \left( \Delta e^*(\sigma^2) \right) \)), and \( \Delta u_i - \Delta u_j \) being sufficiently large. More precisely, concerning the last assumption let \( \Delta \tilde{e} > 0 \) denote the unique intersection between \( g(\cdot; \sigma^2_L) \) and \( g(\cdot; \sigma^2_H) \) at the right-hand side of the distributions, that is \( g(\Delta \tilde{e}; \sigma^2_L) = g(\Delta \tilde{e}; \sigma^2_H) \), which implies that \( g(\cdot; \sigma^2_L) \) lies above (below) \( g(\cdot; \sigma^2_H) \) for arguments smaller (larger) than \( \Delta \tilde{e} \) (see also Figure 1), and let \( \Delta u_i - \Delta u_j \) be so large that \( \Delta e^*(\sigma^2_L) \geq \Delta \tilde{e} \).

Note that, in the given setting, marginally increasing risk from \( \sigma^2_L \) to higher levels unambiguously leads to increased values of \( e^*_i, e^*_j \) and \( \Delta e^* \), and that a non-marginal risk increase from \( \sigma^2_L \) to \( \sigma^2_H \) will also increase these three values: Notice that any intersection \( \Delta \tilde{e} > 0 \) of two normal densities \( g(\cdot; \sigma^2_L) \) and \( g(\cdot; \sigma^2_H) \) with \( \sigma^2_H > \sigma^2_L \) can be explicitly calculated as follows:

\[
g(\Delta \tilde{e}; \sigma^2_L) = g(\Delta \tilde{e}; \sigma^2_H) \Leftrightarrow \Delta \tilde{e} = \sqrt{\frac{\sigma^2_H}{\sigma^2_H - \sigma^2_L} \cdot \ln \left( \frac{\sigma^2_H}{\sigma^2_L} \right)}.
\]

(25)

Further, note that \( \Delta \tilde{e} \) monotonically increases in \( \sigma^2_H \).\(^{16}\) Hence, increasing

\(^{16}\)To see this, substitute \( t \cdot \sigma^2_L \) with \( t > 1 \) for \( \sigma^2_H \) in the expression under the square root
risk marginally from $\sigma_L^2$ to $\sigma_H^2$ shifts both the intersection $\Delta \tilde{e}$ and the equilibrium effort difference $\Delta e^*$ to the right (note that $\Delta e^*(\sigma_L^2) \geq \Delta \tilde{e}$ so that $\text{sign}\left(\frac{\partial g}{\partial \sigma^2}\right) > 0$ for any marginal risk increase) but we have $\Delta \tilde{e} < \Delta e^*$ since $\Delta e^*(\sigma_L^2) \geq \Delta \tilde{e}$ in the initial situation. Altogether, these assumptions guarantee that the necessary condition for the reversed likelihood effect to work, namely $\Delta e^*(\sigma_H^2) > \Delta e^*(\sigma_L^2)$, is always satisfied. Of course, it is not guaranteed that the increased effort difference always dominates the flattening of the cumulative distribution function followed by increased risk. Hence, the imposed restrictions are not sufficient for the existence of the reversed likelihood effect.

Now, we come back to the possible low-risk equilibrium. This equilibrium will exist if and only if

$$\Delta u_i G \left( \Delta e^*(\sigma_L^2) ; \sigma_L^2 \right) - c \left( e_i^* (\sigma_L^2) \right) \geq \Delta u_i G \left( \Delta e^*(\sigma_H^2) ; \sigma_H^2 \right) - c \left( e_i^* (\sigma_H^2) \right)$$

and

$$\Delta u_j G \left( -\Delta e^*(\sigma_L^2) ; \sigma_L^2 \right) - c \left( e_j^* (\sigma_L^2) \right) \geq \Delta u_j G \left( -\Delta e^*(\sigma_H^2) ; \sigma_H^2 \right) - c \left( e_j^* (\sigma_H^2) \right),$$

which can be combined to

$$\frac{c \left( e_i^* (\sigma_H^2) \right) - c \left( e_i^* (\sigma_L^2) \right)}{\Delta u_i} \geq \frac{c \left( e_j^* (\sigma_L^2) \right) - c \left( e_j^* (\sigma_H^2) \right)}{\Delta u_j} \geq G \left( \Delta e^*(\sigma_H^2) ; \sigma_H^2 \right) - G \left( \Delta e^*(\sigma_L^2) ; \sigma_L^2 \right)$$

in (25) which then boils down to $\frac{t}{t-1} \sigma_L^2 \ln t$. This expression is strictly increasing in $t$. 

27
since the distribution is symmetric around zero so that $G(-x) = 1 - G(x)$. The assumptions above imply that $c(e_i^* (\sigma_H^2)) > c(e_j^* (\sigma_L^2))$ and $c(e_j^* (\sigma_H^2)) < c(e_j^* (\sigma_L^2))$. Hence, if the reversed likelihood effect applies (i.e. $G(\Delta e^* (\sigma_H^2); \sigma_H^2) > G(\Delta e^* (\sigma_L^2); \sigma_L^2)$) then the underdog $j$ will never deviate from the low-risk situation but it is not clear whether the favorite $i$ will deviate. This outcome sharply contrasts with the findings for the symmetric setting but is intuitively plausible in our asymmetric setting. In case of a reversed likelihood effect, the underdog benefits from a low-risk situation by both a low effort level (and hence low effort costs) and a rather high winning probability. However, the favorite faces a clear trade-off. On the one hand, the overall low risk ensures that the highly uneven competition characterized by the large difference $\Delta u_i - \Delta u_j$ remains uneven which reduces overall incentives and, therefore, his effort costs. On the other hand, reducing risk from $\sigma_H^2$ to $\sigma_L^2$ implies that $i$’s equilibrium effort level decreases more than $j$’s equilibrium effort\footnote{Recall that $i$ still exerts more effort than $j$ because of $\Delta u_i > \Delta u_j$.} so that $\Delta e^* (\sigma_L^2) < \Delta e^* (\sigma_H^2)$ which results in a decreased winning probability for $i$ due to the reversed likelihood effect. If the reversed likelihood effect does not apply, we have just the opposite situation in which the favorite never deviates from the low-risk situation which exactly describes the findings of Hvide and Kräkel-Sliwka for the symmetric setting.\footnote{However, note that we may have multiple equilibria in this situation.}

To sum up, the last two paragraphs have indicated that a complete equilibrium analysis of the whole two-stage game is impossible without further...
specifying the probability distribution for the underlying noise and/or the players’ cost functions. For this reason, the next subsection considers a framework with discrete effort choice. However, since no further assumptions are made for the specification of the probability distribution the analysis still offers some general results and nicely illustrates the forces at play.

