On the Virtues of Hiring Lemons*

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Abstract

Recruiting high-ability workers and implementing optimal efforts are among the key objectives of a firm’s personnel policy. We show that, if the firm applies a tournament scheme – e.g., a competitive career system – selection and incentive issues are strictly interrelated, thus leading to a fundamental conflict: if the firm is primarily interested in balanced worker competition, there will be a rationale for hiring low-ability workers ("lemons").

Key Words: contest; lemon; recruiting; tournament.

JEL Classification: D21; D44; D86

*Financial support by the Deutsche Forschungsgemeinschaft (DFG), grant SFB/TR 15 ("Governance and the Efficiency of Economic Systems"), is gratefully acknowledged.
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1 Introduction

The aim of a firm’s personnel policy is usually twofold. The firm wants to recruit high-ability workers (or avoid hiring "lemons") and aims at inducing optimal efforts. Both, workers’ abilities and workers’ effort choices, are crucial for the success of a firm. The firm’s optimal policy seems straightforward – first, hiring the most able workers (provided they are not too expensive) and, thereafter, designing optimal incentives for them. However, in this paper we show that selection and incentive issues are strictly interconnected so that the hiring decision may already determine the effectiveness of a firm’s incentive system. In such a situation, recruiting lemons can be optimal from the firm’s point of view.

To induce workers to choose high efforts, some kind of incentive system is needed. In many situations, worker performance is observable within the firm but unverifiable to third parties (e.g., a court). This problem is typical, for example, of managerial tasks that do not lead to direct outputs. Moreover, it is often too costly to measure performance individually so that only relative performance signals exist. In both instances, the firm can rely on a tournament between the workers (e.g., a competitive career system) to incentivize them to exert effort.\(^1\)

\(^1\) If performance information is unverifiable, tournaments have the advantage that the wage bill is fixed and the firm cannot save wages by misrepresenting performance information; see Malcomson (1984). The measurement-cost argument was highlighted by Lazear and Rosen (1981).
ing another low-ability worker, the firm may thus increase incentives in the
tournament. The resulting increase in efforts may overcompensate the loss
in output due to the worker’s lower ability. As a result of this conflict, the
firm may want to hire a low-ability worker even if it could recruit a worker of
higher expected ability at the same wage. To summarize: if balanced tour-
ament competition is of major interest to the firm, there will be a rationale
for violating the selection aim of its personnel policy.

In the following, we present two real-life examples underscoring our ar-
gument that firms incur costs in terms of lost talent to intensify competi-
tion. The first example refers to the problem of overqualification.\(^2\) Bewley
(1999, chapter 15.2) conducts interviews with firms concerning their person-
nel policy and reports that 70\% (10\%) of the firms in his sample are totally
(partially) unwilling to hire overqualified job applicants. At first sight, the
exclusion of overqualified workers seems puzzling since a worker’s produc-
tivity is typically increasing with qualification. Moreover, the hiring of an
overqualified worker, which leads to a mismatch in the short run, may result
in a good match on higher hierarchy levels in the long run. However, as Be-
wley further reports, 78\% of the firms are afraid that overqualified workers
continue to look for a better job with a different employer, leading to costs of
worker turnover. Fifty percent of the firms believe that overqualified work-
ers would be unhappy and demotivated due to less demanding jobs. Note
that this kind of demotivation is mostly based on psychological considera-
tions. Our model offers an economic explanation for the demotivating effect
of hiring overqualified workers. This hiring policy would lead to a rather het-
erogeneous workforce and, consequently, spoil internal worker competition
for wage premiums and job promotions so that both low and high-qualified

\(^2\)See, e.g., the survey article by McGuinness (2003) on overqualification.
workers would be unwilling to exert sufficient effort.

The second example refers to professional team sports leagues. In sports economics, a league is typically considered as a single firm or organization. Often, there even exists a sporting association as a formal organization with the different clubs as single agents.\(^3\) Most professional team sports leagues in North America and Europe have introduced mechanisms, such as salary caps or revenue sharing schemes, with the aim of creating intense competition between the clubs.\(^4\) Without such mechanisms, there exists the threat of a superstar concentration in the rich clubs, whereas less wealthy clubs can only hire the lowest talents. In Europe, in particular, these mechanisms come at a cost. If the clubs from a European country are limited to a revenue sharing agreement, for example, they typically cannot afford to hire the best players so that these often move to other countries, where regulations are less strict. The fact that leagues introduce revenue sharing mechanisms in spite of this cost suggests that balanced competition among rather homogeneous clubs ensuring high incentives can be the dominant motive in regulating sports leagues. This outcome exactly fits the prediction of our model – too much heterogeneity among competing agents should be avoided even if this policy leads to a reduction in overall talent.

Our results have implications for those organizations that primarily make use of tournament incentive schemes. In law firms, in particular, major incentives are created by up-or-out tournaments (Kahn and Huberman 1988, O’Flaherty and Siow 1992, Wilkins and Gulati 1998). Here several lawyers are recruited as a cohort, but only few of them can be promoted to part-

\(^3\)There are strong parallels to the case of a cooperative, founded by a number of profit-maximizing, small firms or individuals. In the case of an association of sport clubs, the single clubs also maximize their individual profits and use the association to coordinate collective behavior.

nership. According to the up-or-out rule, the current partners must choose between two alternatives after a certain span of time – they can either make a lawyer a new partner or dismiss him. The up-or-out rule prevents partners from exploiting the high talents of employed lawyers without adequately paying for their talents. Tournament competition for partnership creates huge incentives for the lawyers. The main result of our paper explains the coexistence of homogeneous elite law firms, on the one hand, and homogeneous law firms consisting of lemons as well as self-employed, low-ability lawyers, on the other.

