Internal Labor Markets and Worker Rents*

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Abstract

We show that establishing an internal labor market by offering combined contracts across hierarchy levels strictly dominates external recruitment when workers are homogeneous. The reason is that only an internal labor market can exploit higher-tier rents for incentive provision on lower tiers. Given unobservable heterogeneity of workers, relying on an internal labor market has the further advantage of improving the selection of high ability workers for higher ranks, which is complemented by rent-based incentive schemes. However, observable worker heterogeneity gives rise to a trade-off between incentive and selection issues and may lead to ports of entry on higher tiers of the hierarchy.

Key Words: bonuses; internal labor market; job promotion; limited liability; tournaments

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1 Introduction

Large corporations often use so-called internal labor markets to govern the long-term employment relationships with their core workers. Internal labor markets are characterized by limited ports of entry, firm-specific training, wages-attached-to-jobs, and well-defined career paths to higher levels of the corporate hierarchy. When deciding whether to install an internal labor market, a firm faces the following trade-off: On the one hand, the rigid structure has important advantages such as supporting the accumulation of firm-specific human capital and improving employer learning about workers’ abilities. On the other hand, members of an internal labor market are widely protected against external labor-market competition for higher-tier jobs, thus earning substantial rents (i.e., their expected utility from staying with the firm exceeds their reservation utility). Consequently, a firm should adopt an internal labor market if the expected benefits exceed the costs in form of worker rents.

However, workers typically earn positive rents irrespective of whether being members of an internal labor market or not. For example, legal minimum wages, wealth constraints, or liability limits often prevent firms from extracting the entire surplus generated in an employment relationship. Moreover, rents may arise from imperfectly competitive labor markets, worker empowerment, or private information of workers. Hence, worker rents are not a specific disadvantage of internal labor markets. Quite the contrary, in this paper we argue that an internal labor market is an effective instrument for utilizing worker rents to improve incentives. In particular, we abstract from the well-known advantages of internal labor markets and show that the pure presence of worker rents along a firm’s hierarchy makes a strong case for internal promotion procedures to fill vacancies on higher tiers.

We consider a moral-hazard model where a firm needs to implement performance pay to provide its workers with effort incentives. Workers earn rents on each tier of the
firm’s hierarchy because they are protected by limited liability.\textsuperscript{6} We show that, in such a situation, the firm establishes an internal labor market to complement incentive contracts for lower-tier employees by the prospect of future rents in case of promotion. Moreover, when an internal labor market is adopted and positions on upper tiers are primarily filled with internal candidates, the firm voluntarily offers internal recruits higher rents than it pays to occasional external hires, thereby improving overall incentives. Hence, we show that rents are not a downside of internal recruiting but, on the contrary, both a rationale for the existence of internal labor markets and a measure for further increasing their effectiveness.

We derive our results by comparing two different kinds of employment contracts. The first contract corresponds to a stylized internal labor market. We call this contractual form a \textit{combined contract} because it interlinks the incentive schemes on the firm’s different hierarchical tiers by a strict internal promotion rule. The second kind of contract focuses on spot contracting and allows for external hiring on higher hierarchy levels. We call this alternative \textit{separate contracts} because the incentive schemes from the different hierarchical tiers do not interact: The firm chooses an optimal contract for each tier of the hierarchy.

First, we focus on the case of homogeneous workers and show that an internal labor market in form of a combined contract dominates separate contracts. The rationale for this result is as follows. As mentioned above, we consider a setting where workers are protected by limited liability and, thus, earn positive rents under any contract form. These rents lead to a specific advantage of an internal labor market in comparison to separate contracts: With an internal labor market, worker rents can be used to enhance effort incentives for lower-level workers, which is impossible under separate contracts. Under the optimal combined contract the additional incentives via expected rents replace direct incentives on lower hierarchy levels and, hence, improve effort supply at higher levels and possibly also at lower tiers of the hierarchy. As a consequence, with the combined contract, the firm optimally leaves larger rents to higher-level workers than under separate contracts but still incurs lower overall costs of inducing effort. Surprisingly, combined contracts may even lead to the implementation of first-best effort on higher tiers. Contract theorists as Tirole (1999) and Schmitz (2005a) have pointed out that

\textsuperscript{6}We choose this standard approach to generate worker rents. However, our rent-based argument for the existence of internal labor markets would apply to other sources of rents as well.
optimal bonus payments that lead to positive rents can be reinterpreted as efficiency wages. Since rents are usually strictly increasing in effort in single-agent hidden action models with continuous effort, the implemented effort level is inefficiently small. By contrast, in our model the firm may induce first-best effort although it is associated with a strictly positive rent, which also monotonically increases in effort. Hence, in our context we obtain efficiency wages in a more literal sense.

