Competitive Careers as a Way to Mediocrity
Matthias Kräkel*

University of Bonn,
Adenauerallee 24-42, D-53113 Bonn, Germany,

Abstract
I show that in competitive careers based on individual performance the least productive individuals may have the highest probabilities to be promoted to top positions. These individuals have the lowest fall-back positions and, hence, the highest incentives to succeed in career contests. This detrimental incentive effect exists irrespective of whether effort and talent are substitutes or complements in the underlying contest-success function. However, in case of complements the incentive effect can be outweighed by a productivity effect that favors high effort choices by the more talented individuals. Switching from wages-attached-to-jobs to pay-for-performance at top career positions can be a solution to the mediocrity problem.

* tel: +49 228 733914, fax: +49 228 739210, e-mail: m.kraekel@uni-bonn.de.

JEL Classification: D72; J44; J45; M51

Key Words: career competition; contest; mediocrity; politicians.
1 Introduction

A society where people with average or less than average talent are assigned to key positions (e.g., in politics) can be called a mediocracy. Career systems that are not based on individual performance but on criteria that are unrelated to talent (e.g., like seniority) may end up in such society. At first sight, one might suspect that competitive career systems in which the most effective individuals climb the ladder should lead to a significantly better outcome. However, in this paper I show that competitive career systems may tend to promote the least productive individuals, thus leading to mediocracy. The intuition for this result comes from the fact that more productive people have better fall-back positions than less productive ones when failing in the competition for top positions. Hence, highly productive people have only moderate incentives to win the competition for top jobs, whereas individuals with low productivity have strong incentives to avoid their rather unattractive fall-back positions.

I use a contest model to analyze competitive careers of heterogeneous individuals. At the beginning, all individuals have the chance to reach the top position with the highest possible career income.\(^1\) However, during the career there are losers and winners, where winners still compete for the top job but losers strive towards less important positions in a so-called consolation match.\(^2\) Winning such consolation match defines the fall-back position of an individual. The fact that more productive individuals have better fall-back positions than less productive individuals does not depend on the underlying contest-success technology: In any kind of consolation contest, a more productive player can either achieve the same winning probability at less effort costs or a higher winning probability at the same effort costs compared to a less productive opponent. Hence, the expected utility of an individual when participating in a consolation contest will always be positively correlated with his productivity. As natural consequence, the individuals with the

---

\(^1\)Another motive for career competition is typically an increase in status. See Frank (1985) on the role of status as motivation in individual careers. Moldovanu et al. (2007) discuss competition for status in a formal contest model.

\(^2\)See, for example, Uehara (2009) on consolation contests in a Japanese corporation. For a theoretical analysis of consolation contests see Kiyotaki (2004).
lowest productivities have the highest incentives to climb the ladder in order to avoid their respective fall-back positions.

In a first step, I analyze a setting with effort and talent (or productivity) being substitutes in the contest-success function of an individual. The results show that the individuals with the worst fall-back positions will be most likely to win the contest for the top position, due to the incentive effect mentioned above. In a second step, I consider a career model where talent and effort are complements in the contest-success function. Again, there is a detrimental incentive effect that makes winning of the top position by the least productive individuals most likely. However, now an additional effect works into the opposite direction: The larger an individual’s productivity the more effective the exertion of effort to win the contest will be. For this reason, a highly talented individual may prefer to spend high effort despite an attractive fall-back position. If this productivity effect is dominated by the incentive effect, society will still tend to a mediocracy. Otherwise, the most talented people will succeed in the career competition and will be assigned the top positions. For the class of linear impact functions and time-invariant talents of individuals, I can show that in the complements model the productivity effect always dominates the incentive effect, thus eliminating the mediocracy problem.

Aside from the application to society in general, the model also offers insights for competitive careers in a more concrete situation. The best example comes from politics. During his life, a politician faces different career opportunities: He can try to reach the top political position as a leader of a political party and/or the president of country. If such top position cannot be achieved, the politician may still struggle for less influential political positions (e.g., as a minister). Alternatively, the politician can return to his original occupation, where he worked previously before becoming a full-time politician. Typically, if a politician does not succeed to reach the top, he has to consider a less influential political position or his original occupation as possible fall-back positions. In either of these alternatives, the politician will be more successful the higher his talent. Consider, for example, a politician that is highly talented and possesses a legal qualification. This individual will have a larger success probability when choosing a secondary
political career path or a fall-back position as lawyer than a less talented one with a legal qualification. Consequently, less talented individuals should have significantly stronger incentives to become the political leader of a country than highly talented individuals.

In Section 5, I will discuss the robustness of the mediocracy result by referring to an extended career ladder that consists of two steps and by referring to the possibility of sandbagging. Moreover, I will consider pay-for-performance as a possible solution to the mediocracy problem. If, for example, the income of a politician at a certain leading position is not determined by a wages-attached-to-jobs policy but depends on individual performance, top political positions should be quite attractive for the most talented candidates. Combining top positions with pay-for-performance would lead to strong career incentives for highly productive individuals, who expect sufficiently large incomes when reaching the top. By this wage policy, the detrimental incentive effect stated above could be turned into a beneficial one. However, wages should be attached to subordinate political jobs since pay-for-performance at lower career levels may aggravate the mediocracy problem.

My paper is related to two strands of the literature. First of all, it is related to the work on contests. Congleton et al. (2008a, 2008b) and Konrad (2009) offer a comprehensive overview of the most frequently used contest-success functions, applications of contest models and the most important results in contest theory. There are three widely used classes of contest-success functions (csf) – the ratio-form csf, the difference-form csf, and the all-pay auction.\(^3\) My paper builds on the ratio-form csf, which was introduced by Tullock (1980) and has often been applied to the discussion of rent seeking and distribution conflicts.\(^4\) The ratio-form model has been extended in various directions. Ryvkin (2010) compares a ratio-form contest with private information about players’ marginal costs with the standard case of public information. Eggert and Kolmar (2006) and Lee (2007) consider a rent-seeking contest where the rent increases with the number of contestants.


\(^{4}\)See Aidt and Hillman (2008) on a dynamic rent-seeking setting.
Skaperdas (1996) offers a generalized and axiomatized ratio-form csf. The ratio-form model is particularly used for the discussion of collective rent seeking where different groups compete against each other, see, for example, Lee and Kang (1998), Lee and Cheong (2005), Baik et al. (2006), Baik and Lee (2007), and Cheikbossian (2008).