In a broader context, our paper also contributes to the extensive debate on the opportunities and threats of organizational diversity (Kondra and Hinings 1998, Lazear 1999, Carroll and Hannan 2000, Jackson, Joshi and Erhardt 2003). The results of our paper show that organizational diversity may be harmful if the respective firm uses competitive mechanisms such as forced-distribution systems, relative performance pay, or job-promotion tournaments. In these cases, the firm’s hiring and career policies should aim at generating rather homogeneous groups or departments if creating high incentives is of major interest.

Our paper is related to two strands of the tournament literature. The first combines selection issues with tournament competition. Most of these papers analyze whether tournaments can be used to identify workers of high ability. For a tournament where individuals can only choose risk, Hvide and Kristiansen (2003) show, for example, that reducing the number and the average quality of contestants may increase selection efficiency. Münster (2007) allows for sabotage in tournaments. He finds that high-ability individuals are sabotaged most heavily, which leads to equal winning probabilities across contestants and may prevent high-ability types from participating.
Clark and Riis (2007) show that efficient selection becomes possible under a scheme of endogenous winner and loser prizes. Schöttner and Thiele (2010) combine a job-promotion tournament with a piece-rate scheme and address both selection and incentives issues. However, they concentrate on internal candidates filling vacancies and not on candidates from external worker pools. In contrast to all these papers, the current paper focuses on the hiring process preceding the tournament. The paper shows that firms may voluntarily decrease the quality of their workforce, while the other papers analyze how tournaments can be structured to increase the expected ability of the tournament’s winner.

The second strand of the literature addresses the problem of uneven competition. In their seminal paper, Lazear and Rosen (1981) show that tournament competition between heterogeneous workers can lead to an inefficient outcome. Lynch (2005) uses field data from horse racing to empirically investigate uneven tournament competition. His findings show that jockeys’ efforts decrease in the degree of uneven competition, thus supporting tournament theory. Brown (2008) analyzes the impact of heterogeneity on tournament competition by using panel data from professional golfers. Her results show that the presence of a superstar like Tiger Woods leads to significantly lower overall performance, compared to a situation where superstars are absent. Kräkel and Schöttner (2010) consider technology choice under worker heterogeneity and tournament competition. They show that firms may forgo a better production technology if workers’ abilities are complementary to technology. In contrast to our contribution, these papers do not address the possible conflict between selection and incentives issues when using a tournament system.

The remainder of the paper is organized as follows. Section 2 introduces
the model. In Section 3, we solve the model, present the main effects and illustrate them by an example. Section 4 demonstrates that the main effects continue to hold if there is little noise in the tournament so that the workers play mixed strategies. Section 5 concludes.

2 The Model

Consider a firm using a tournament to induce incentives for its workers to put forth effort. There are two types of workers, one type with low ability $t_L$ and the other with high ability $t_H (> t_L)$. Define $\Delta t \equiv t_H - t_L$. The firm currently employs a worker of low ability $t_L$, and needs to hire a second worker to fill a vacancy.\(^5\) The firm can hire the worker from different worker pools (indexed by $j$), each of which is characterized by the fraction $p_j \in (0, 1)$ of high-ability workers. When the firm decides to hire a worker, the worker’s ability is unobservable to all players (including the worker himself). However, when the tournament starts, the ability uncertainty is resolved to some degree. In particular, the two competing workers are assumed to observe their own and the opponent’s ability. We normalize a worker’s reservation value to zero and assume it to be independent of the pool the worker is hired from. Note that this assumption strengthens our results since it implies that workers from a pool of workers with higher expected ability are not more expensive than workers from a pool with lower expected ability.

After the vacancy is filled, the firm organizes a tournament between the current worker and the newly hired one. During the tournament, each worker $i \ (= 1, 2)$ exerts unobservable effort $e_i \in \mathbb{R}_+$. Exerting effort, a worker produces an observable but unverifiable output $q_i = h(e_i, t_i) + \varepsilon_i$. Unverifiability

\(^5\)We assume that it is not in the firm’s interest to fire its current worker and hire two new workers. Possible reasons are addressed in the concluding section.
of \( q_i \) implies that individual incentive schemes based on that signal do not work. However, tournaments do work because of their self-commitment property (Malcomson 1984). The production function \( h: \mathbb{R}_+ \times \{t_L, t_H\} \to \mathbb{R} \) is monotonically increasing and concave in effort \( e_i \) and monotonically increasing in ability \( t_i \). Let \( \frac{\partial^2 h}{\partial e_i \partial t_i} \geq 0 \), thereby allowing for both kinds of worker heterogeneity considered in the tournament literature:\(^6\) (1) If \( \frac{\partial^2 h}{\partial e_i \partial t_i} > 0 \), effort and abilities are strict complements. (2) If \( \frac{\partial^2 h}{\partial e_i \partial t_i} = 0 \), ability does not influence marginal productivity but still has an absolute impact on production (since \( \partial h(e_i, t_i)/\partial t_i > 0 \)); in that case, a worker can choose higher effort to substitute for a lack of ability. \( \varepsilon_i \) denotes a random variable with zero mean, which accounts for luck or measurement error. We follow the standard assumption in tournament theory that the difference \( \varepsilon_2 - \varepsilon_1 \) is distributed according to a pdf \( g \) that has a unique mode at zero.\(^7\) Denote the corresponding cdf by \( G \). Effort entails cost \( c(e_i) \) for a worker, \( c: \mathbb{R}_+ \to \mathbb{R}_+ \) is a strictly increasing and strictly convex function satisfying \( c(0) = 0, c'(0) = 0 \) and \( c''(e_i) > 0 \) for all \( e_i \in \mathbb{R}_+ \).\(^8\)