4.3 The Discrete Case

In this subsection, the two players choose between discrete risk levels and between discrete effort levels which allows for a complete equilibrium analysis of the whole two stage-game. At stage 1, players $i$ and $j$ simultaneously choose $\sigma_i^2, \sigma_j^2 \in \{\hat{\sigma}_L^2, \hat{\sigma}_H^2\}$ (with $\hat{\sigma}_L^2 < \hat{\sigma}_H^2$) so that overall risk $\sigma^2 = \sigma_i^2 + \sigma_j^2$ is either low ($\sigma^2 = \sigma_L^2 := 2\hat{\sigma}_L^2$), or of medium size ($\sigma^2 = \sigma_M^2 := \hat{\sigma}_L^2 + \hat{\sigma}_H^2$), or high ($\sigma^2 = \sigma_H^2 := 2\hat{\sigma}_H^2$) with $\sigma_L^2 < \sigma_M^2 < \sigma_H^2$. At stage 2, both players observe $\sigma^2$ and then simultaneously choose their efforts $e_i(\sigma^2), e_j(\sigma^2) \in \{0, \hat{e}\}$ with $\hat{e} > 0$. Whereas zero effort is for free, exerting positive effort $\hat{e}$ leads to positive costs $\hat{c} > 0$ for the respective player. Let, w.l.o.g., $\Delta u_i > \Delta u_j$ so that player $i$ is the favorite and $j$ the underdog. We assume that $\hat{c} \neq (G(\hat{e}; \sigma^2) - \frac{1}{2}) \Delta u_i$ and $\hat{c} \neq (G(\hat{e}; \sigma^2) - \frac{1}{2}) \Delta u_j, \forall \sigma^2$, so that both players have strict preferences when choosing effort in stage 2. Finally, we assume that increasing risk shifts probability mass from the mean to the tails so that $G(\hat{e}; \sigma_L^2) > G(\hat{e}; \sigma_M^2) > G(\hat{e}; \sigma_H^2)$.

We solve the game by backward induction starting with the players’ effort choices at stage 2 for a given overall risk $\sigma^2$. If both players exert zero effort,
their respective payoffs will be \( u_i (w_2) + \frac{\Delta u_i}{2} \) and \( u_j (w_2) + \frac{\Delta u_j}{2} \). If both choose positive efforts, payoffs are given by \( u_i (w_2) + \frac{\Delta u_i}{2} - \hat{c} \) and \( u_j (w_2) + \frac{\Delta u_j}{2} - \hat{c} \).

If player \( i \) chooses \( e_i = \hat{e} \) but player \( j \) the effort level \( e_j = 0 \), then \( i \) gets \( u_i (w_2) + \Delta u_i G (\hat{e}; \sigma^2) - \hat{c} \) whereas \( j \)'s payoff is \( u_j (w_2) + \Delta u_j G (-\hat{e}; \sigma^2) = u_j (w_2) + \Delta u_j [1 - G (\hat{e}; \sigma^2)] \).\(^{19}\) Interchanging the subscripts "\( i \)" and "\( j \)" in the last two payoffs describes the outcome when \( i \) chooses zero effort and \( j \) a positive effort level. Let

\[
\Delta G (\sigma^2) := G (\hat{e}; \sigma^2) - \frac{1}{2} > 0
\]

denote a player's additional winning probability from exerting positive effort when his opponent chooses zero effort. Note that \( \Delta G (\sigma^2_L) > \Delta G (\sigma^2_M) > \Delta G (\sigma^2_H) \).

Comparing the four possible outcomes of the game at the effort stage, shows that there are three unique equilibria \( (e^*_i, e^*_j) \) depending on the given parameter constellation:

\[
(e^*_i, e^*_j) = \begin{cases} 
(\hat{e}, \hat{e}) & \text{if } \hat{c} < \Delta G (\sigma^2) \Delta u_j \\
(\hat{e}, 0) & \text{if } \Delta G (\sigma^2) \Delta u_j < \hat{c} < \Delta G (\sigma^2) \Delta u_i \\
(0, 0) & \text{if } \hat{c} > \Delta G (\sigma^2) \Delta u_i.
\end{cases}
\]

If effort costs are low for both players relative to the expected additional utility of winning the tournament, each player exerts high effort. However, both will choose zero effort if even for the favorite the expected gain of choosing high effort does not cover his additional effort costs. For intermediate

\(^{19}\)Note that \( G (-\hat{e}; \sigma^2) < \frac{1}{2} < G (\hat{e}; \sigma^2), \forall \sigma^2.\)
cost levels only the favorite prefers a high effort level whereas the underdog chooses zero effort.

At stage 1, both players choose risk while anticipating one of the possible equilibria \((e_i^*, e_j^*)\) for a given overall risk \(\sigma^2\). We obtain the following results:\(^{20}\)

**Proposition 5** (i) If \(\hat{c} < \Delta G (\sigma^2_H) \Delta u_j \) or \(\hat{c} > \Delta G (\sigma^2_L) \Delta u_i \), then any combination of risk choices \((\sigma^2_i, \sigma^2_j)\) will be an equilibrium at stage 1. (ii) If \(\hat{c} \in (\Delta G (\sigma^2_H) \Delta u_j, \Delta G (\sigma^2_M) \Delta u_j)\), then \((\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H)\) will be equilibria. (iii) If \(\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_j, \Delta G (\sigma^2_M) \Delta u_i)\), then \((\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H)\) is the unique equilibrium. (iv) If \(\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_i, \Delta G (\sigma^2_L) \Delta u_i)\), then \((\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_L, \hat{\sigma}^2_L)\) will be equilibria. (v) If \(\Delta G (\sigma^2_L) \Delta u_j > \Delta G (\sigma^2_M) \Delta u_i\) and \(\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_i, \Delta G (\sigma^2_L) \Delta u_j)\), then \((\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_L, \hat{\sigma}^2_L)\) will be an equilibrium.