The paper is also related to the public economics literature on political careers and bad politicians. This literature offers an explanation for why political leaders and presidents often have rather low qualifications or are of low quality. Contrary to my paper, they do not consider a contest framework but follow either the political-agency or the citizen-candidate approach (see, among many others, Caselli and Morelli (2004), Messner and Polborn (2004), Mattozzi and Merlo (2007)). In the model by Caselli and Morelli (2004), bad and dishonest candidates have rather low opportunity costs and extract more money from their political positions. These two effects make them more likely to run for office. Messner and Polborn (2004) consider a similar approach, but in their setting political candidates care for both their salaries and the impact of the politician’s ability on the quality of the political office. Mattozzi and Merlo (2007) show that, due to competition for highly talented individuals between the political sector and the lobbying sector, a political party may prefer not to recruit the best candidates although they would accept a contract offer. However, all these papers do not address the individuals’ incentives and endogenous investments or activities during their careers, which take center stage in my model.

The paper is organized as follows. Section 2 introduces the model. In Section 3, I consider the case of activities and productivities being substitutes in an individual’s contest-success function. Section 4 turns to a situation with activities and productivities as complements. Section 5 further discusses the robustness of the main findings and pay-for-performance as a possible way out of the mediocrity problem. Section 6 concludes.

2 The Model

I consider a career game between \( N \geq 3 \) risk neutral individuals (e.g., full-time politicians). Similar to Lazear and Rosen (1981), the model sketches the long-term
career of the individuals without using a dynamic setting with multiple periods. The game starts with the $N$ players simultaneously choosing productive activities (e.g., efforts) $a_{i,m} \geq 0$ ($i = 1, \ldots, N$) to become the winner of the major career contest $m$. This winner receives a high long-term career income $Y_H$. The game is finished for this player, who has achieved the top position. The $N - 1$ other players enter a consolation match $c$ where they compete for less attractive positions by choosing activities $a_{i,c} \geq 0$. The winner of this consolation contest earns $Y_M$ in the long run, whereas the $N - 2$ losers get $Y_L$ with $0 < Y_L < Y_M < Y_H$. I assume that the three career incomes are sufficiently large so that all players choose positive efforts in equilibrium.

In both, the major contest for $Y_H$ and the consolation contest each player has to bear the costs of his activity (e.g., disutility of effort or opportunity costs of time). For simplicity, when player $i$ exerts activity level $a_{i,\kappa}$ in contest $\kappa \in \{m, c\}$ let his costs be equal to $a_{i,\kappa}$. Moreover, in both contests, players face the same kind of contest-success function based on the one suggested by Skaperdas (1996): If player $i$ chooses $a_{i,\kappa}$ and the other players $j \neq i$ choose activity levels $a_{j,\kappa}$, then $i$’s probability of winning contest $\kappa \in \{m, c\}$ is given by

$$p_{i,\kappa} = \frac{f_\kappa (a_{i,\kappa}, \delta_{i,\kappa})}{f_\kappa (a_{i,\kappa}, \delta_{i,\kappa}) + \sum_{j \neq i} f_\kappa (a_{j,\kappa}, \delta_{j,\kappa})}$$

with $f_\kappa$ as strictly positive impact function. $f_\kappa$ is assumed to be monotonically increasing and concave in $a_{i,\kappa}$. Hence, each player increases his winning probability by exerting more effort, but the other contestants’ efforts as well as luck or measurement error also influence the outcome of the contest. I assume that $f_\kappa$ is decreasing in the exogenous parameter $\delta_{i,\kappa} > 0$ that indicates player $i$’s talent or productivity in contest $\kappa$. The lower the value of $\delta_{i,\kappa}$ the more productive will be the player. If, for example, all players choose identical activity levels (i.e., $a_{i,\kappa} = a$,

---

5 Throughout the paper, the first subscript indicates the individual, whereas the second one stands for the kind of contest.

6 Note that in the model career incomes are exogenously given, but as can be seen below the general effects still exist under incomes being endogenously chosen by a contest designer.

7 As the player’s activity is assumed to be productive, $f_\kappa (a_{i,\kappa}, \delta_{i,\kappa})$ may describe a politician’s output.
∀i), the most productive player will have the highest winning probability, whereas the player with the lowest productivity is least likely to win the contest.

I assume that the ranking between the parameters δ₁,m, ..., δ₉,m is identical to the ranking between δ₁,c, ..., δ₉,c with respect to the first subscript, which denotes the individual; that is, if δᵢ,m < δⱼ,m then δᵢ,c < δⱼ,c, and vice versa. However, δᵢ,m ≠ δᵢ,c (i ∈ {1, ..., N}) is possible. In words, the individuals differ in talent or productivity and the productivity ranking is consistent between career levels. Thus, tasks on adjacent career levels are related so that an individual who is more productive on a lower career level is also more productive at a higher level. However, individual talent may have a different impact on different career levels.

On the one hand, we can imagine a situation where individual talent similarly matters for the major contest and the consolation contest (δᵢ,m = δᵢ,c), since in both contests players have to fulfill nearly identical tasks. On the other hand, there may be a situation where in one of the two contests winning depends rather on personality than on talent (δᵢ,m ≠ δᵢ,c).

In the following, I will analyze two different forms of the contest-success function (1). In Section 3, I assume that activities and productivities are substitutes in the sense of

\[ f_κ(αᵢ, δᵢ, κ) = f(αᵢ - δᵢ, κ). \] (2)

Hence, an individual can substitute one unit of productivity by one unit of effort.

In Section 4, activities and productivities are assumed to be complements:

\[ f_κ(αᵢ, δᵢ, κ) = f\left(\frac{αᵢ}{δᵢ, κ}\right). \] (3)

Now, lower values of δᵢ,κ (i.e., higher productivity levels) make higher activity levels αᵢ,κ more effective. In both (2) and (3), f describes the same strictly positive, monotonically increasing and concave function.

I assume that the productivity parameters δ₁,κ, ..., δ₉,κ (κ = m, c) are common knowledge. This assumption can be justified for at least two reasons. First, players typically observe each others’ qualifications, which are positively correlated with productivity. Second, note that the common-knowledge assumption is
only introduced for the contestants. These players often observe each other when working close together in the same market or at the same organization every day. For example, the two arguments hold for a situation where politicians compete for becoming the leader of their political party.

Although I do not explicitly specify a welfare function, I assume that welfare will be highest if the most productive player (i.e., the one with parameter value \( \min\{\delta_{1,m}, \ldots, \delta_{N,m}\} \)) wins the major contest and is assigned to the most important position, which is associated with career income \( Y_H \).

### 3 Activities and Productivities as Substitutes

Career paths in politics and bureaucracies constitute good examples for the substitutes case. Here, individuals typically have to solve routine problems, so that an individual with a high productivity generates the same output at a certain task as a less productive individual who spends sufficiently more time on the same task. Hence, less talent can be easily compensated by additional working time.