The worker with the higher output is declared the tournament’s winner and receives a prize \( w_1 \), while the loser receives \( w_2 \leq w_1 \). Define \( \Delta w \equiv w_1 - w_2 \). The workers are protected by limited liability, hence \( w_2 \geq 0 \). We assume the existence of pure-strategy equilibria characterized by the first-order conditions to the workers’ maximization problems.\(^9\)

\(^6\)See, e.g., O’Keeffe, Viscusi, and Zeckhauser (1984), Schotter and Weigelt (1992). The first setting is labeled "uneven contest," whereas the second setting is referred to as "unfair contest."

\(^7\)For example, Dixit (1987) and Hvide (2002). If \( \varepsilon_1 \) and \( \varepsilon_2 \) are i.i.d. and follow a normal (uniform) distribution, the convolution \( g \) will be normal (triangular), symmetric about zero, and unimodal; see, e.g., Wolfstetter (1999), p. 306.

\(^8\)We need the assumption \( c''(e_i) > 0 \) (\( e_i > 0 \)) to guarantee the existence of an interior solution for the firm.

\(^9\)As already emphasized by the seminal paper of Lazear and Rosen (1981), p. 845, fn. 2, existence is not guaranteed in general. For the case of a linear production technology and effort and ability being additive, see Gürtler (2011) on the conditions that ensure the
The timing of the model is as follows: The firm first decides from which pool to hire its worker. Then it offers a tournament contract both to its current worker and the potential new worker. If (at least) one of the worker rejects, the game ends. Otherwise, workers compete in the tournament and prizes are paid.

3 Solution to the Model

As the model is solved by backward induction, we start with the tournament stage. Since the new worker’s talent is unknown when he is hired, we may either have a homogeneous tournament between two low-ability workers or a heterogeneous tournament with one worker of low ability and one worker of high ability. Denote the firm’s current worker as worker 1. In the tournament, this worker chooses effort $e_1$ so as to maximize

$$EU_1 = w_2 + G(h(e_1, t_L) - h(e_2, t_2)) \Delta w - c(e_1)$$

with $t_2 \in \{t_L, t_H\}$. Similarly, the newly hired worker, hereafter worker 2, maximizes

$$EU_2 = w_2 + [1 - G(h(e_1, t_L) - h(e_2, t_2))] \Delta w - c(e_2).$$

existence of pure-strategy equilibria.
As indicated above, the optimal efforts can be characterized by the first-order conditions to the workers’ maximization problems:

\[
\Delta w g \left( h(e_1, t_L) - h(e_2, t_2) \right) \frac{\partial h(e_1, t_L)}{\partial e_1} - c'(e_1) = 0, \tag{1}
\]

\[
\Delta w g \left( h(e_1, t_L) - h(e_2, t_2) \right) \frac{\partial h(e_2, t_2)}{\partial e_2} - c'(e_2) = 0. \tag{2}
\]

Let \((e_1^*, e_2^*)\) denote equilibrium efforts in the tournament game, being described by (1) and (2). Obviously, \(e_1^* = e_2^*\) for \(t_2 = t_L\) and/or \(\frac{\partial^2 h}{\partial e_i \partial t_i} = 0\), but \(e_1^* < e_2^*\) otherwise. In any case, each worker’s marginal winning probability \(g(h(e_1, t_L) - h(e_2, t_2))\) is highest in a homogeneous match (i.e., \(t_2 = t_L\)) and decreases in \(|h(e_1, t_L) - h(e_2, t_2)|\) due to \(g\)’s unique mode at zero. Hence, in equilibrium the workers’ incentives should be rather high in a homogeneous match. They should be rather low in a heterogeneous match and further decrease in the degree of heterogeneity, \(\Delta t\).\(^{10}\) Intuitively, a heterogeneous match between a \(t_L\)-worker and a \(t_H\)-worker results in the discouragement of the less able worker, so that the more able worker optimally reacts by reducing effort as well. This competition effect yields that, for maximum efforts under given tournament prizes, we must have that \(t_2 = t_L\) if effort and abilities are substitutes in the sense of \(\frac{\partial^2 h}{\partial e_i \partial t_i} = 0\).

At the second stage, the firm chooses \(w_1\) and \(w_2\) to maximize expected profit. In doing so, it has to consider several constraints. On the one hand, it needs to consider the workers’ incentive constraints (1) and (2). On the other hand, it needs to make sure that the workers take part in the tournament. Because of the limited liability assumption, however, the latter constraint can