In a second step, we enrich our model by introducing unobserved worker heterogeneity so that neither the firm nor the workers themselves perfectly learn individual worker productivity. We show that an internal labor market in form of a combined contract still dominates separate contracts. In addition to the incentive advantage, internal labor markets now exhibit the extra benefit of improved selection quality compared to separate contracts when filling positions on higher hierarchy levels.\(^7\) We point out that, if effort and ability of workers are complements, the rent-based incentive effect and the well-known selection advantage of internal labor markets reinforce each other. Finally, we consider observed heterogeneity of workers, i.e., the firm learns workers’ abilities during the beginning of their careers. The firm can then use this information to improve worker selection, which may now include external recruitment on higher hierarchy levels. In this case, we find that external hires face lower-powered incentives and get lower rents than internally promoted workers.

Our findings partially depart from the traditional view of strict internal labor markets. However, our results are in line with the following three empirical observations that also contradict strict internal labor markets and, in addition, standard theory of job-promotion tournaments:\(^8\)

- First, there is considerable variation in pay on hierarchy levels, which contradicts the important prerequisite of tournaments that wages must be attached to jobs in order to generate incentives.

- Second, promotion premiums that are paid to workers when moving to higher levels in the hierarchy can explain only part of the hierarchical wage differences in firms.

\(^7\)This is well-established in the literature. See, e.g., Gifford and Kenney (1986), Rosen (1988), p. 83, and our more detailed discussion in Section 4.1.

• Third, we can observe external recruiting on higher hierarchy levels, which would erase incentives from internal job-promotion tournaments.

In our model of a two-tier hierarchy, promoted workers earn high or low bonuses depending on success or failure on the second hierarchy level. Thus, we have a natural variation in pay on the second tier, which is in line with the first observation. As a promoted worker earns both relative performance pay and bonuses, hierarchical wage increases are only in part determined by job promotion, hence leading to the second observation. In this respect, our model fits quite well to one of the empirical findings by Dohmen et al. (2004). Contrary to other studies, they are able to determine the exact point in time when a worker realizes a pay increase, and they find that promotion and wage increase are often not simultaneous. Finally, under observed worker heterogeneity, the firm may hire externally with some positive probability to improve selection, which is in line with the third stylized fact on ports of entry on higher hierarchy levels. Thus, by studying the combination of a job-promotion scheme with additional incentive devices – a practice frequently observed in organizations – we obtain results that are in accordance with well-established empirical evidence on internal labor markets.

In his seminal paper, Rosen (1986) shows that, to keep effort constant throughout a multi-round elimination tournament, rewards need to be concentrated in the top ranks. Intuitively, in the early stages of the tournament, winning one round also gives the option to continue on to all successive ranks. This option value, however, decreases as one moves up the ranks. Effort incentives can thus be maintained only if the reward for climbing the top rank is sufficiently large. Rosen considers a dynamic game with a finite number of stages so that incentives in higher-rank tournaments suffer from the well-known end-game problem. By considerably increasing the prize spread between the highest and the second highest ranks of the hierarchy, the contestants’ perceived time horizon will become infinitely large as if they will compete forever. Technically, the optimal final-round spread converts the option value on any rank into an infinite geometric series of constant rewards. 9

We offer an alternative explanation for concentrating rewards on the top tier of a corporate hierarchy. In our contract-theoretic model, workers are protected by limited liability, implying positive rents on each level of a two-tier hierarchy. The firm can

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provide individual effort incentives for each tier. It is optimal, however, to concentrate incentive pay and corresponding worker rents on the second tier. Because incentives spill down the hierarchy, a firm that raises bonuses for top-tier employees can reduce incentive pay on lower tiers and, at the same time, increase effort incentives along the hierarchy. Effectively, large employee rents on the top tier are not costly and, hence, not detrimental to the firm. This effect relaxes the firm’s fundamental trade-off between inducing high incentives and paying large worker rents.