Proposition 5 shows that risk taking does not play any role for the two contestants if effort is either very cheap \((\hat{c} < \Delta G (\sigma^2_H) \Delta u_j)\) or very expensive \((\hat{c} > \Delta G (\sigma^2_L) \Delta u_i)\). More interestingly, in those cases where the players have strict preferences over different risk levels we can isolate various forces that determine a player’s risk taking. First of all, we can observe the reversed likelihood effect. Consider, for example, a low-risk situation with \(\sigma^2_i = \sigma^2_j = \hat{\sigma}^2_L\) and effort not being too costly. In particular, let \(\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_j, \min\{\Delta G (\sigma^2_L) \Delta u_j, \Delta G (\sigma^2_M) \Delta u_i\})\). According to (26), favorite \(i\) would induce \((e_i^*, e_j^*) = (\hat{e}, \hat{e})\) if sticking to low risk \(\sigma^2_i = \hat{\sigma}^2_L\), but he

\(^{20}\)The proof is omitted for brevity but can be requested from the author.
would induce \((e_i^*, e_j^*) = (\hat{e}, 0)\) if deviating to high risk \(\sigma_i^2 = \hat{\sigma}_H^2\). Comparison of the respective payoffs shows that deviation to higher risk is profitable for the favorite if

\[
    u_i (w_2) + \Delta u_i G (\hat{e}; \sigma^2_{M}) - \hat{e} > u_i (w_2) + \frac{\Delta u_i}{2} - \hat{e} \Leftrightarrow \\
    G (\hat{e}; \sigma^2_{M}) > \frac{1}{2}
\]

which is obviously true. Hence, by choosing a higher risk, the favorite discourages the underdog\(^{21}\) and increases the equilibrium effort spread \(\Delta e^* (\sigma^2) = e_i^* (\sigma^2) - e_j^* (\sigma^2)\) from \(\Delta e^* (\sigma^2_L) = 0\) to \(\Delta e^* (\sigma^2_M) = \hat{e}\) which increases his winning probability from \(\frac{1}{2}\) to \(G (\hat{e}; \sigma^2_{M})\). In this situation, indeed a higher risk is advantageous for the favorite as it leads to a higher likelihood of winning. Note that such effect is impossible under a purely symmetric setting at the effort stage with \(\Delta e^* (\sigma^2) = 0, \forall \sigma^2\). Furthermore, note that the same reversed likelihood effect applies if the underdog chooses high risk \(\sigma_j^2 = \hat{\sigma}_H^2\) and \(\hat{c} \in (\Delta G (\sigma^2_H) \Delta u_j, \min \{\Delta G (\sigma^2_M) \Delta u_j, \Delta G (\sigma^2_H) \Delta u_i\})\). By choosing high risk instead of low risk, the favorite again induces \((\hat{e}, 0)\) instead of \((\hat{e}, \hat{e})\) at the effort stage which increases the equilibrium effort spread by \(\hat{e}\) and his winning probability by \(\Delta G (\sigma^2_H) = G (\hat{e}; \sigma^2_{H}) - \frac{1}{2}\).

The analysis also highlights the effort effect which can be observed in a purely symmetric setting, too. Consider, for example, a low-risk situation

\(^{21}\) Increasing risk makes the outcome of the tournament be more determined by luck. Now it does not pay any longer for the underdog to choose positive effort since the additional expected gains from choosing \(\hat{e}\) do not longer cover the respective effort costs \(\hat{c}\).
\[ \sigma_i^2 = \sigma_j^2 = \hat{\sigma}_L^2 \] with \[ \Delta G (\sigma_{M}^2) \Delta u_i < \hat{c} < \Delta G (\sigma_{L}^2) \Delta u_j. \] Here, sticking to low risk would lead to \((e_i^*, e_j^*) = (\hat{e}, \hat{c})\) at the effort stage (see (26)). However, since unilaterally deviating to high risk now yields \((e_i^*, e_j^*) = (0, 0)\) at stage 2 – while leaving winning probabilities unchanged –, any player is interested in reducing incentives and, therefore, effort costs by making use of the effort effect. Deviating to high risk implies a kind of implicit collusion for the effort stage: The choice of \(\hat{\sigma}_L^2\) by one of the players leads to a credible commitment for both contestants to exert zero effort at the next stage of the game as the high risk has completely erased all incentives.\(^22\)

Finally, we can observe the pure likelihood effect which is also at work in a symmetric setting. Suppose that, again, we have initially \(\sigma_i^2 = \sigma_j^2 = \hat{\sigma}_L^2\), and let now \(\Delta G (\sigma_{L}^2) \Delta u_j < \hat{c} < \Delta G (\sigma_{M}^2) \Delta u_i\). Irrespective of whether one player unilaterally deviates to high risk or not, we would have \((e_i^*, e_j^*) = (\hat{e}, 0)\) at the effort stage. However, the favorite would never deviate to higher risk as this would decrease his winning probability from \(G (\hat{e}; \sigma_{L}^2)\) to \(G (\hat{e}; \sigma_{M}^2)\). But, of course, the underdog would be interested in raising his likelihood of winning by switching to \(\sigma_j^2 = \hat{\sigma}_H^2\).\(^23\)

\(^{22}\)Note that the same effort effect would apply if initially one player has chosen low risk and the other one high risk (i.e. \(\sigma^2 = \sigma_{M}^2\)) given \(\Delta G (\sigma_{H}^2) \Delta u_i < \hat{c} < \Delta G (\sigma_{M}^2) \Delta u_j\). Here, the player who has chosen low risk prefers to increase his risk in order to decrease equilibrium efforts from \((\hat{e}, \hat{c})\) to \((0, 0)\).