#### 3.1 Consolation Contest

I solve the substitutes model with impact function (2) backwards, beginning with the stage-two contest \( c \) between those \( N - 1 \) players that have failed in the major contest \( m \) and, therefore, enter the consolation stage. Here, the winner earns career income \( Y_M \) but the \( N - 2 \) losers end up with the lower income \( Y_L \). Player \( i \) chooses activity \( a_{i,c} \) to maximize expected utility

\[
EU_{i,c}(a_{i,c}) = Y_L + (Y_M - Y_L) \frac{f(a_{i,c} - \delta_{i,c})}{f(a_{i,c} - \delta_{i,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c})} - a_{i,c}.
\]

Since this objective function is strictly concave and, by assumption, career incomes are sufficiently large to prevent a corner solution where neither player chooses positive effort, the equilibrium is described by the \( N - 1 \) first-order conditions

\[
\frac{(Y_M - Y_L) f'(a_{i,c} - \delta_{i,c}) \sum_{j \neq i} f(a_{j,c} - \delta_{j,c})}{[f(a_{i,c} - \delta_{i,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c})]^2} = 1,
\]
which can be rearranged to

\[
\frac{Y_M - Y_L}{\left[ \sum_j f(a_{j,c} - \delta_{j,c}) \right]^2} = \frac{1}{f'(a_{i,c} - \delta_{i,c}) \left[ f(a_{x,c} - \delta_{x,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c}) \right]}
\]

with player \( x \) denoting an arbitrary opponent of player \( i \). In analogy, the first-order condition of player \( x \neq i \) can be written as

\[
\frac{Y_M - Y_L}{\left[ \sum_j f(a_{j,c} - \delta_{j,c}) \right]^2} = \frac{1}{f'(a_{x,c} - \delta_{x,c}) \left[ f(a_{i,c} - \delta_{i,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c}) \right]}
\]

Combining the right-hand sides of both equations yields

\[
\frac{f(a_{i,c} - \delta_{i,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c})}{f'(a_{i,c} - \delta_{i,c})} = \frac{f(a_{x,c} - \delta_{x,c}) + \sum_{j \neq i} f(a_{j,c} - \delta_{j,c})}{f'(a_{x,c} - \delta_{x,c})}
\]

As both sides describe the same monotonically increasing function of \( a_{i,c} - \delta_{i,c} \) and \( a_{x,c} - \delta_{x,c} \), respectively, we have

\[
a_{i,c} - \delta_{i,c} = a_{x,c} - \delta_{x,c}, \quad \text{for all } i, x \in \{1, \ldots, N\}.
\]

Hence, the higher the productivity of a player the lower will be his equilibrium effort. Intuitively, a highly productive player prefers to save effort costs by choosing a low activity level since activities and productivity parameters are substitutes in the impact function \( f \). By inserting \( a_{i,c} - \delta_{i,c} = a_{x,c} - \delta_{x,c} \) (for all \( i, x \in \{1, \ldots, N\} \)), in player \( i \)'s first-order condition, the equilibrium activity level \( a_{i,c}^* \) is described by

\[
\frac{(N - 2)(Y_M - Y_L)}{(N - 1)^2 f(a_{i,c}^* - \delta_{i,c})} = \frac{1}{f'(a_{i,c}^* - \delta_{i,c})} \iff a_{i,c}^* = A \left( \frac{(N - 2)(Y_M - Y_L)}{(N - 1)^2} \right) + \delta_{i,c}
\]

with \( A(\cdot) \) denoting the monotonically increasing inverse function of \( f/f' \). Equilibrium activity increases in the size of the career income difference \( Y_M - Y_L \), because any contestant earns at least \( Y_L \) in the consolation round. Furthermore, \( a_{i,c}^* \) is non-increasing and for \( N > 3 \) strictly decreasing in the number of contestants. This effect can be labeled discouragement effect: Each player exerts less effort.
when the number of opponents increases since his relative impact on the outcome of the contest becomes smaller.

Finally, I insert equilibrium activities in the objective function $EU_{i,c}(a_{i,c})$. Thus, in equilibrium player $i$’s expected utility is given by

$$EU_{i,c}^*(a_{i,c}) = Y_L + \frac{Y_M - Y_L}{N - 1} - A \left( \frac{(N - 2) (Y_M - Y_L)}{(N - 1)^2} \right) - \delta_{i,c}. \quad (4)$$

The equation shows that a player strictly benefits from higher productivity since expected utility $EU_{i,c}^*(a_{i,c})$ decreases in the parameter $\delta_{i,c}$ in equilibrium. In other words, an individual’s fall-back position monotonically increases in his productivity or talent.

### 3.2 Major Contest

A player will earn the highest career income $Y_H$, if he passes the highest hurdle and wins the major contest. In case of losing, he will be relegated to the consolation contest associated with expected utility $EU_{i,c}^*(a_{i,c})$. Hence, player $i$’s objective function in the major contest can be written as

$$EU_{i,m}(a_{i,m}) = EU_{i,c}^*(a_{i,c}) + \frac{(Y_H - EU_{i,c}^*(a_{i,c}))}{f(a_{i,m} - \delta_{i,m})} \cdot f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m}) - a_{i,m}. \quad (5)$$

In analogy to the consolation contest, the first-order conditions of two arbitrary players $i$ and $x$ are given by

$$\frac{Y_H - EU_{i,c}^*(a_{i,c})}{\sum_j f(a_{j,m} - \delta_{j,m})^2} = \frac{1}{f'(a_{i,m} - \delta_{i,m}) [f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})]} \quad (5)$$

and

$$\frac{Y_H - EU_{x,c}^*(a_{x,c})}{\sum_j f(a_{j,m} - \delta_{j,m})^2} = \frac{1}{f'(a_{x,m} - \delta_{x,m}) [f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})]}. \quad (6)$$
Hence, if $\delta_{i,c} < \delta_{x,c}$, then we have $EU^*_{i,c}(a^*_{i,c}) > EU^*_{x,c}(a^*_{x,c})$ and

$$\frac{1}{f'(a_{i,m} - \delta_{i,m}) \left[ f(a_{x,m} - \delta_{x,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m}) \right]} < \frac{1}{f'(a_{x,m} - \delta_{x,m}) \left[ f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m}) \right]} \iff$$

$$\frac{f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})}{f'(a_{i,m} - \delta_{i,m})} < \frac{f(a_{x,m} - \delta_{x,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})}{f'(a_{x,m} - \delta_{x,m})},$$

which implies $a_{i,m} - \delta_{i,m} < a_{x,m} - \delta_{x,m}$ since $[f(y) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})]/f'(y)$ is a monotonically increasing function of $y$. Let $\delta_{(1),m} < \delta_{(2),m} < \cdots < \delta_{(N),m}$ denote the order of the players’ productivity parameters (i.e., player (1) is the most productive one), $a^*_{(1),m}, a^*_{(2),m}, \ldots, a^*_{(N),m}$ the respective equilibrium efforts and $p^*_{(1),m}, p^*_{(2),m}, \ldots, p^*_{(N),m}$ the winning probabilities in the major contest. Then we obtain the following result:

**Proposition 1.** The players’ winning probabilities in the major contest satisfy $p^*_{(1),m} < p^*_{(2),m} < \cdots < p^*_{(N),m}$ and the corresponding equilibrium efforts $a^*_{(1),m} < a^*_{(2),m} < \cdots < a^*_{(N),m}$.