Our paper is related to the literature on external recruitment versus internal job promotion. Chan (1996) points out that allowing external workers to apply for a vacant position reduces the career incentives of internal workers. To restore worker motivation, the firm can choose a handicap for discriminating against the external applicants. Tsoulouhas et al. (2007) also analyze handicaps, but include sorting of workers. Waldman (2003) focuses on the time-inconsistency problem of job assignment: To maximize work incentives, a firm should ignore external applications when filling a vacancy. However, ex post it should assign the most able worker to the vacant position irrespective of whether this worker is an insider or an outsider. Internal workers anticipate this time-inconsistency problem and reduce their efforts. The firm can install an internal labor market as a commitment device to solve this problem. Chen (2005) shows that external recruitment can be optimal from a pure incentive perspective. If internal workers can invest in sabotage, the firm will benefit from allowing external applications. In contrast to the previous literature, our paper stresses an advantage of internal job promotion under unobserved worker heterogeneity: When effort and ability are complements, the selection quality of internal promotion is strictly better than that of external recruiting. Under observed worker heterogeneity, however, the time-inconsistency problem pointed out by Waldman (2003) applies. The firm then hires from outside with a positive probability. We show that external hires earn lower rents than internal recruits.

This paper is also related to the principal-agent literature on multi-period incentive schemes under moral hazard and limited liability (or, similarly, wealth constraints and wage floors). Schmitz (2005b), Ohlendorf and Schmitz (2012), and Kräkel and Schöttner (2010) analyze how a principal can optimally organize different stages of a production process with wealth-constraint agents. While these authors also highlight that second-period rents can be used to motivate the first-period task, their models and applications
differ from ours. In Schmitz (2005b), the principal has to decide whether to assign two stages of the production process to the same agent or to two different agents. In contrast to our framework, success in the first stage decreases the rent necessary to induce high effort in the second stage. Like us, Ohlendorf and Schmitz (2012) consider two technologically independent production stages. However, for exogenous reasons, both stages have to be performed by the same agent. Moreover, the principal is integrated in the production process and can make an investment in each of the two periods. Kräkel and Schöttner (2010) show that, under certain conditions, the presence of minimum wages lead to more than first-best effort.

Furthermore, our paper sheds some light on the optimal interaction of rank-order tournaments with additional incentive schemes. This topic is of high practical relevance because firms usually combine different incentive devices for motivation and selection purposes. However, related work is still scarce. Tsoulouhas et al. (2007) analyze optimal handicapping of internal and external candidates in a contest to become CEO. They consider a promotion tournament where the prize is given by the incentive contract on the next hierarchy level, but do not allow for worker rents and long-term contracting. Schöttner and Thiele (2010) also investigate incentive contracting within a two-tier hierarchy. They examine the optimal combination of piece rates for level-1 workers and a promotion tournament to the next tier when both motivation and selection is an issue. In contrast to the present paper, the firm has to fill second-tier positions with internal candidates because firm-specific human capital is of particular importance. Moreover, workers are not protected by limited liability.

The remainder of the paper is organized as follows. In the next section, we introduce our basic model. Section 3 offers a solution to this model, comparing a combined contract with two separate contracts under homogeneous workers. In Section 4, we extend the basic model by introducing worker heterogeneity. Section 5 concludes.

2 The Basic Model

We consider two representative periods in the lifespan of a firm that consists of two hierarchy levels. In the first period, the firm needs to hire two workers for hierarchy level 1. In the second period, the firm has to fill one position on hierarchy level 2. The tasks
to be performed on the two hierarchy levels differ in their nature. For example, in a manufacturing firm, level 1 is the stage where workers fulfill production tasks while on level 2 a managerial task has to be performed. For clarity, we use the notation of worker and manager to refer to the agents on level 1 and 2, respectively, throughout the paper. However, the firm could also operate in the service industry.\(^\text{10}\)

Initially, we assume that all workers share the same abilities in both tasks. In Section 4, we also discuss the case of heterogeneous workers. Furthermore, all players are risk neutral. Agents are protected by limited liability, i.e., the firm cannot exact payments from them. On both tiers of the hierarchy, agents have zero reservation values. For simplicity, we neglect discounting.