\(^{23}\)An analogous situation applies if initially \(\sigma^2 = \sigma_{M}^2\) given \(\Delta G (\sigma_{H}^2) \Delta u_j < \hat{c} < \Delta G (\sigma_{M}^2) \Delta u_i\).
5 Conclusion

In rank-order tournaments, players often can choose both risk and effort (or investment). Previous papers on risk taking in two-stage tournaments have focused on the case of symmetric equilibrium efforts at the second stage. The findings of this paper point out that switching to asymmetric behavior at the effort stage can change players’ risk taking substantially.

In general, there are two effects that are decisive for a player’s risk choice – an effort effect (i.e. more risk strengthens or weakens effort incentives) and a likelihood effect (i.e. more risk increases or decreases a player’s winning probability). Whereas both players may be interested in the same risk choice in order to decrease total efforts and hence effort costs, their interests strictly differ concerning the likelihood effect since winning probabilities sum up to one.

The previous literature on risk taking has considered either rank-order tournaments with homogeneous players or so-called "unfair tournaments". In both cases, we always have a symmetric equilibrium at the effort stage which unambiguously separates the effort effect from the likelihood effect. However, if the players’ utility functions at least slightly differ, only asymmetric pure-strategy equilibria can exist at the effort stage. The analysis of this paper shows that this asymmetry has important implications for players’ risk taking since now effort effect and likelihood effect are strictly interrelated and may lead to a reversed likelihood effect: In "unfair tournaments", the
favorite (underdog) always prefers low (high) risk concerning the likelihood effect. If heterogeneous players compete within an "uneven tournament", these preferences may be exactly reverse.
Appendix

Computation of (20) and (21):

An interior solution at stage 2, \((e^*_i, e^*_j)\), is characterized by (5) and (6):

\[
\frac{\Delta u_i}{\sigma \sqrt{2\pi}} \cdot \exp \left\{ -\frac{(e^*_i - e^*_j)^2}{2\sigma^2} \right\} = c \cdot \exp \{ c \cdot e^*_i \} \tag{A1}
\]

\[
\frac{\Delta u_j}{\sigma \sqrt{2\pi}} \cdot \exp \left\{ -\frac{(e^*_i - e^*_j)^2}{2\sigma^2} \right\} = c \cdot \exp \{ c \cdot e^*_j \} \tag{A2}
\]

with \(\sigma = \sqrt{\sigma_i^2 + \sigma_j^2}\) yielding \(e^*_k = \frac{1}{c} \cdot \ln \left( \frac{\Delta u_k}{\Delta u_j} \phi \left( \frac{\Delta e^*}{\sigma} \right) \right) \quad (k = i, j) \tag{A3}\)

\((\phi (\cdot)\) denotes the density of the standardized normal distribution) and

\[
\Delta e^* = e^*_i - e^*_j = \frac{1}{c} \cdot \ln \left( \frac{\Delta u_i}{\Delta u_j} \right). \tag{A4}
\]

Concerning the first stage, from (13)–(16) we know that

\[
\frac{\partial EU_i}{\partial \sigma_i^2} = \Delta u_i \left( -g(\Delta e^*) \frac{\partial e^*_j}{\partial \sigma_i^2} + \frac{\partial G(\Delta e^*)}{\partial \sigma_i^2} \right) \tag{A5}
\]

\[
\frac{\partial EU_j}{\partial \sigma_j^2} = \Delta u_j \left( -g(\Delta e^*) \frac{\partial e^*_i}{\partial \sigma_j^2} + \frac{\partial G(-\Delta e^*)}{\partial \sigma_j^2} \right). \tag{A6}
\]

In the case of normally distributed noise we obtain from (A3) and (A4)

\[
\frac{\partial e^*_k}{\partial \sigma_l^2} = -\frac{1}{2c\sigma^2} + \frac{(\Delta e^*)^2}{2c\sigma^4} \quad (k = i, j; l = i, j). \tag{A7}
\]

Moreover, we have \(g(\Delta e^*) = \frac{1}{\sigma} \phi \left( \frac{\Delta e^*}{\sigma} \right)\) and \(G(\Delta e^*) = \Phi \left( \frac{\Delta e^*}{\sigma} \right)\) with \(\frac{\partial G(\Delta e^*)}{\partial \sigma_i^2} = \frac{\Delta e^*}{2\sigma^3} \phi \left( \frac{\Delta e^*}{\sigma} \right)\) \((\Phi (\cdot)\) denotes the cdf of the standardized normal distribution).

Similarly, we obtain \(G(-\Delta e^*) = \Phi \left( -\frac{\Delta e^*}{\sigma} \right)\) with \(\frac{\partial G(-\Delta e^*)}{\partial \sigma_j^2} = \frac{\Delta e^*}{2\sigma^3} \phi \left( \frac{\Delta e^*}{\sigma} \right)\) \((\phi (x) = \phi (-x), \forall x)\). Putting all together, (A5) and (A6) can be rewritten as (20) and (21).
References


Example 3: Normally distributed noise and heterogeneous exponential costs

The first-order conditions for an interior solution at the effort stage, (5) and (6), leads to

\[ \frac{c_i'(e_i)}{\Delta u_i} = \frac{c_j'(e_j)}{\Delta u_j}. \]

Using the exponential cost functions yields

\[ \frac{c_i \exp(c_i e_i)}{\Delta u_i} = \frac{c_j \exp(c_j e_j)}{\Delta u_j} \Leftrightarrow e_j = \frac{\ln \left( \frac{\Delta u_i c_i}{\Delta u_i c_j} \right)}{c_j} + \frac{c_i}{c_j} e_i \]

and

\[ \Delta e^* = e_i - e_j = -\frac{\ln \left( \frac{\Delta u_i c_i}{\Delta u_i c_j} \right)}{c_j} + \frac{c_j - c_i}{c_j} e_i \]

which is positive as \( \Delta u_i c_i < \Delta u_i c_j \).