\(^{10}\)In general, given a heterogeneous match with \(t_1 < t_2\), we obtain \(\frac{\partial c_1}{\partial t_2}, \frac{\partial c_2}{\partial t_2} < 0\) for \(\frac{\partial^2 h}{\partial e_i \partial t_i} = 0\) due to less balanced competition; if \(\frac{\partial^2 h}{\partial e_i \partial t_i} > 0\), then the same comparative statics will hold, when the negative effect of more uneven competition outweighs extra incentives for the leading worker from increased ability. See Appendix A on a detailed analysis.
be ignored: regardless of the size of prizes, a worker can always ensure himself a nonnegative payoff by choosing zero effort. If instead the worker chooses positive effort, he must be even better off so that his equilibrium payoff always exceeds his reservation value of zero. A direct consequence is that the firm always finds it optimal to choose \( w_2 = 0 \). A higher loser prize would clearly not be optimal because it would increase wage costs and, at the same time, decrease the workers’ incentive to put forth effort. Hence, \( w_1 = \Delta w \) and we can write \( e_1^* = e_1^* (w_1, t_L, t_2) \) and \( e_2^* = e_2^* (w_1, t_2) \). Expected profit of the firm is then given by

\[
E\pi = p_j \left[ h \left( e_1^* (w_1, t_L, t_H), t_L \right) + h \left( e_2^* (w_1, t_L, t_H), t_H \right) \right] \\
+ (1 - p_j) \left[ h \left( e_1^* (w_1, t_L, t_L), t_L \right) + h \left( e_2^* (w_1, t_L, t_L), t_L \right) \right] - w_1.
\]

Let the firm’s problem be well behaved\(^\text{11}\) so that the optimal winner prize is determined by the first-order condition

\[
p_j \frac{\partial h \left( e_1^* (w_1, t_L, t_H), t_L \right) \partial e_1^* (w_1, t_L, t_H)}{\partial w_1} + p_j \frac{\partial h \left( e_2^* (w_1, t_L, t_H), t_H \right) \partial e_2^* (w_1, t_L, t_H)}{\partial w_1} \\
+ (1 - p_j) \frac{\partial h \left( e_1^* (w_1, t_L, t_L), t_L \right) \partial e_1^* (w_1, t_L, t_L)}{\partial w_1} \\
+ (1 - p_j) \frac{\partial h \left( e_2^* (w_1, t_L, t_L), t_L \right) \partial e_2^* (w_1, t_L, t_L)}{\partial w_1} = 1.
\]

If specific functional forms were assumed for functions \( h \) and \( c \), this condition could be solved explicitly for the optimal prize. We revisit this point later when we illustrate our findings with a specific example. Before, however, we turn to the first stage of the model. Here, the firm must choose from which

\(^{11}\)In the example below, the firm has a strictly concave objective function.
pool to hire its worker. Recall that all workers have the same reservation value, hence hiring a worker with high expected ability would not be more expensive than hiring a worker whose expected ability is low. As we will see below, though, the firm may find it profitable to hire a worker with low expected ability.

Note that the optimal winner prize, as determined by (3), depends on $p_j$ and thus on the quality of the pool from which the worker is hired. In the following, we therefore write the optimal winner prize as $w_1^* = w_1^*(p_j)$. Using this notation, the firm’s optimal expected profit can be written as

$$E^{\pi^*} = p_j \left[ h(e_1^* (w_1^* (p_j), t_L, t_H), t_L) + h(e_2^* (w_1^* (p_j), t_L, t_H), t_H) \right]$$

$$+ (1 - p_j) \left[ h(e_1^* (w_1^* (p_j), t_L, t_L), t_L) + h(e_2^* (w_1^* (p_j), t_L, t_L), t_L) \right] - w_1^* (p_j),$$

which immediately leads to our main result:

**Proposition 1** The derivative $\frac{dE^{\pi^*}}{dp_j}$ is negative iff

$$[h(e_1^* (w_1^* (p_j), t_L, t_H), t_L) - h(e_1^* (w_1^* (p_j), t_L, t_L), t_L)] +$$

$$[h(e_2^* (w_1^* (p_j), t_L, t_H), t_H) - h(e_2^* (w_1^* (p_j), t_L, t_L), t_L)] < 0.$$

**Proof.** In a more general form, optimal expected profit can be written as $E^{\pi^*} = E^{\pi^*} (p_j, w_1^* (p_j))$. Hence, we have $\frac{dE^{\pi^*}}{dp_j} = \frac{\partial E^{\pi^*}}{\partial p_j} + \frac{\partial E^{\pi^*}}{\partial w_1^*} \frac{d w_1^*}{dp_j}$. In the optimum, $\frac{\partial E^{\pi^*}}{\partial w_1^*} = 0$ so that the envelope theorem applies: $\frac{dE^{\pi^*}}{dp_j} = \frac{\partial E^{\pi^*}}{\partial p_j}$. Using (4), computation of $\frac{dE^{\pi^*}}{dp_j} < 0$ yields condition (5). ■

Proposition 1 shows that there are situations in which the firm prefers to hire a worker from a low-quality worker pool. The first line in (5) is unambiguously negative due to the competition effect (i.e., low-type worker 1 chooses strictly less effort when matched with a high-type worker 2 than with
another low-type one). The sign of the second line in (5) is not clear: On the one hand, again the competition effect applies, which makes worker 2 choose lower effort in a heterogeneous match compared to a homogeneous one. On the other hand, however, a second effect works in the opposite direction, which can be labeled ability effect: if worker 2 has a high ability, this leads directly to higher expected output since, by assumption, the production function $h$ monotonically increases in ability. Similarly, if $\partial^2 h (e_i, t_i) / \partial e_i \partial t_i > 0$, the higher ability makes the worker more productive on the margin and induces him to choose a higher effort. If the ability effect is stronger than the competition effect for worker 2, the second line of condition (5) will be positive. Summarizing, Proposition 1 shows under which circumstances the competition effect dominates and the firm prefers to strategically reduce the quality of its workforce.