**Proof.** The first part immediately follows from the fact that $\delta_{i,c} < \delta_{x,c}$ ($\kappa = m, c$) implies $a_{i,m} - \delta_{i,m} < a_{x,m} - \delta_{x,m}$ and that $p_{i,m} = f(a_{i,m} - \delta_{i,m})/[f(a_{i,m} - \delta_{i,m}) + \sum_{j \neq i} f(a_{j,m} - \delta_{j,m})]$ is strictly increasing in $a_{i,m} - \delta_{i,m}$. The second part follows from $a_{i,m} - \delta_{i,m} < a_{x,m} - \delta_{x,m} \iff a_{i,m} - a_{x,m} < \delta_{i,m} - \delta_{x,m} < 0$.

Proposition 1 shows that the more productive a player, the less likely he will win the major contest and the less effort he will choose. The intuition for the first result comes from the players’ different fall-back positions in the major contest. If a player has a large productivity (i.e., a small $\delta_{i,c}$), then he will also be a strong player in the consolation match, which will guarantee him a large expected utility $EU^*_{i,c}(a^*_{i,c})$ as a kind of fall-back position. This fact reduces his incentives in the major contest so that we have a tendency to mediocrity where key positions are filled by less productive individuals. In other words, participation in the consolation match is rather unattractive for the less productive players so that they have very strong
incentives to win the major contest.

Note that the results of Proposition 1 do not depend on whether \( \delta_{i,m} = \delta_{i,c} \) or \( \delta_{i,m} \neq \delta_{i,c} \) \((i \in \{1, \ldots, N\})\). In particular, whether individual talent is more decisive for the outcome of the major contest \((\delta_{i,m} \geq \delta_{i,c})\) or the consolation contest \((\delta_{i,m} \leq \delta_{i,c})\) is irrelevant for the mediocrity result in the substitutes model. Moreover, although I assume career incomes to be exogenously given, the results of Proposition 1 will qualitatively hold under incomes or prizes that are endogenously chosen by a contest designer before the competition starts. The career incomes of course influence the levels of all players’ equilibrium efforts, but they neither have an impact on the players’ effort differences \(a_{i,c} - a_{x,c}\) in the consolation match nor an impact on the ranking between \(a_{i,m} - \delta_{i,m}\) and \(a_{x,m} - \delta_{x,m}\) in the major contest.\(^8\)

The result on effort ranking \(a_{(1),m}^* < a_{(2),m}^* < \ldots < a_{(N),m}^*\) stems from the intuition before together with the fact that effort and productivity are substitutes in the impact function. Hence, even if all players had identical fall-back positions, the order \(a_{(1),m}^* < a_{(2),m}^* < \ldots < a_{(N),m}^*\) would not change. Welfare was solely defined via the career decision based on the outcome of the major contest. However, if the activity levels \(a_{i,\kappa}^*\) \((i \in \{1, \ldots, N\}, \kappa = m, c)\), which are productive by assumption, were also important for welfare considerations, we might have a second source for welfare losses since the most productive individuals choose the lowest activity levels.

4 Activities and Productivities as Complements

So far I have assumed that activities \((a_{i,\kappa})\) and productivity parameters \((\delta_{i,\kappa})\) are substitutes in the players’ impact function \(f_\kappa\) \((\kappa = m, c)\). This assumption drives part of the previous results, in particular the findings that players with higher productivities (i.e., lower values of \(\delta_{i,\kappa}\)) choose lower efforts in equilibrium. However, this paper does not focus on effort choice but on the probability that the most productive player is not assigned to the top career position. In this section, I will check the robustness of the mediocrity result by assuming that activities and

\(^8\)However, if contest prizes are endogenous in the sense of endogenously created rents that are increasing in a player’s performance, the mediocrity result may quite depend on the magnitude of the contest prizes. Note that, in this case, there are parallels to pay-for-performance discussed in Section 5.
productivities are complementary in the impact function $f$. This complementarity assumption is rather realistic for jobs with innovative and creative tasks like in science and art, but may also apply to leading positions in business and politics. To simplify the analysis, I assume that the winner of the major contest is replaced in the consolation contest by a player with the same productivity, which allows to include the additional case $N = 2$. Therefore, in the major contest $m$ the set of player-types is given by $\{\delta_{1,m}, \ldots, \delta_{N,m}\}$ and in the consolation contest by $\{\delta_{1,c}, \ldots, \delta_{N,c}\}$.

In this section, I consider the impact function (3), which leads to player $i$’s contest-success function

$$
\hat{p}_{i,m} = \frac{f\left(\frac{a_{i,m}}{\delta_{i,m}}\right)}{f\left(\frac{a_{j,m}}{\delta_{j,m}}\right) + \sum_{j \neq i} f\left(\frac{a_{j,m}}{\delta_{j,m}}\right)}
$$

with $f$ denoting the same strictly positive, increasing and concave function as before. Again, the smaller $\delta_{i,m}$ the more productive will be the respective player, and for the case of identical activity levels by all players the most productive one has the highest winning probability. However, comparison of (2) and (3) shows that the contest-success functions in the substitutes model and the complements model differ significantly. Now the activity variable and the productivity parameter are complements in the sense that lower values of $\delta_{i,m}$ make higher activity levels $a_{i,m}$ more effective.

As in Section 3, the two-stage game is solved by backward induction. In the stage with the consolation contest $c$, player $i$ maximizes $EU_{i,c}(a_{i,c}) = Y_L + (Y_M - Y_L) \cdot \hat{p}_{i,c} - a_{i,c}$. The first-order conditions of two arbitrary players $i$ and $x$.

\(^9\)Note that this additional assumption is not used for the substitutes model since $EU_{i,c}(a_{i,c})$ is independent of $\delta_{j,c} (j \neq i)$ (see (4)).

\(^{10}\)Activities and productivities are complements in the impact function in the sense of $\frac{\partial^2}{\partial a_{i,c} \partial a_{x,c}} \left(\frac{a_{i,c}}{\delta_{i,c}}\right) > 0$. 