By inserting the expression for \( \Delta e^* \) into \( i \)'s first-order condition (5) and using the assumptions on the noise distribution and the cost functions we obtain

\[ \frac{\Delta u_i}{\sigma \sqrt{2\pi}} \exp \left\{ \left( \frac{\ln \left( \frac{\Delta u_i c_i}{\Delta u_i c_j} \right)}{c_j} + \frac{c_j - c_i}{c_j} e_i \right)^2 \right\} = c_i \exp \{ c_i e_i \} \Leftrightarrow \]

\[ \ln \left( \frac{\Delta u_i}{c_i \sigma \sqrt{2\pi}} \right) + \frac{1}{2\sigma^2} \left( -\frac{\ln \left( \frac{\Delta u_i c_i}{\Delta u_i c_j} \right)}{c_j} + \frac{c_j - c_i}{c_j} e_i \right)^2 = c_i e_i \Leftrightarrow \]
This quadratic equation has the two solutions
\[ e_i = \frac{1}{c_j - c_i} \left[ \ln \left( \frac{\Delta u_j c_i}{\Delta u_i c_j} \right) + \frac{\sigma^2 c_i c_j^2}{c_j - c_i} \right] \]
\[ \pm \sqrt{\frac{2 \ln \left( \frac{\Delta u_j c_i}{\Delta u_i c_j} \right) \sigma^2 c_i}{c_j - c_i} + \frac{\sigma^4 c_i^2 c_j^2}{(c_j - c_i)^2} + 2 \sigma^2 \ln \left( \frac{\Delta u_i c_j}{c_i \sigma \sqrt{2\pi}} \right)} \]
\[ = \frac{1}{(c_j - c_i)^2} \left[ (c_j - c_i) \ln \left( \frac{\Delta u_j c_i}{\Delta u_i c_j} \right) + \sigma^2 c_i c_j^2 \right] \]
\[ \pm \sigma c_j \sqrt{2 (c_j - c_i) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma \sqrt{2\pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma \sqrt{2\pi}} \right) \right) + \sigma^2 c_i^2 c_j^2} \]
with
\[ e_i = \frac{1}{(c_j - c_i)^2} \left[ (c_j - c_i) \ln \left( \frac{\Delta u_j c_i}{\Delta u_i c_j} \right) + \sigma^2 c_i c_j^2 \right] \]
\[ - \sigma c_j \sqrt{2 (c_j - c_i) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma \sqrt{2\pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma \sqrt{2\pi}} \right) \right) + \sigma^2 c_i^2 c_j^2} \]

Corresponding to \( i \)'s maximum. Inserting into the expression for \( \Delta e^* \) gives
\[ \Delta e^* (\sigma^2) \]
\[ = \frac{\sigma}{c_j - c_i} \left[ \sigma c_i c_j - \sqrt{2 (c_j - c_i) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma \sqrt{2\pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma \sqrt{2\pi}} \right) \right) + \sigma^2 c_i^2 c_j^2} \right]. \]

By using this expression, the condition for the reversed likelihood effect,
\[ \Delta e^* (\sigma^2_H) > \frac{\sigma_H}{\sigma_L} \Delta e^* (\sigma^2_L), \]
\[ c_j \left( \sigma_H - \sigma_L \right) > \sqrt{2 \left( c_j - c_i \right) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma_H \sqrt{2\pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma_H \sqrt{2\pi}} \right) \right) + \sigma_H^2 c_i^2 c_j^2 \]

\[ - \sqrt{2 \left( c_j - c_i \right) \left( c_i \ln \left( \frac{\Delta u_j}{c_j \sigma_L \sqrt{2\pi}} \right) - c_j \ln \left( \frac{\Delta u_i}{c_i \sigma_L \sqrt{2\pi}} \right) \right) + \sigma_L^2 c_i^2 c_j^2 \].

Proof of Proposition 5:

According to (26), given a certain risk level \( \sigma^2 \) there exist two cut-offs for \( \hat{c} \) when players exert efforts at stage 2. Since each player chooses one of two possible risk levels at stage 1, in sum we have four possible cut-offs for the effort stage depending on \( (\sigma_i^2, \sigma_j^2) \). However, the lowest possible cut-off is \( \Delta G (\sigma_H^2) \Delta u_j \), and the highest possible one is \( \Delta G (\sigma_L^2) \Delta u_i \). For all \( \hat{c} < \Delta G (\sigma_H^2) \Delta u_j \) both players will choose positive efforts, whereas for all \( \hat{c} > \Delta G (\sigma_L^2) \Delta u_i \) both will choose zero efforts irrespective of the given risk \( \sigma^2 \). Since in the case of identical effort choices each player’s winning probability is always \( \frac{1}{2} \) and does not depend on \( \sigma^2 \), any combination \( (\sigma_i^2, \sigma_j^2) \) forms an equilibrium at stage 1 in these situations, which proves result (i).

Now consider a possible high-risk equilibrium \( (\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_H^2, \hat{\sigma}_H^2) \). If no player deviates, we will have overall risk \( \sigma^2 = \sigma_H^2 \). However, if one of the players deviates we have \( \sigma^2 = \sigma_M^2 \). Hence, there are four cut-offs \( \Delta G (\sigma_H^2) \Delta u_j, \Delta G (\sigma_H^2) \Delta u_i, \Delta G (\sigma_M^2) \Delta u_j \) and \( \Delta G (\sigma_M^2) \Delta u_i \) for \( \hat{c} \). Whereas it is clear that \( \Delta G (\sigma_H^2) \Delta u_j (\Delta G (\sigma_M^2) \Delta u_i) \) is the lowest (highest) one, there
is no clear ranking between the two other cut-offs. Figures 2a and 2b illustrate the two possible scenarios.