Finally, by using a certain specification for $h$ and $c$, we will demonstrate that condition (5) can indeed be satisfied. Let $h (e_i, t_i) = e_i + t_i$ and $c (e_i) = e_i^3 / 3$. For this specification the first-order conditions (1) and (2), together with $\Delta w = w_1$, yield $w_1 g (e_1 + t_L - e_2 - t_2) = c_1^2 - c_2^2$ such that in a homogeneous match ($t_2 = t_L$), both workers exert effort $e_{ho} = \sqrt{w_1 g (0)}$ in equilibrium, whereas in a heterogeneous match ($t_2 = t_H$), both choose $e_{he} = \sqrt{w_1 g (-\Delta t)}$. Note that both workers choose strictly higher effort levels in the homogeneous match because of the competition effect. Technically, $g (-\Delta t) < g (0)$ due to unimodality of the convolution $g$, implying $e_{he} < e_{ho}$. At the second stage of the game, the firm solves

$$\max_{w_1} p_j \left[ 2\sqrt{w_1 g (-\Delta t) + t_L + t_H} + (1 - p_j) \right] 2\sqrt{w_1 g (0) + 2t_L} - w_1.$$ 

Since the firm’s objective function is strictly concave, the optimal winner prize is characterized by the first-order condition: $w_1^* (p_j) = (p_j \sqrt{g (-\Delta t) + \Delta t}) +$
After applying the envelope theorem to $E \pi^*$ and inserting for $w_1^* (p_j)$, straightforward calculations lead to the following result:

**Corollary 1** Given $h (e_i, t_i) = e_i + t_i$ and $c (e_i) = e_i^3 / 3$, the derivative $\frac{dE \pi^*}{dp_j}$ is negative iff

$$\Delta t < 2 \left( \sqrt{g (0)} - \sqrt{g (-\Delta t)} \right) \left( \sqrt{g (-\Delta t)} p_j + (1 - p_j) \sqrt{g (0)} \right).$$

The right-hand side of the condition in Corollary 1 describes the competition effect, whereas the left-hand side characterizes the ability effect. If the competition effect dominates the ability effect, the firm will optimally hire an external worker from a worker pool that offers relatively low expected ability.

### 4 Mixed-Strategy Equilibria

In our model, we have assumed that there is sufficient exogenous noise so that a pure-strategy equilibrium in the tournament game exists. However, in case of insufficient noise either the two workers will exert zero effort if their abilities differ substantially or there are only equilibria in mixed strategies. Consider, for example, a setting with zero noise, which transforms the tournament game into an all-pay auction, and let again $h (e_i, t_i) = e_i + t_i$. Define, in a heterogeneous match, $e_L - \Delta t$ as the low-ability worker’s relative score and $e_H + \Delta t$ as the high-ability worker’s relative score. Further note that, due to limited liability and zero reservation values of the workers, the firm again optimally chooses a zero loser prize and a non-negative winner prize $w_1$. We obtain the following result:

\[12\] This happens, e.g., if $\bar{\varepsilon}_2 - \bar{\varepsilon}_1$ is normally distributed with zero mean and variance $\sigma^2 = \frac{1}{3}$, while $\Delta t = 1$ and $p_j = 0.5$.\]
Proposition 2  Let worker $i$’s output be described by $q_i = e_i + t_i$. (i) In a homogeneous match, there exists a unique equilibrium in mixed strategies where each worker chooses effort according to the cdf $G(e) = c(e)/w_1$ with $e \in [0, \bar{e}]$. (ii) In a heterogeneous match, if $w_1 < c(\Delta t)$, both workers exert zero effort in equilibrium; if $w_1 > c(\Delta t)$, there is a mixed-strategy equilibrium with the low-ability worker randomizing his relative score $e_L - \Delta t$ according to cdf

$$G_L(x) = \begin{cases} 
1 - \frac{c(\bar{e} - \Delta t)}{w_1} & \text{for } x \in [0, \bar{e} - \Delta t] \\
1 - \frac{c(\bar{e} - \Delta t)}{w_1} + \frac{c(x)}{w_1} & \text{for } x > \bar{e} - \Delta t,
\end{cases}$$

whereas the high-ability one chooses his relative score $e_H + \Delta t$ according to the cdf

$$G_H(x) = \begin{cases} 
\frac{c(x)}{w_1} & \text{for } x \in [\Delta t, \bar{e}] \\
1 & \text{for } x > \bar{e}
\end{cases}$$

with $G_L(x) := P(e_L - \Delta t \leq x)$, $G_H(x) := P(e_H + \Delta t \leq x)$ and $c(\bar{e}) = w_1$.

Proof. See Appendix B. □

Part (ii) of Proposition 2 shows that, if the workers’ degree of heterogeneity – measured by $\Delta t$ – is sufficiently large relative to incentives, both players will drop out. Intuitively, in that case it does not pay for the low-ability worker to catch up with his more able co-worker. In order to save effort costs, the low-ability worker chooses zero effort. The high-ability worker’s best response then is to choose zero effort as well. If the degree of heterogeneity is not too large relative to given incentives (i.e., $w_1 > c(\Delta t)$), both workers choose zero effort with a strictly positive probability.$^{13}$ The drop-out probability of the low-ability worker, $1 - \frac{c(\bar{e} - \Delta t)}{w_1}$, monotonically increases in

$^{13}$Note that if workers differ in their cost functions instead of their production functions, the low-ability worker’s drop-out probability also increases in worker heterogeneity, see Baye, Kovenock and de Vries (1996).
worker heterogeneity. To sum up, we have qualitatively the same kind of competition effect as in Section 3 with pure-strategy equilibria.