The tasks associated with the two hierarchical tiers lead to qualitatively different performance signals. On hierarchy level 1, a worker’s effort does not generate an individually attributable output. Accordingly, the firm has only coarse information about the workers’ relative contribution to firm success. Technically, it observes a non-verifiable relative performance signal that provides information about which worker has performed better. By contrast, the managerial task on level 2 is accompanied by personal responsibility and, thus, leads to an individual performance measure that is verifiable. These assumptions fit well with stylized empirical facts. Nowadays, many firms employ holistic work organization on lower hierarchy levels where the blurring of occupational barriers makes the collection of verifiable, individual performance signals difficult. Instead, firms frequently rely on subjective performance evaluations by workers’ superiors.\(^\text{11}\) Sharper demarcation lines between jobs resulting in contractible individual performance signals are often more likely to arise on managerial ranks of the hierarchy. For example, the position on level 2 may be head of a department or a division.\(^\text{12}\)

On the first hierarchy level, each of the two workers \(i = (A, B)\) exerts effort \(\hat{e}_i \geq 0\). The effort has a non-verifiable monetary value \(\hat{v}(\hat{e}_i)\) to the firm with \(\hat{v}' > 0\) and \(\hat{v}'' \leq 0\). The firm neither observes \(\hat{e}_i\) nor \(\hat{v}(\hat{e}_i)\), but receives a non-verifiable ordinal signal \(\hat{s} \in \{\hat{s}_A, \hat{s}_B\}\) about the relative performance of the two workers. The signal \(\hat{s} = \hat{s}_A\) (\(\hat{s} = \hat{s}_B\)) indicates that worker \(A\) (worker \(B\)) has performed better than his co-worker. The

\(^{10}\)For example, in a large law firm, on level 1 associates solve cases in teams whereas on level 2 the partners’ most important task is to bring in business (Wilkins and Gulati 1998, p. 1597 and p. 1623).

\(^{11}\)See, e.g., Ichmiowski et al. (1997) and Lindbeck and Snower (2000) on holistic organizations.

\(^{12}\)We use this particular information structure because it often arises in practice. It is, however, not crucial for deriving our rent-based explanation for the existence of internal labor markets.
probability of the event \( \hat{s} = \hat{s}_A \) is given by \( \hat{p}(\hat{e}_A, \hat{e}_B) \) and that of \( \hat{s} = \hat{s}_B \) by \( 1 - \hat{p}(\hat{e}_A, \hat{e}_B) \).

We assume that the probability function \( \hat{p}(\hat{e}_A, \hat{e}_B) \) exhibits the properties of the well-known contest-success function introduced by Dixit (1987):\(^{13}\)

(i) \( \hat{p}(\cdot, \cdot) \) is symmetric, i.e. \( \hat{p}(\hat{e}_i, \hat{e}_j) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i) \),

(ii) \( \hat{p}_1 > 0, \hat{p}_{11} < 0, \hat{p}_2 < 0, \hat{p}_{22} > 0 \),

(iii) \( \hat{p}_{12} > 0 \iff \hat{p} > 0.5 \).

According to (ii), exerting effort has positive but decreasing marginal returns. Property (iii) implies that if, initially, player \( A \) has chosen higher effort than \( B \), a marginal increase in \( \hat{e}_B \) will make it more attractive to \( A \) to increase \( \hat{e}_A \) as well, due to the more intense competition the increase of \( \hat{e}_B \) has caused.

Spending effort \( \hat{e}_i \) leads to private costs \( \hat{c}(\hat{e}_i) \) for worker \( i \) (\( i = A, B \)) with \( \hat{c}(0) = \hat{c}'(0) = 0 \) and \( \hat{c}'(\hat{e}_i) > 0, \hat{c}''(\hat{e}_i) > 0 \) for all \( \hat{e}_i > 0 \). Furthermore, to guarantee some regularity conditions, we make the following technical assumptions. To ensure concavity of the firm’s objective function, we assume that \( \hat{c}'(\hat{e})/\hat{p}_1(\hat{e}, \hat{e}) \) is convex for all \( \hat{e} > 0 \).\(^{14}\)

Finally, to obtain an interior solution, we assume that \( \hat{c}''(0) = 0 \).

On the second hierarchy level, a manager’s effort generates an individual performance signal. Following the binary-signal model by Demougin and Garvie (1991) and Demougin and Fluet (2001), we assume that the manager chooses effort \( e \geq 0 \) leading to a contractible performance signal \( s \in \{ s^L, s^H \} \) with \( s^H > s^L \). The observation \( s = s^H \) is favorable information about the worker’s effort choice in the sense of Milgrom (1981). Let the probability of this favorable outcome be \( p(e) \) with \( p'(e) > 0 \) (strict monotone likelihood ratio property) and \( p''(e) < 0 \) (convexity of the distribution function condition).