13
can be combined to

\[
\frac{(Y_M - Y_L)}{\left[ \sum_j f \left( \frac{a_{i,c}}{\delta_{j,c}} \right) \right]^2} = \frac{\delta_{i,c}}{\delta_{x,c}} = f\left( \frac{a_{i,c}}{\delta_{i,c}} \right) \left[ f \left( \frac{a_{x,c}}{\delta_{x,c}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,c}}{\delta_{j,c}} \right) \right] \]

which yields the following equilibrium outcome:

**Lemma 1.** *In the consolation contest, if \( \delta_{i,c} < \delta_{x,c} \) then (i) \( \hat{p}_{i,c}^* > \hat{p}_{x,c}^* \) and (ii) \( EU_{i,c}^*(a_{i,c}^*) > EU_{x,c}^*(a_{x,c}^*) \).*

**Proof.** Part (i) can be shown by contradiction. Suppose that

\[
\hat{p}_{i,c}^* \leq \hat{p}_{x,c}^* \iff \frac{a_{i,c}}{\delta_{i,c}} \leq \frac{a_{x,c}}{\delta_{x,c}}. \tag{9}
\]

From the first-order conditions (8) we obtain

\[
\frac{\delta_{i,c}}{\delta_{x,c}} \left[ f \left( \frac{a_{i,c}}{\delta_{i,c}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,c}}{\delta_{j,c}} \right) \right] = \frac{\delta_{x,c}}{\delta_{x,c}} \left[ f \left( \frac{a_{x,c}}{\delta_{x,c}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,c}}{\delta_{j,c}} \right) \right]. \tag{10}
\]

Since \( [f(y) + \sum_{j \neq i,x} f \left( \frac{a_{j,c}}{\delta_{j,c}} \right)] / f'(y) \) is a monotonically increasing function of \( y \), (9) and (10) can only be satisfied at the same time if \( \delta_{i,c} \geq \delta_{x,c} \), a contradiction.

(ii) Since player \( i \) can always choose the same effort level as \( x \) so that he has the same effort costs but a higher winning probability we must have \( EU_{i,c}^*(a_{i,c}^*) > EU_{x,c}^*(a_{x,c}^*) \) in equilibrium.

Lemma 1 points out that, in the consolation match, more productive players have higher winning probabilities and larger expected utilities than less productive players. Result (ii) is also important for the major contest, where players compete for the top position with income \( Y_H \). Due to the positive correlation between productivity and expected utility, more productive players have better fall-back positions \( EU_{i,c}^*(a_{i,c}^*) \) in the major contest, leading to less incentives. This effect also works in the model with substitutes (Section 3) and will be called incentive
In the major contest, player $i$ maximizes $EU_{i,m} (a_{i,m}) = EU_{i,c}^* (a_{i,c}^*) + (Y_H - EU_{i,c}^* (a_{i,c}^*)) \cdot \tilde{p}_{i,m} - a_{i,m}$. The first-order conditions of two players $i$ and $x$,

$$
\frac{Y_H - EU_{i,c}^* (a_{i,c}^*)}{\sum_j f \left( \frac{a_{i,m}}{\delta_{j,m}} \right)^2} = \frac{\delta_{i,m}}{f' \left( \frac{a_{i,m}}{\delta_{i,m}} \right)} \left[ f \left( \frac{a_{i,m}}{\delta_{i,m}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right) \right],
$$

and

$$
\frac{Y_H - EU_{x,c}^* (a_{x,c}^*)}{\sum_j f \left( \frac{a_{x,m}}{\delta_{j,m}} \right)^2} = \frac{\delta_{x,m}}{f' \left( \frac{a_{x,m}}{\delta_{x,m}} \right)} \left[ f \left( \frac{a_{x,m}}{\delta_{x,m}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right) \right],
$$

(11)

together with $EU_{i,c}^* (a_{i,c}^*) > EU_{x,c}^* (a_{x,c}^*)$ imply

$$
\frac{\delta_{i,m} \left[ f \left( \frac{a_{i,m}}{\delta_{i,m}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right) \right]}{f' \left( \frac{a_{i,m}}{\delta_{i,m}} \right)} < \frac{\delta_{x,m} \left[ f \left( \frac{a_{x,m}}{\delta_{x,m}} \right) + \sum_{j \neq i,x} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right) \right]}{f' \left( \frac{a_{x,m}}{\delta_{x,m}} \right)}. \tag{12}
$$

The inequality shows that solutions of type $\frac{a_{i,m}}{\delta_{i,m}} < \frac{a_{x,m}}{\delta_{x,m}}$ are still possible. Such outcomes coincide with the findings above where more productive players are less likely to obtain the top career position. However, the parameters $\delta_{i,m}$ and $\delta_{x,m}$ in the numerators of the two sides in (12) indicate that we cannot rule out solutions with $\frac{a_{i,m}}{\delta_{i,m}} > \frac{a_{x,m}}{\delta_{x,m}}$ if $\delta_{i,m}$ is sufficiently small and $\delta_{x,m}$ sufficiently large. This effect can be labeled productivity effect. Hence, if activities and productivity parameters are complements in the impact function, there will be two effects that work into opposite directions. Coming back to the question regarding efficient assignment at the top gives the following result:

Lemma 2. Consider the major contest for the top position with income $Y_H$ and let $\delta_{i,\kappa} < \delta_{x,\kappa}$ ($\kappa = m, c$). If

$$
\frac{\delta_{x,m}}{\delta_{i,m}} < \frac{Y_H - EU_{x,c}^* (a_{x,c}^*)}{Y_H - EU_{i,c}^* (a_{i,c}^*)},
$$

(13)

then $\tilde{p}_{i,m} < \tilde{p}_{x,m}$.
Proof. Combining the first-order conditions (11) yields
\[
\frac{f \left( \frac{a_{i,m}}{\delta_{i,m}} \right) + \sum_{j \neq i} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right)}{f' \left( \frac{a_{i,m}}{\delta_{i,m}} \right)} = \frac{\delta_{x,m} \left[ Y_H - EU_{i,c}^* (a_{i,c}^*) \right] f \left( \frac{a_{x,m}}{\delta_{x,m}} \right) + \sum_{j \neq i} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right)}{f' \left( \frac{a_{x,m}}{\delta_{x,m}} \right)}.
\]

If (13) is satisfied, we will have
\[
\frac{f \left( \frac{a_{i,m}}{\delta_{i,m}} \right) + \sum_{j \neq i} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right)}{f' \left( \frac{a_{i,m}}{\delta_{i,m}} \right)} < \frac{f \left( \frac{a_{x,m}}{\delta_{x,m}} \right) + \sum_{j \neq i} f \left( \frac{a_{j,m}}{\delta_{j,m}} \right)}{f' \left( \frac{a_{x,m}}{\delta_{x,m}} \right)}
\]
and, thus, \( \frac{a_{i,m}}{\delta_{i,m}} < \frac{a_{x,m}}{\delta_{x,m}} \), which implies \( \tilde{\rho}_{i,m}^* < \tilde{\rho}_{x,m}^* \) for the winning probabilities in equilibrium according to (7).

Condition (13) points out the two opposing effects. While the right-hand side describes the incentive effect, the left-hand side characterizes the productivity effect. If the incentive effect dominates the productivity effect, more productive players will have lower winning probabilities in the major contest than less productive ones. In this case, the mediocracy result under substitutes qualitatively still holds for activities and productivities being complements.