Again, we can construct reasonable examples where the firm’s expected profits decrease in the average quality of the worker pool from which the firm hires its new workers. Let effort costs be quadratic: \( c(e) = e^2 \). In that case, \( \bar{e} = \sqrt{w_1} \) and expected efforts in a homogeneous match between two low-type workers amount to

\[
2 \int_0^{\sqrt{w_1}} e \frac{2e}{w_1} de = \frac{4w_1^{\frac{3}{2}}}{3}.
\]

In a heterogeneous match with sufficiently high incentives (i.e., \( w_1 > c(\Delta t) \)), total expected efforts are

\[
\int_0^{\sqrt{w_1}} (x + \Delta t) \frac{2x}{w_1} dx + \int_{\Delta t}^{\sqrt{w_1}} (x - \Delta t) \frac{2x}{w_1} dx = \frac{2}{3w_1} \left( 2w_1^{\frac{3}{2}} - 3\Delta tw_1 + \Delta t^3 \right).
\]

Note that RHS(6) > RHS(7) \( \Leftrightarrow 3w_1 > \Delta t^2 \) is satisfied since we must have \( w_1 > \Delta t^2 \) in case of a mixed-strategy equilibrium. Hence, the competition effect also works in the all-pay auction setting. When deciding on the optimal winner prize, the firm maximizes expected profits

\[
E\pi = p_j \left( \frac{2}{3w_1} \left( 2w_1^{\frac{3}{2}} - 3\Delta tw_1 + \Delta t^3 \right) + t_H + t_L \right) + (1 - p_j) \left( \frac{4}{3}w_1^{\frac{1}{2}} + 2t_L \right) - w_1.
\]

The derivative with respect to \( w_1 \) yields

\[
\frac{\partial E\pi}{\partial w_1} = \frac{1}{3w_1^{\frac{3}{2}}} \left( -2p_j \Delta t^3 + w_1^{\frac{3}{2}} \left( 2 - 3\sqrt{w_1} \right) \right).
\]

\footnote{For the case of an all-pay auction, we can skip the assumption \( c''(e_i) > 0 \) for all \( e_i \in \mathbb{R}_+ \).}
Recall that for the existence of the asymmetric mixed-strategy equilibrium at the tournament stage and, thus, for positive incentives in a heterogeneous match, we must have that \( w_1 > \Delta t^2 \iff \sqrt{w_1} > \Delta t \). Obviously, if \( \Delta t > 2/3 \) then \( \partial E \pi/\partial w_1 < 0 \) (i.e., the degree of worker heterogeneity is so large that generating incentives is too expensive for the firm). Hence, let \( \Delta t = 1/4 \). Furthermore, to show that \( dE \pi^*/dp_j \) can be negative, let \( p_j = 1/4, t_L = 1 \) and \( t_H = 5/4 \). In this example, the optimal winner prize is approximately \( w_1^* = 0.45 \). Moreover,

\[
\frac{dE \pi^*}{dp_j} \bigg|_{p_j = \frac{1}{4}, w_1^* = 0.45} = -0.22685,
\]

that is, due to the competition effect, the firm prefers to fill its vacancy by hiring from a worker pool with low average quality.

5 Conclusion

In this paper, we have shown that a firm using a competitive career system to induce incentives, sometimes prefers to hire a worker with low expected ability, although it could hire a more able worker at the same wage. The hiring decision is determined by the interplay of two effects. Due to the ability effect, the firm should hire a high-ability worker since the worker’s higher ability contributes to firm output and makes the worker more productive on the margin. If the firm currently employs a worker with low ability, the competition effect works in the opposite direction: a heterogeneous match between the current worker and a new high-ability worker would result in uneven career competition in which the workers choose relatively low effort. Accordingly, this effect makes hiring a worker of low ability more attractive. We have identified situations where the second effect dominates and the firm
indeed wants to hire a low-ability worker.

The results depend on the assumption that one position in the firm is already occupied by a low-ability worker and the firm is not able to fire this worker or does not want to. This assumption, however, is quite natural in many real-world settings. Workers are often protected by labor laws so that a firm can dismiss a worker only at high costs. Moreover, workers may have acquired firm-specific human capital while working for a firm. This human capital would be lost if a worker left the firm. Accordingly, the firm may want to retain the worker even if he is of low ability.

Note that handicaps cannot completely solve the problem of insufficient incentives in a situation where the firm hires a worker from a high-ability worker pool. This is because the firm does not know the worker’s exact ability. Thus, even if handicaps were admitted, the effects described in this paper would continue to hold.
Appendix