If the firm assigns worker \( i \) to the management task, \( i \)'s effort choice \( e \) yields the firm a non-verifiable monetary value \( v(e) \) with \( v' > 0 \) and \( v'' \leq 0 \). Again, neither \( e \) nor its monetary value is observable by the firm. The manager’s private costs of exerting effort \( e \) are \( c(e) \) with \( c(0) = c'(0) = 0 \) and \( c'(e) > 0, c''(e) > 0 \) for all \( e > 0 \). Furthermore, analogous to the technical assumptions for the first hierarchy level, \( c'(e)/p'(e) \) is assumed to be convex for all \( e > 0 \),\(^{15}\) and \( c''(0) = 0 \).

In the given setting, the firm can use three different instruments to provide incentives:

First, it can employ relative performance pay (i.e., a rank-order tournament) on hierarchy

\(^{13}\)Subscripts of \( \hat{p}(\cdot, \cdot) \) denote partial derivatives.

\(^{14}\)This condition holds for a wide range of functions, e.g., \( c(\hat{e}) = \hat{e}^2 \) and \( \hat{p}(\hat{e}_A, \hat{e}_B) = \hat{e}_A/(\hat{e}_A + \hat{e}_B) \).

\(^{15}\)For example, feasible functions are \( c(e) = e^2 \) and \( p(e) = e^\theta \) with \( \theta \in (0, 1) \).
level 1. Under relative performance pay, the better performing worker receives a high wage $w_H$ whereas the other worker obtains a low wage $w_L$. Hence, worker $i$ earns $w_H$ if $\hat{s} = s_i$. Otherwise, he obtains $w_L$. Second, the firm can install a bonus scheme on hierarchy level 2. In case of a favorable signal ($s = s^H$) the manager gets a high bonus $b_H$, whereas he receives a low bonus $b_L$ if the signal is bad news ($s = s^L$). Due to limited liability, payments must always be non-negative ($w_L, w_H, b_L, b_H \geq 0$). Third, the firm can interlink the two hierarchy levels by committing to establish an internal labor market (or job-promotion scheme) where the better performing worker from level 1 will be promoted to level 2 at the end of the first period. Note that, even though $\hat{s}$ is unverifiable, the firm does not have an incentive to renege on such a promotion rule because workers are homogeneous in the basic model. Consequently, it does not matter which worker is assigned to the management position. The job-promotion scheme thus exhibits the important self-commitment property pointed out by Malcomson (1984).

According to these incentive devices, the firm can offer one of the following two types of contracts. Under the first type, the firm designs separate contracts for each tier of the hierarchy. In this case, the assignment of a worker to the level-2 job is purely random: The candidate can either be hired from the external labor market or be chosen from the internal pool of workers. This kind of contract clearly differs from the ideal of an internal labor market that allows for a port of entry only on the lowest hierarchy level. By contrast, the second type of contract follows the idea of an internal labor market by combining both hierarchy levels via a job-promotion scheme (combined contract). The contract details are specified in Section 3, where we analyze incentives and worker behavior under each contractual form.

3 Homogeneous Workers

3.1 Separate Contracts

We start our analysis of the basic model with the case of separate contracts. The time schedule of the game is as follows. First, the firm offers two workers a one-period contract specifying relative performance pay $(w_L, w_H)$ for employment on hierarchy level 1. Provided that the workers accept the contract, they exert efforts $\hat{e}_A$ and $\hat{e}_B$, respectively. Afterwards, $\hat{s}$ is observed. The workers then obtain $w_L$ or $w_H$, respectively, whereas the
firm receives $\hat{v}(\hat{e}_A) + \hat{v}(\hat{e}_B)$. Next, the firm has to hire an agent for the management job on hierarchy level 2. The firm offers a randomly chosen internal or external worker a one-period contract $(b_L, b_H)$ based on the performance signal $s$ on hierarchy level 2. After acceptance of the contract, the manager on level-2 chooses effort $e$, yielding signal $s$ and either a low or a high bonus payment. The firm earns $v(e)$. The timing is summarized in Figure 1.

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<td>firm offers $(w_L, w_H)$; workers</td>
<td>firm offers $(b_L, b_H)$; worker</td>
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Figure 1: Timing under separate contracts.