[Figures 2a and 2b]

In scenario 1, we have $\Delta G (\sigma^2_M) \Delta u_i < \Delta G (\sigma^2_H) \Delta u_i \iff \frac{\Delta G (\sigma^2_M)}{\Delta G (\sigma^2_H)} < \frac{\Delta u_i}{\Delta u_j}$ so that the impact of heterogeneity dominates that of the likelihood effect.

However, in scenario 2, we have just the opposite constellation. First, we can check whether favorite $i$ has an incentive to deviate from $\sigma^2_i = \hat{\sigma}^2_H$ in stage 1 given that underdog $j$ chooses $\sigma^2_j = \hat{\sigma}^2_H$:

- If $\hat{c} \in (\Delta G (\sigma^2_H) \Delta u_j, \Delta G (\sigma^2_M) \Delta u_j)$ in scenario 1 or $\hat{c} \in (\Delta G (\sigma^2_H) \Delta u_i, \Delta G (\sigma^2_M) \Delta u_i)$ in scenario 2, player $i$ will induce $(e^*_i, e^*_j) = (\hat{e}, 0)$ when choosing $\sigma^2_i = \hat{\sigma}^2_H$, but $(e^*_i, e^*_j) = (\hat{e}, \hat{e})$ when choosing $\sigma^2_i = \hat{\sigma}^2_L$ (see again (26)). Hence, $i$ will not deviate from $\sigma^2_i = \hat{\sigma}^2_H$ if $u_i (w_2) + \Delta u_i G (\hat{e}; \sigma^2_H) - \hat{c} \geq u_i (w_2) + \frac{\Delta u_i}{2} - \hat{c}$, which is true.

- If $\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_i, \Delta G (\sigma^2_M) \Delta u_i)$ in scenario 1 or $\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_j, \Delta G (\sigma^2_M) \Delta u_j)$ in scenario 2, player $i$ will induce $(e^*_i, e^*_j) = (0, 0)$ when choosing $\sigma^2_i = \hat{\sigma}^2_H$, but $(e^*_i, e^*_j) = (\hat{e}, 0)$ when choosing $\sigma^2_i = \hat{\sigma}^2_L$ (see again (26)). Hence, $i$ will not deviate from $\sigma^2_i = \hat{\sigma}^2_H$ if $u_i (w_2) + \Delta u_i G (\hat{e}; \sigma^2_M) - \hat{c} \geq u_i (w_2) + \Delta u_i G (\hat{e}; \sigma^2_M) - \hat{e} \iff \hat{e} \geq \Delta G (\sigma^2_M) \Delta u_i$, which is false. Hence, in this situation $i$ deviates to $\sigma^2_i = \hat{\sigma}^2_L$.

- If $\hat{c} \in (\Delta G (\sigma^2_M) \Delta u_j, \Delta G (\sigma^2_H) \Delta u_i)$ in scenario 1, both $\sigma^2_i = \hat{\sigma}^2_H$ and $\sigma^2_i = \hat{\sigma}^2_L$ will induce $(e^*_i, e^*_j) = (\hat{e}, 0)$ in stage 2. Therefore, $i$ will stick
to \( \sigma_i^2 = \hat{\sigma}_H^2 \) if \( u_i(w_2) + \Delta u_i G(\hat{e}; \sigma_H^2) - \hat{c} \geq u_i(w_2) + \Delta u_i G(\hat{e}; \sigma_M^2) - \hat{c} \Leftrightarrow G(\hat{e}; \sigma_H^2) \geq G(\hat{e}; \sigma_M^2) \) which is false; so \( i \) deviates to \( \sigma_i^2 = \hat{\sigma}_L^2 \).

- If \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_i, \Delta G(\sigma_M^2) \Delta u_j) \) in scenario 2, \( \sigma_i^2 = \hat{\sigma}_H^2 \) induces \( (e_i^*, e_j^*) = (0, 0) \), but \( \sigma_i^2 = \hat{\sigma}_L^2 \) leads to \( (e_i^*, e_j^*) = (\hat{e}, \hat{e}) \) in the effort stage. Hence, \( i \) clearly sticks to \( \sigma_i^2 = \hat{\sigma}_H^2 \) in order to save effort costs.

- Altogether, the analysis has shown that favorite \( i \) will not deviate from \( (\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_H^2, \hat{\sigma}_H^2) \) if

\[
\hat{c} < \Delta G(\sigma_M^2) \Delta u_j
\]

Next, we consider underdog \( j \) given that favorite \( i \) chooses \( \sigma_i^2 = \hat{\sigma}_M^2 \). Note that we have the same effort choices for both players as above when again checking all possible cases:

- If \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_i, \Delta G(\sigma_M^2) \Delta u_j) \) in scenario 1 or \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_j, \Delta G(\sigma_M^2) \Delta u_i) \) in scenario 2, player \( j \) will prefer \( \sigma_j^2 = \hat{\sigma}_H^2 \ (\to (\hat{e}, 0)) \) to \( \sigma_j^2 = \hat{\sigma}_L^2 \ (\to (\hat{e}, 0)) \) if \( u_j(w_2) + \Delta u_j G(-\hat{e}; \sigma_H^2) \geq u_j(w_2) + \frac{\Delta u_j}{2} - \hat{c} \Leftrightarrow \hat{c} \geq \frac{\Delta u_j}{2} - \Delta u_j G(-\hat{e}; \sigma_H^2) = \Delta G(\sigma_H^2) \Delta u_j, \) which is true.

- If \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_i, \Delta G(\sigma_M^2) \Delta u_j) \) in scenario 1 or \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_j, \Delta G(\sigma_M^2) \Delta u_i) \) in scenario 2, player \( j \) will prefer \( \sigma_j^2 = \hat{\sigma}_H^2 \ (\to (0, 0)) \) to \( \sigma_j^2 = \hat{\sigma}_L^2 \ (\to (\hat{e}, 0)) \) if \( u_j(w_2) + \frac{\Delta u_j}{2} \geq u_j(w_2) + \Delta u_j G(-\hat{e}; \sigma_M^2) \Leftrightarrow \frac{1}{2} \geq G(-\hat{e}; \sigma_M^2), \) which again is true.