Comparative statics show under which parameter constellations the incentive effect dominates the productivity effect and vice versa. If players’ productivities are very similar in the major contest (i.e., \( \delta_{x,m} \approx \delta_{i,m} \)) but strictly differ in the consolation match (i.e., \( \delta_{x,c} > \delta_{i,c} \)), the left-hand side of (13) will tend to 1 whereas the right-hand side of (13) is strictly larger than 1. In this situation, dominance of the incentive effect leads to mediocrity. However, if winning the major contest is sufficiently attractive, mediocrity will be eliminated: For given values of \( \delta_{x,m}, \delta_{i,m}, EU_{x,c}^* (a_{x,c}^*) \) and \( EU_{i,c}^* (a_{i,c}^*) \), there exists a threshold value \( \tilde{Y}_H \) so that a mediocrity outcome is impossible if \( Y_H > \tilde{Y}_H \).

Contestants’ heterogeneity in the consolation match influences condition (13) via \( EU_{i,c}^* (a_{i,c}^*) \) and \( EU_{x,c}^* (a_{x,c}^*) \). Note that an individual’s expected utility \( EU_{i,c}^* (a_{i,c}^*) \) decreases in \( \delta_{i,c} \) (i.e., the more productive a player the higher will be his expected utility in equilibrium), because a low value of \( \delta_{i,c} \) enables player \( i \) to realize the
same winning probability at lower costs compared to a situation where $\delta_{i,c}$ is large. Hence, if in the consolation contest individuals are sufficiently heterogeneous in terms of productivity, condition (13) is likely to be satisfied.

Although condition (13) offers clear comparative static results, so far it is not clear whether this condition is satisfied under concrete specifications of the impact function (3). In order to check the condition in this respect, let the impact function (3) be parameterized as

$$f_\alpha (a_{i,\kappa}, \delta_{i,\kappa}) = \frac{a_{i,\kappa}}{\delta_{i,\kappa}} + 1, \quad (\kappa = m, c)$$

(14)

which is linear (and hence concave) and monotonically increasing in $a_{i,\kappa}$, and strictly positive for all feasible effort levels. The following results can be obtained:

**Proposition 2.** Let the impact function be given by (14). In the consolation match, individual $i$ chooses effort

$$a_{i,c}^* = (N - 1) \delta_{i,c} (Y_M - Y_L) \frac{\left( \sum_{j \neq i} \delta_{j,c} \right) - (N - 2) \delta_{i,c}}{\left( \sum_{j=1}^N \delta_{j,c} \right)^2} - \delta_{i,c}$$

(15)

in equilibrium ($i = 1, \ldots, N$), leading to expected utilities

$$EU_{i,c}^* (a_{i,c}^*) = Y_L + (Y_M - Y_L) \frac{\left( \sum_{j \neq i} \delta_{j,c} \right) - (N - 2) \delta_{i,c}}{\left( \sum_{j=1}^N \delta_{j,c} \right)^2} + \delta_{i,c}.$$  

(16)

(i) If $\delta_{i,c} = \delta_{i,m}$ ($i = 1, \ldots, N$), then it is impossible that, for two players $i$ and $x$, the productivity ranking $\delta_{i,\kappa} < \delta_{x,\kappa}$ ($\kappa = m, c$) leads to $\bar{p}_{i,m}^* < \bar{p}_{x,m}^*$. (ii) If $\delta_{i,c} \neq \delta_{i,m}$ ($i = 1, \ldots, N$), there exist parameter constellations for which $\delta_{i,\kappa} < \delta_{x,\kappa}$ ($\kappa = m, c$) and $\bar{p}_{i,m}^* < \bar{p}_{x,m}^*$ hold at the same time.

Proof. See the appendix.

The proposition shows that the productivity effect will unambiguously dominate the incentive effect if players have identical productivity levels in the different career contests. This dominance renders a mediocrity outcome impossible for the
natural case of time-invariant productivity parameters. However, if a player’s productivity parameters $\delta_{i,c}$ and $\delta_{i,m}$ differ across contests and if player heterogeneity in terms of productivity is stronger in the consolation contest than in the major contest, a mediocrity outcome will be still possible.

5 Discussion

In the game analyzed above with the major contest in stage 1 and the consolation contest in stage 2, an individual must win only one winner-takes-all competition in order to reach the top. However, if individuals have to pass more than one hurdle to get the top position, it will be crucial whether productivity has a higher impact on the fall-back position or the future career and how dissipative the competition is on different career levels. In Kräkel (2010), a career model is analyzed where an individual must win two sequential contests in order to reach the top. In case of political careers, first a politician has to achieve the leadership of his political party. When being successful, the new political leader competes against the leader of another political party for presidency. As crucial difference to the one-hurdle career model considered in this paper, now a politician has to succeed two times before reaching the top and his productivity has an influence on both his fall-back position and the future contest for presidency. In that model, if consolation contests are sufficiently dissipative (i.e., expected utility from participating in these contests is rather low), fall-back positions will be quite unattractive, but most unattractive for the least productive individuals. As a consequence, the less productive individuals are most likely to reach the top, so that the mediocrity result still holds.

Furthermore, there are situations where highly talented individuals can either compete against other strong opponents for the top positions or play against a significantly weaker field for less attractive jobs. If the pay (and status) gap between the top positions and the less attractive ones is not too large and the competition against other strong opponents really exhausting it may be optimal for talented individuals not to strive towards the top positions. Such sandbagging at a preceding career stage to avoid promotion to the major contest has three advantages for an individual. First, by withholding effort the high-ability player saves effort costs at the preceding career stage. Second, he avoids strong opponents
and, hence, a relatively low winning probability in the major contest. Third, he avoids a rather homogeneous field in the major contest, which would lead to strong competition and high effort costs. Sandbagging in competitive careers would further explain the deficit of top-class candidates for top positions. Note that this sandbagging strategy may be optimal for either high-potential individual so that they face a coordination problem: If there are two highly talented players and both prefer to sandbag, then they will end up in a consolation contest with strong competition. Of course, such failed coordination would exacerbate the mediocracy problem.\(^{11}\)

So far, I have assumed that all players have the same career incomes as contest prizes. This assumption is realistic for those cases where wages are attached to jobs. Such wage policy can be often observed in politics and (public) bureaucracies. Here, we have a clear bundle of tasks that is assigned to a certain job. These tasks determine the job holder’s qualification as well as his salary. Moreover, wages-attached-to-jobs is one of the key assumptions within the concept of internal labor markets.\(^ {12}\) Finally, if workers’ performance signals are unverifiable, tying wages to jobs is necessary for a firm to use job-promotion tournaments as credible incentive schemes.\(^{13}\)

However, there are also jobs with verifiable performance signals, allowing to apply pay-for-performance. In these cases, a more able player has a higher expected career income at a certain position than a less able one. The combination of pay-for-performance at the top position and wages-attached-to-jobs for subordinate positions can be used as a solution for the mediocracy problem: Pay-for-performance at the top implies large expected career incomes for high-productivity players when winning the major contest, thus leading to a very strong motivation for them to climb the top position. Hence, the detrimental incentive effect of

\(^{11}\)See Kräkel (2010).
\(^{12}\)See Doeringer and Piore (1971), Williamson et al. (1975).
\(^{13}\)See Malcomson (1984, 1986). Without the self-commitment property of wages-attached to jobs, the employer would always promote the worker with the lowest promised salary for the vacant job in order to save labor costs. Since such opportunistic behavior is anticipated by the workers, job-promotion tournaments can only create incentives if salaries are linked with positions.
Sections 3 and 4 turns into a beneficial one, which makes excellent candidates more likely to win major contests at the top. A wages-attached-to-jobs policy for subordinate positions works against high fall-back positions for highly productive individuals.