Appendix A: Impact of Heterogeneity on Equilibrium Efforts

Applying the implicit-function theorem to the set of first-order conditions (1) and (2) yields

\[
\begin{pmatrix}
\frac{\partial \Delta B_1}{\partial e_1} - e''(e_1) \\
\frac{\partial \Delta B_2}{\partial e_1} \\
\frac{\partial \Delta B_1}{\partial e_2} - e''(e_2) \\
\frac{\partial \Delta B_2}{\partial e_2}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial e_1}{\partial t_2} \\
\frac{\partial e_2}{\partial t_2}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{\partial \Delta B_1}{\partial t_2} \\
\frac{\partial \Delta B_2}{\partial t_2}
\end{pmatrix}
\]

with \( \Delta B_i := \Delta w g \left( h(e_1, t_1) - h(e_2, t_2) \right) \frac{\partial h(e_i, t_i)}{\partial e_i} \) denoting worker \( i \)'s marginal benefit from increasing effort. Note that \( \det J \) describes the Jacobian determinant, which is strictly positive since \( \frac{\partial \Delta B_2}{\partial e_1} = -\frac{\partial \Delta B_1}{\partial e_2} \), and \( \frac{\partial \Delta B_i}{\partial e_i} - c''(e_i) < 0 \) by the strict concavity of each worker's objective function. Define

\[
J_1 := \begin{pmatrix}
-\frac{\partial \Delta B_1}{\partial e_2} & \frac{\partial \Delta B_1}{\partial e_1} \\
-\frac{\partial \Delta B_2}{\partial e_2} & \frac{\partial \Delta B_2}{\partial e_1} - c''(e_2)
\end{pmatrix}
\quad \text{and} \quad
J_2 := \begin{pmatrix}
\frac{\partial \Delta B_1}{\partial e_1} - c''(e_1) & -\frac{\partial \Delta B_1}{\partial e_2} \\
\frac{\partial \Delta B_2}{\partial e_1} & -\frac{\partial \Delta B_2}{\partial e_2}
\end{pmatrix}.
\]

Then, by Cramer's rule,

\[
\frac{\partial e_1}{\partial t_2} = \frac{\det J_1}{\det J} = \frac{-\frac{\partial \Delta B_1}{\partial e_2} \left( \frac{\partial \Delta B_2}{\partial e_2} - c''(e_2) \right) + \frac{\partial \Delta B_2}{\partial e_2} \frac{\partial \Delta B_1}{\partial e_1}}{\det J}
\]

\[
\frac{\partial e_2}{\partial t_2} = \frac{\det J_2}{\det J} = \frac{-\frac{\partial \Delta B_2}{\partial e_2} \left( \frac{\partial \Delta B_1}{\partial e_1} - c''(e_1) \right) + \frac{\partial \Delta B_1}{\partial e_1} \frac{\partial \Delta B_2}{\partial e_2}}{\det J}.
\]

Let \( t_2 > t_1 \). In that case, (1) and (2), together with \( \frac{\partial^2 h}{\partial e_i \partial t_i} \geq 0 \), imply \( e_1 \leq e_2 \) in equilibrium. Moreover, we have \( h(e_1, t_1) - h(e_2, t_2) < 0 \) and, due to \( g \)'s unimodality, \( g' \left( h(e_1, t_1) - h(e_2, t_2) \right) < 0 \), which implies \( \frac{\partial \Delta B_1}{\partial e_2} < 0 \) and \( \frac{\partial \Delta B_2}{\partial e_1} > 0 \). Intuitively, if the leading worker exerts more effort, uneven competition will become even more uneven so that it pays less for the trailing
worker to exert additional effort. However, if the trailing worker catches up, competition will become more balanced, and the leading worker’s marginal benefit of exerting more effort increases. To obtain the sign of \( \frac{\partial e_1}{\partial t_2} \) and \( \frac{\partial e_2}{\partial t_2} \), we finally have to compute \( \frac{\partial \Delta B_i}{\partial t_2} \) for \( i = 1, 2 \):

\[
\frac{\partial \Delta B_1}{\partial t_2} = -\frac{\partial h(e_2, t_2)}{\partial t_2} \Delta wg'(h(e_1, t_1) - h(e_2, t_2)) \frac{\partial h(e_1, t_1)}{\partial e_1} < 0
\]
\[
\frac{\partial \Delta B_2}{\partial t_2} = -\frac{\partial h(e_2, t_2)}{\partial t_2} \Delta wg'(h(e_1, t_1) - h(e_2, t_2)) \frac{\partial h(e_2, t_2)}{\partial e_2} < 0
\]

\[
+ \Delta wg(h(e_1, t_1) - h(e_2, t_2)) \frac{\partial h(e_2, t_2)}{\partial e_2 \partial t_2} \geq 0
\]

Hence, for both workers the uneven competition becomes yet more uneven. However, for the leading worker there is an additional effect that works in the opposite direction if effort and ability are strict complements: in that case, enhanced ability increases his marginal productivity, which, in turn, increases his marginal benefit from exerting additional effort. Altogether, we unambiguously have \( \frac{\partial e_1}{\partial t_2}, \frac{\partial e_2}{\partial t_2} < 0 \) for \( \frac{\partial^2 h}{\partial e_i \partial t_i} = 0 \) due to less balanced competition (competition effect); if \( \frac{\partial^2 h}{\partial e_i \partial t_i} > 0 \), then the same comparative statics will only hold if the competition effect outweighs extra incentives for the leading worker from increased ability.
Appendix B: Proof of Proposition 2

(i) In case of a homogeneous match (i.e., $t_1 = t_2 = t_L$), worker $i$ wins against worker $j$ if $e_i > e_j$. Hence, the game describes an all-pay auction with complete information, which does not have an equilibrium in pure strategies (see, e.g., Baye, Kovenock and de Vries 1996). However, Baye, Kovenock, and de Vries (1996) show that for the given setting there is a unique symmetric mixed-strategy equilibrium where each worker is indifferent between (i) exerting zero effort and winning the zero loser prize for sure, (ii) exerting maximum effort $\bar{e}$ with $w_1 = c(\bar{e})$, and (iii) choosing an effort between 0 and $\bar{e}$ according to the cdf $G$ with $w_1 \cdot G(e_i) - c(e_i) = 0$. In equilibrium, each worker’s mixed strategy $G(e)$ is therefore given by $G(e) = c(e)/w_1$ with $e \in [0, \bar{e}]$.\(^{15}\)