- If \( \hat{c} \in (\Delta G(\sigma_H^2) \Delta u_j, \Delta G(\sigma_M^2) \Delta u_i) \) in scenario 1, both \( \sigma_j^2 = \hat{\sigma}_H^2 \) and \( \sigma_j^2 = \hat{\sigma}_L^2 \) will lead to \( (e_i^*, e_j^*) = (\hat{e}, 0) \) in stage 2. \( j \) will stick to \( \sigma_j^2 = \hat{\sigma}_H^2 \)
if \( u_j(w_2) + \Delta u_j G(-\hat{\epsilon}; \sigma_H^2) \geq u_j(w_2) + \Delta u_j G(-\hat{\epsilon}; \sigma_M^2) \Leftrightarrow G(\hat{\epsilon}; \sigma_H^2) \leq G(\hat{\epsilon}; \sigma_M^2) \) which is true.

- If \( \hat{\epsilon} \in (\Delta G(\sigma_H^2) \Delta u_i, \Delta G(\sigma_M^2) \Delta u_j) \) in scenario 2, \( j \) will prefer \( \hat{\sigma}_H^2 \) \((\to (0,0))\) to \( \hat{\sigma}_L^2 \) \((\to (\hat{\epsilon}, \hat{\epsilon}))\) in order to save effort costs.

- To sum up, underdog \( j \)'s best response to \( \sigma_i^2 = \hat{\sigma}_H^2 \) is always choosing high risk, too. Hence, \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_H^2, \hat{\sigma}_H^2)\) is an equilibrium at the risk stage 1 if and only if condition (*) is satisfied or \( \hat{\epsilon} < \Delta G(\sigma_H^2) \Delta u_j \) or \( \hat{\epsilon} > \Delta G(\sigma_M^2) \Delta u_i \). Under the last two conditions, both players are indifferent between choosing high or low risk.

Now we consider the case of a low-risk situation with \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_L^2)\). First, we have again to differentiate between two possible scenarios which are illustrated by Figures 3a and 3b.

[Figures 3a and 3b]

We start with the best responses of favorite \( i \) supposing that \( \sigma_j^2 = \hat{\sigma}_L^2 \).

- If \( \hat{\epsilon} \in (\Delta G(\sigma_M^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_j) \) in scenario 1 or \( \hat{\epsilon} \in (\Delta G(\sigma_M^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_i) \) in scenario 2, player \( i \) will prefer \( \sigma_i^2 = \hat{\sigma}_H^2 \) \((\to (0,0))\) to \( \sigma_i^2 = \hat{\sigma}_L^2 \) \((\to (\hat{\epsilon}, \hat{\epsilon}))\) if \( u_i(w_2) + \Delta u_i G(\hat{\epsilon}; \sigma_M^2) - \hat{\epsilon} \geq u_i(w_2) + \frac{\Delta u_i}{2} - \hat{\epsilon} \Leftrightarrow G(\hat{\epsilon}; \sigma_M^2) \geq \frac{1}{2}, \) which is true; so \( i \) deviates from \( \hat{\sigma}_L^2 \) to \( \hat{\sigma}_H^2 \).

- If \( \hat{\epsilon} \in (\Delta G(\sigma_M^2) \Delta u_i, \Delta G(\sigma_L^2) \Delta u_i) \) in scenario 1 or \( \hat{\epsilon} \in (\Delta G(\sigma_L^2) \Delta u_j, \Delta G(\sigma_M^2) \Delta u_i) \) in scenario 2, player \( i \) will prefer \( \sigma_i^2 = \hat{\sigma}_H^2 \) \((\to (0,0))\) to
\[ \sigma_i^2 = \hat{\sigma}_L^2 \rightarrow (\hat{e}, 0) \] if \( u_i(w_2) + \frac{\Delta u_i}{\Delta} \geq u_i(w_2) + \Delta u_i(\hat{e}; \sigma_L^2) - \hat{e} \Leftrightarrow \hat{e} \geq \Delta G(\sigma_L^2) \Delta u_i, \) which is false. Hence, \( i \) sticks to the low-risk situation.

- If \( \hat{e} \in (\Delta G(\sigma_M^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_i) \) in scenario 1, both \( \sigma_i^2 = \hat{\sigma}_H^2 \) and \( \sigma_j^2 = \hat{\sigma}_L^2 \) will lead to \((e_i^*, e_j^*) = (\hat{e}, 0)\) in stage 2. \( i \) will prefer \( \sigma_i^2 = \hat{\sigma}_H^2 \) if \( u_i(w_2) + \Delta u_i(\hat{e}; \sigma_M^2) - \hat{e} \geq u_i(w_2) + \Delta u_i(\hat{e}; \sigma_L^2) - \hat{e} \Leftrightarrow G(\hat{e}; \sigma_M^2) \geq G(\hat{e}; \sigma_L^2) \) which is false. Hence, again \( i \) sticks to \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_L^2)\).

- If \( \hat{e} \in (\Delta G(\sigma_M^2) \Delta u_i, \Delta G(\sigma_L^2) \Delta u_j) \) in scenario 2, \( i \) will prefer \( \sigma_H^2 \rightarrow (0, 0) \) to \( \sigma_L^2 \rightarrow (\hat{e}, \hat{e}) \) in order to save effort costs.

- Altogether, favorite \( i \) will not deviate from \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_L^2)\) if

\[ \hat{e} \geq \Delta G(\sigma_L^2) \Delta u_j \] (***)

Next we turn to the risk choice of underdog \( j \). Suppose that \( \sigma_j^2 = \hat{\sigma}_L^2 \).

- If \( \hat{e} \in (\Delta G(\sigma_M^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_j) \) in scenario 1 or \( \hat{e} \in (\Delta G(\sigma_M^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_i) \) in scenario 2, player \( j \) will prefer \( \sigma_j^2 = \hat{\sigma}_H^2 \rightarrow (\hat{e}, 0) \) to \( \sigma_j^2 = \hat{\sigma}_L^2 \rightarrow (\hat{e}, \hat{e}) \) if \( u_j(w_2) + \Delta u_j(\hat{e}; \sigma_M^2) \geq u_j(w_2) + \frac{\Delta u_j}{\Delta} - \hat{e} \Leftrightarrow \hat{e} \geq \Delta G(\sigma_M^2) \Delta u_j, \) which is true.