Besides reversing the incentive effect, the combination of pay-for-performance at the top and wages-attached-to-jobs for less important positions can make heterogeneous individuals self-sort into different career paths. In that case, highly talented individuals prefer career tracks to the top whereas less talented ones are better off choosing subordinate careers with guaranteed incomes that are independent of talent. Applying the suggested combination to politics would lead to a remuneration system that honors good performance and penalizes bad performance of top politicians, while attaching fixed payments to less important positions in the political hierarchy and the public bureaucracies. For example, it may be worthwhile thinking about pay for performance of the president of a country (and the ministers) that depends on voter satisfaction being evaluated in regular time intervals.

Another possible solution to the mediocrity problem can be the destruction of the highly productive individuals’ fall-back positions. If a senior partner of a law firm starts a political career, he may credibly commit himself to the new career track by selling his portion of the partnership, so that he cannot profitably come back to his original occupation as a lawyer when failing as a politician. However, if fall-back positions are mainly based on human capital, credible commitment not to make use of an attractive fall-back position needs additional precautions. For example, an individual can forgo to educate himself, so that his human capital becomes obsolete during his political career. In addition, institutional arrangements such as up-or-out rules can be used to destroy consolation matches for losers of a major contest and, hence, their respective fall-back positions. Such arrangements are useful if individuals’ outside options differ less than their internal fall-back positions from participating in a consolation match.

6 Conclusion
At first sight, one might expect that career contests perfectly correspond to the
well-known phrase "survival of the fittest". According to Darwin, there should be a natural selection among heterogeneous individuals so that the best suited ones will win the competition for reproduction. Relating to career competition, the most talented or most productive players should win and be promoted to the top positions within structured career paths. However, in this paper I show that, contrary to the Darwinian view, the least productive players may have the highest probability of winning career competition. The intuition comes from the fact that the phrase "survival of the fittest" implicitly assumes that all individuals choose the same activity level. Maybe, in biology this crucial assumption holds. In an economic context, however, equilibrium behavior of heterogeneous players usually differs. Since the least productive players have the lowest fall-back positions, these individuals are strongly motivated to win career competition and, thereby, to avoid their unattractive fall-back options. In this sense, there is a natural tendency that the least productive players succeed. It is important to emphasize that under identical activity levels the model would replicate the Darwinian outcome: The individuals with the highest productivities would most likely win the career contest.

In this paper, I used a game-theoretic perspective to show how the detrimental effect of fall-back positions may lead to adverse career outcomes for society. Switching to a contract-theoretic view would not qualitatively alter the results. Allowing for optimal career incomes that are optimally chosen by a principal would probably lead to a change in the levels of equilibrium activities. However, as mentioned in Section 3 the ranking between the players’ activities and winning probabilities would not change in the substitutes model. Introducing reservation utilities for the players may even reinforce the results. If the most productive players have also the highest reservation utilities, these players may prefer their outside options and decide not to participate in the consolation contest. If the fall-back positions are now determined by the players’ different reservation utilities, we will have the same natural tendency that the most productive players have the lowest incentives to succeed in a given career contest. All these considerations are based on the assumption that the contest designer cannot use a mechanism to reveal the players’ types and then choose type-dependent contest prizes to adjust individual
incentives. However, as has been emphasized by Malcomson (1984, 1986), identical prizes for all contestants are important if individuals only have unverifiable but observable performance signals. In that case, under different prizes the principal would ex-post always claim that the player with the lowest winner prize has performed best, thereby saving labor costs. Since the players can anticipate such opportunistic behavior, contest incentives would break down if prizes differ.

According to the Peter Principle, individuals are promoted until they reach their level of incompetence. This observation was made by Peter and Hull (1969). The outcome of the Peter Principle is based on two rules – currently good performance is rewarded by job promotion and demotions are not possible. In the subsequent economic work, the Peter Principle has been used as a synonym for the misallocation of managers at (high) hierarchy levels. For example, Prendergast (1992) explains such misallocation by the personnel policy of hiding good talents, whereas the explanation of Lazear (2004) is based on temporary luck. Fairburn and Malcomson (2001) point out that with risk averse workers the trade-off between incentives and insurance can result into inefficiently many promotions. In the light of my model, misallocation at higher hierarchy levels can be explained by a detrimental incentive effect that gives less talented individuals strong incentives to succeed in competitive careers.

Acknowledgements
I would like to thank Oliver Gürtler, Daniel Müller, Petra Nieken, Anja Schöttner, the participants of the economics workshop of the Leibniz University of Hannover, in particular Hendrik Hakenes, Vilen Lipatov and Georgios Katsenos, the participants of the annual meeting of the "Unternehmenstheoretischer und -politischer Ausschuss" of the German Economic Association (Verein für Socialpolitik), particularly Jürgen Eichberger, Dieter Pfaff, Andreas Pfingsten, Kerstin Pull, Ulrike Stefani and Siegfried Trautmann, and two anonymous referees for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft (DFG), grant SFB/TR 15, is gratefully acknowledged.
Appendix: Proof of Proposition 2

For the impact function (14), player $i$’s expected utility from participating in the consolation match is given by

$$EU_{i,c}(a_{i,c}) = Y_L + (Y_M - Y_L) \cdot \frac{\left(\frac{a_{i,c}}{\delta_{i,c}} + 1\right)}{\left(\frac{a_{i,c}}{\delta_{i,c}} + 1\right) + \sum_{j \neq i} \left(\frac{a_{j,c}}{\delta_{j,c}} + 1\right)} - a_{i,c}$$

yielding the first-order condition

$$(Y_M - Y_L) \frac{\frac{1}{\delta_{i,c}} \sum_{j \neq i} \left(\frac{a_{j,c}}{\delta_{j,c}} + 1\right)}{\left[\sum_{j=1}^{N} \left(\frac{a_{j,c}}{\delta_{j,c}} + 1\right)\right]^2} = 1.$$  