(ii) In the case of a heterogeneous match (i.e., $t_1 = t_L < t_2 = t_H$), worker 1 wins if $e_1 - \Delta t > e_2$, but worker 2 wins if $e_2 + \Delta t > e_1$. Define $s_L := e_1 - \Delta t$ as worker 1’s relative score and $s_H := e_2 + \Delta t$ as worker 2’s relative score. The tournament game between the two workers for the given winner prize $w_1$ can be solved in analogy to Lemma 1 in Konrad (2002, p. 1526): if $w_1 < c(\Delta t)$, it does not pay for trailing worker 1 to catch up with leading worker 2. Hence, worker 1 optimally chooses zero effort and worker 2’s best response is to choose zero effort as well. However, if $w_1 > c(\Delta t)$, then again a mixed-strategy equilibrium exists. Worker 1 will not choose any effort in the interval $(0, \Delta t)$ because such an effort level would imply strictly positive costs without leading to a positive winning probability. Instead, worker 1 drops out (i.e., chooses zero effort) with a certain probability and, with the rest of the probability mass, randomizes his effort choice over the interval $[\Delta t, \bar{e}]$ with $\bar{e}$

\(^{15}\)See, e.g., Konrad (2009), p. 27, on the case of convex costs. See Siegel (2011) for the general case of more than two players.
being defined in the paragraph before. Thus, in terms of worker 1’s score, (a) 

\[ s_L = -\Delta t \]

with some probability \( P(s_L = -\Delta t) > 0 \), (b) \( s_L \neq (\Delta t, 0) \), and (c) \( s_L \in [0, \bar{e} - \Delta t] \) with probability \( 1 - P(s_L = -\Delta t) \). Following Konrad (2002), worker 2 also chooses zero effort with positive probability since he already has a lead of size \( \Delta t \). Within his mixed equilibrium strategy, he will not choose a higher score than \( s_H = \bar{e} \) because, in that case, he could still win the tournament for sure but save effort cost by reducing his score. Thus, worker 2 optimally randomizes his score \( s_H \) over the interval \([\Delta t, \bar{e}]\).

In a next step, we can derive the workers’ mixed equilibrium strategies. The situation for worker 1 is similar to that of the two low-type workers in the homogeneous match. In the mixed equilibrium, worker 1 must be indifferent between (i) earning zero income when choosing \( s_L = -\Delta t \), (ii) earning zero income with \( s_L = \bar{e} - \Delta t \), and (iii) choosing a score \( s_L \) between 0 and \( \bar{e} - \Delta t \) leading to expected income \( w_1 \cdot G_{s_H}(\bar{e}_L) - c(\bar{e}_L) \) where \( G_{s_H} \) denotes worker 2’s equilibrium cdf. Therefore, we must have that \( 0 = w_1 \cdot G_{s_H}(\bar{e}_L) - c(\bar{e}_L) \), yielding \( G_{s_H}(x) = c(x)/w_1 \) \((x \in [\Delta t, \bar{e}]\)) as worker 2’s mixed strategy in equilibrium. With probability

\[
1 - \int_{\Delta t}^{\bar{e}} G'_{s_H}(x) \, dx = 1 - \int_{\Delta t}^{\bar{e}} \frac{c'(x)}{w_1} \, dx = 1 - \frac{c(\bar{e})}{w_1} + \frac{c(\Delta t)}{w_1} = \frac{c(\Delta t)}{w_1}
\]

worker 2 exerts zero effort (i.e., \( s_H = \Delta t \)) in equilibrium. Worker 1’s equilibrium cdf can be derived by looking at 2’s problem. Worker 2 must be indifferent between just outperforming worker 1, approximately leading to income \( w_1 - c(\bar{e} - \Delta t) \), and any effort level with corresponding score \( s_H \in [\Delta t, \bar{e}] \), yielding expected income \( w_1 \cdot G_{s_L}(\bar{e}_H) - c(\bar{e}_H) \) with \( G_{s_L} \) as worker 1’s equilib-
rium cdf. Thus, $G_{sL}$ is given by $G_{sL}(x) = 1 - \frac{c(\bar{e} - \Delta t)}{w_1} + \frac{c(x)}{w_1}$. With probability

$$P(s_L = -\Delta t) = 1 - \int_0^{\bar{e}} - \Delta t G_{sL}'(x) \, dx = 1 - \frac{c(\bar{e} - \Delta t)}{w_1}$$

worker 1 exerts zero effort. Note that this probability is strictly positive due to $c(\bar{e}) = w_1$. Summarizing, for $w_1 > c(\Delta t)$ there is a mixed-strategy equilibrium being described by

$$G_{sL}(x) = \begin{cases} 
1 - \frac{c(\bar{e} - \Delta t)}{w_1} & \text{for } x \in [-\Delta t, 0) \\
1 - \frac{c(\bar{e} - \Delta t)}{w_1} + \frac{c(x)}{w_1} & \text{for } x \in [0, \bar{e} - \Delta t] \\
1 & \text{for } x > \bar{e} - \Delta t
\end{cases}$$

and

$$G_{sH}(x) = \begin{cases} 
\frac{c(x)}{w_1} & \text{for } x \in [\Delta t, \bar{e}] \\
1 & \text{for } x > \bar{e}.
\end{cases}$$
References