- If \( \hat{e} \in (\Delta G(\sigma_M^2) \Delta u_i, \Delta G(\sigma_L^2) \Delta u_j) \) in scenario 1 or \( \hat{e} \in (\Delta G(\sigma_L^2) \Delta u_j, \Delta G(\sigma_L^2) \Delta u_i) \) in scenario 2, player \( j \) will prefer \( \sigma_j^2 = \hat{\sigma}_H^2 \rightarrow (0, 0) \) to \( \sigma_j^2 = \hat{\sigma}_L^2 \rightarrow (\hat{e}, 0) \) if \( u_j(w_2) + \frac{\Delta u_j}{\Delta} \geq u_j(w_2) + \Delta u_j(-\hat{e}; \sigma_L^2) \Leftrightarrow \frac{1}{2} \geq G(-\hat{e}; \sigma_L^2), \) which is also true.

46
• If \( \hat{c} \in (\Delta G(\sigma^2_H) \Delta u_j, \Delta G(\sigma^2_M) \Delta u_i) \) in scenario 1, both \( \sigma^2_j = \hat{\sigma}^2_H \) and \( \sigma^2_j = \hat{\sigma}^2_L \) will lead to \( (e^*_i, e^*_j) = (\hat{c}, 0) \) in stage 2. \( j \) will prefer \( \sigma^2_j = \hat{\sigma}^2_H \) if \( u_j(w_2) + \Delta u_jG(-\hat{e}; \sigma^2_M) \geq u_j(w_2) + \Delta u_jG(-\hat{e}; \sigma^2_L) \Leftrightarrow G(\hat{e}; \sigma^2_M) \leq G(\hat{e}; \sigma^2_L) \) which is again true.

• If \( \hat{c} \in (\Delta G(\sigma^2_M) \Delta u_i, \Delta G(\sigma^2_L) \Delta u_j) \) in scenario 2, \( j \) will prefer \( \hat{\sigma}^2_H \) (\( \rightarrow (0, 0) \)) to \( \hat{\sigma}^2_L \) (\( \rightarrow (\hat{e}, \hat{e}) \)) in order to save effort costs.

We can see that \( j \)'s strategic behavior is just analogous to his behavior in the high-risk situation with \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H) \): He always prefers the higher risk \( \sigma^2_j = \hat{\sigma}^2_H \) as a best response to \( i \)'s given risk choice. Hence, choosing \( \sigma^2_j = \hat{\sigma}^2_H \) is dominant for underdog \( j \) at stage 1.

This finding has two immediate implications: (1) \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_L, \hat{\sigma}^2_L) \) can only be an equilibrium if both players are indifferent between high and low risk, that is if \( \hat{c} < \Delta G(\sigma^2_M) \Delta u_j \) or \( \hat{c} > \Delta G(\sigma^2_L) \Delta u_i \). (2) Similarly, \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H) \) can only be an equilibrium at stage 1 if \( j \) is indifferent between both risk levels. From the discussion of \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H) \) we know that this is impossible within the range \( \hat{c} \in (\Delta G(\sigma^2_M) \Delta u_j, \Delta G(\sigma^2_M) \Delta u_i) \). The discussion of scenario 1 in the low-risk situation with \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_L, \hat{\sigma}^2_L) \) shows that, in this scenario, \( (\sigma^2_i, \sigma^2_j) = (\hat{\sigma}^2_H, \hat{\sigma}^2_H) \) can also be no equilibrium as we have \( \Delta G(\sigma^2_L) \Delta u_j < \Delta G(\sigma^2_M) \Delta u_i \) together with condition (**). However, in the corresponding scenario 2 we have \( \Delta G(\sigma^2_L) \Delta u_j > \Delta G(\sigma^2_M) \Delta u_i \) so that \( j \) is indifferent between both risk levels within the range \( \hat{c} \in (\Delta G(\sigma^2_M) \Delta u_i, \Delta G(\sigma^2_L) \Delta u_j) \) whereas \( i \) strictly prefers high risk in that interval as we
know from the derivation of condition (**). This proves result (v) of Proposition 5.

Finally, we have to consider the possible equilibrium \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_H^2)\). Since the choice of \(\sigma_j^2 = \hat{\sigma}_H^2\) is dominant for the underdog, he will not deviate from \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_H^2)\). However, the situation is different for \(i\). From the discussion of \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_H^2, \hat{\sigma}_H^2)\) we know that \(i\) prefers overall risk \(\sigma_M^2\) to \(\sigma_H^2\) if condition (*) does not hold. Hence, \((\sigma_i^2, \sigma_j^2) = (\hat{\sigma}_L^2, \hat{\sigma}_H^2)\) will be an equilibrium in stage 1 if \(\hat{c} > \Delta G (\sigma_M^2) \Delta u_j\).

Note that we have the cut-off ranking \(\Delta G (\sigma_H^2) \Delta u_j < \Delta G (\sigma_M^2) \Delta u_j < \Delta G (\sigma_M^2) \Delta u_i < \Delta G (\sigma_L^2) \Delta u_i\) which completes the proof of Proposition 5.
Figure 1: Reversed likelihood effect under discrete risk choice
\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ji}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ji}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]

**Figure 2a:** High-risk situation (scenario 1: heterogeneity is dominant)

\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]

**Figure 2b:** High-risk situation (scenario 2: likelihood effect is dominant)

\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]

**Figure 3a:** Low-risk situation (scenario 1: heterogeneity is dominant)

\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ij}^2) \Delta u_j \quad \Delta G(\sigma_{ij}^2) \Delta u_i \]
\[ \Delta G(\sigma_{ii}^2) \Delta u_j \quad \Delta G(\sigma_{ii}^2) \Delta u_i \]

**Figure 3b:** Low-risk situation (scenario 2: likelihood effect is dominant)