Inserting for $a_{j,c}$ according to (15) yields

$$\frac{1}{\delta_{j,c}} \sum_{j \neq i} \left(\frac{a_{j,c}}{\delta_{j,c}} + 1\right) \left(\frac{N}{\delta_{j,c}} \left(N - 1\right) \left(Y_M - Y_L\right) \left(\frac{\sum_{k \neq j} \delta_{k,c}}{\sum_{k=1}^{N} \delta_{k,c}}\right)^2 - \left(\frac{\sum_{k \neq j} \delta_{k,c}}{\sum_{k=1}^{N} \delta_{k,c}}\right)^2 \right) = 1 \iff$$

$$\left(\sum_{k=1}^{N} \delta_{k,c}\right)^2 \frac{1}{\delta_{i,c}} \left(\frac{\sum_{j \neq i} \left(\sum_{k \neq j} \delta_{k,c}\right) - \left(N - 2\right) \sum_{j \neq i} \delta_{j,c}}{\sum_{j=1}^{N} \left(\sum_{k \neq j} \delta_{k,c}\right) - \left(N - 2\right) \sum_{j=1}^{N} \delta_{j,c}}\right)^2 = N - 1.$$  

Since

$$\sum_{j=1}^{N} \left(\sum_{k \neq j} \delta_{k,c}\right) = \left(N - 1\right) \sum_{j=1}^{N} \delta_{j,c} \quad \text{and} \quad \sum_{j \neq i} \left(\sum_{k \neq j} \delta_{k,c}\right) = \delta_{i,c} + \left(N - 2\right) \sum_{j=1}^{N} \delta_{j,c},$$

the equation boils down to

$$\left(\sum_{k=1}^{N} \delta_{k,c}\right)^2 \frac{1}{\delta_{i,c}} \left(\delta_{i,c} + \left(N - 2\right) \delta_{i,c}\right) \left[\sum_{j=1}^{N} \delta_{j,c}\right]^2 = N - 1,$$

which is true. Hence, (15) describes the equilibrium in the consolation match.
Inserting equilibrium efforts (15) into $EU_{i,c}^* (a_{i,c}^*)$ gives

$$EU_{i,c}^* (a_{i,c}^*) = Y_L + (Y_M - Y_L) \cdot \frac{(\sum_{k \neq i} \delta_{k,c}) - (N - 2) \delta_{i,c}}{(N - 1) \sum_{j=1}^{N} \delta_{j,c} - (N - 2) \sum_{j=1}^{N} \delta_{j,c}}$$

$$- (N - 1) \delta_{i,c} (Y_M - Y_L) \frac{(\sum_{k \neq i} \delta_{k,c}) - (N - 2) \delta_{i,c}}{(\sum_{k=1}^{N} \delta_{k,c})^2} + \delta_{i,c}$$

$$= Y_L + (Y_M - Y_L) \left( \frac{\sum_{k \neq i} \delta_{k,c}}{\sum_{j=1}^{N} \delta_{j,c}} \right) - (N - 2) \delta_{i,c} \left[ \frac{\sum_{j=1}^{N} \delta_{j,c}}{\sum_{j=1}^{N} \delta_{j,c}} \right] + \delta_{i,c}$$

$$= Y_L + (Y_M - Y_L) \left( \frac{\left( \sum_{k \neq i} \delta_{k,c} \right) - (N - 2) \delta_{i,c}}{\left( \sum_{j=1}^{N} \delta_{j,c} \right)^2} \right) + \delta_{i,c}$$

Result (i) is true if condition (13) cannot hold for $\delta_{i,c} = \delta_{i,m} =: \delta_i (i = 1, \ldots, N)$. Inserting for $EU_{x,c}^* (a_{x,c}^*)$ and $EU_{i,c}^* (a_{i,c}^*)$ in (13) and rewriting yields

$$\frac{Y_H - Y_L}{Y_M - Y_L} (\delta_x - \delta_i)$$

$$< \delta_x \frac{\left( \sum_{k \neq i} \delta_k \right) - (N - 2) \delta_i}{\left( \sum_{j=1}^{N} \delta_j \right)^2} - \delta_i \frac{\left( \sum_{k \neq x} \delta_k \right) - (N - 2) \delta_x}{\left( \sum_{j=1}^{N} \delta_j \right)^2}.$$

For this condition to hold, we must have that

$$\delta_x \frac{\left( \sum_{k \neq i} \delta_k \right) - (N - 2) \delta_i}{\left( \sum_{j=1}^{N} \delta_j \right)^2} - \delta_i \frac{\left( \sum_{k \neq x} \delta_k \right) - (N - 2) \delta_x}{\left( \sum_{j=1}^{N} \delta_j \right)^2} > \delta_x - \delta_i \iff$$

24
\[
\delta_x \left( \sum_{k=1}^{N} \delta_k \right) - (N - 1) \delta_i \right)^2 - \delta_i \left( \sum_{k=1}^{N} \delta_k \right) - (N - 1) \delta_x \right)^2 \\
> \delta_x \left( \sum_{j=1}^{N} \delta_j \right)^2 - \delta_i \left( \sum_{j=1}^{N} \delta_j \right)^2.
\]

Expanding the two quadratic expressions at the left-hand side and rearranging immediately leads to \( \delta_x < \delta_i \). Thus, conditions (13) and \( \delta_{i,\kappa} < \delta_{x,\kappa} \) (\( \kappa = m, c \)) cannot hold at the same time.

To prove result (ii), suppose that the consolation contest only consists of players 1 and 2 with \( \delta_{1,\kappa} < \delta_{2,\kappa} \) (\( \kappa = m, c \)). In this case, \( \text{EU}_{i,c}^* (a_{i,c}^*) = Y_L + \frac{\alpha_{i,c}^* (Y_M - Y_L)}{(\delta_{1,c} + \delta_{2,c})^2} + \delta_{i,c} \) (\( i, j = 1, 2; i \neq j \)). The mediocrity condition (13) now reads as follows:

\[
\frac{\delta_{2,m}}{\delta_{1,m}} < \frac{(Y_H - Y_L) - \frac{\delta_{2,c}^2 (Y_M - Y_L)}{(\delta_{1,c} + \delta_{2,c})^2} - \delta_{2,c}}{(Y_H - Y_L) - \frac{\delta_{2,c}^2 (Y_M - Y_L)}{(\delta_{1,c} + \delta_{2,c})^2} - \delta_{1,c}}.
\]

This condition is satisfied if \( (Y_H - Y_L) \) and \( (Y_M - Y_L) \) do not differ too much. Therefore, let \( (Y_H - Y_L) = 100 \) and \( (Y_M - Y_L) = 95 \). It can be easily checked that the inequality holds for small differences \( \delta_{2,m} - \delta_{1,m} \) and large differences \( \delta_{2,c} - \delta_{1,c} \) (e.g., for \( \delta_{2,c} = 1, \delta_{2,m} = 0.5, \delta_{1,m} = 0.4, \) and \( \delta_{1,c} = 0.1 \)).
References


27