We solve the game by backwards induction and thus first analyze hierarchy level 2. For this tier, the firm’s optimization problem is

$$\max_{b_L, b_H, e} \{ v(e) - b_L - p(e) \cdot (b_H - b_L) \}$$

$$\text{s.t. } e = \arg \max_z \{ b_L + p(z) \cdot (b_H - b_L) - c(z) \}$$

$$b_L + p(e) \cdot (b_H - b_L) - c(e) \geq 0$$

$$b_L, b_H \geq 0.$$  \hspace{1cm} (1) \hspace{1cm} (2) \hspace{1cm} (3)

The firm maximizes its profit net of wage payments taking into account the incentive constraint (1), the participation constraint (2), and the limited-liability constraints (3). Due to the monotone likelihood ratio property and the convexity of the distribution function condition, the incentive constraint (1) is equivalent to its first-order condition

$$b_H - b_L = \frac{c'(e)}{p'(e)}.$$  \hspace{1cm} (4)
Using this relationship, the firm’s problem can be transformed to

\[
\max_{b_L, e} \left\{ v(e) - b_L - p(e) \cdot \frac{c'(e)}{p'(e)} \right\}
\]

s.t. \( b_L + p(e) \cdot \frac{c'(e)}{p'(e)} - c(e) \geq 0 \)

\( b_L \geq 0. \) (5)

Regarding the participation constraint, we can make the following observation, which is important for our further analysis.

**Lemma 1** The term

\[ r(e) := p(e) \frac{c'(e)}{p'(e)} - c(e) \]  

is strictly positive and strictly monotonically increasing for all \( e > 0. \)

**Proof.** \( r(e) > 0 \) can be rewritten as \( c(e) - c'(e) \frac{p(e)}{p'(e)} < 0. \) Note that \( p(e) - ep'(e) > 0 \) since \( p \) is strictly concave and thus \( \frac{p(e)}{p'(e)} > e. \) We therefore have \( c(e) - c'(e) \frac{p(e)}{p'(e)} < c(e) - ec'(e) < 0 \) from strict convexity of \( c. \) By strict concavity of \( p \) and strict convexity of \( c, \)

\[ r'(e) = p(e) \left[ \frac{c''(e)p'(e) - p''(e)c'(e)}{p'(e)^2} \right] > 0 \]  

for all \( e > 0. \) Hence, given \( e, \) the transformed participation constraint (5) is satisfied for all \( b_L \geq 0. \)

Therefore, the firm optimally sets \( b_L^* = 0, \) where superscript “s” denotes optimal contract parameters under separate contracts. After substituting \( b_L^* \) into the firm’s objective function, we obtain that the firm induces the effort level \( e^s > 0 \) given by\(^{16}\)

\[ e^s = \arg \max_e \{ v(e) - r(e) - c(e) \}. \]

Since the manager is protected by limited liability, the firm has to leave him a rent \( r(e). \)

As a result, the firm’s costs for inducing effort \( e \) are composed of the manager’s effort costs and rent.

Now we turn to hierarchy level 1. Here, two workers compete in a tournament for relative performance pay \( w_H \) and \( w_L. \) We first characterize the workers’ effort choices.

\(^{16}\)Due to our technical assumptions, the objective function is strictly concave; \( r''(e) > 0 \) follows from \( c'(e)/p'(e) \) being convex. Furthermore, the assumption \( c''(0) = 0 \) ensures an interior solution.
Given the wages $w_H$ and $w_L$, worker $A$ and worker $B$ choose their effort levels to solve

\[
\max_{\hat{e}_A} w_L + \hat{p}(\hat{e}_A, \hat{e}_B) \cdot [w_H - w_L] - \hat{c}(\hat{e}_A),
\]

\[
\max_{\hat{e}_B} w_L + [1 - \hat{p}(\hat{e}_A, \hat{e}_B)] \cdot [w_H - w_L] - \hat{c}(\hat{e}_B),
\]

respectively. The equilibrium effort levels must satisfy the first-order conditions

\[
(w_H - w_L) \hat{p}_1 (\hat{e}_A, \hat{e}_B) = \frac{\partial}{\partial \hat{e}_A} (\hat{e}_A)
\quad \text{and} \quad
-(w_H - w_L) \hat{p}_2 (\hat{e}_A, \hat{e}_B) = \frac{\partial}{\partial \hat{e}_B} (\hat{e}_B).
\]

Recall that, due to the symmetry property (i) of the probability function $\hat{p}$, we have $\hat{p}(\hat{e}_B, \hat{e}_A) = 1 - \hat{p}(\hat{e}_A, \hat{e}_B)$. Differentiating both sides with respect to $\hat{e}_B$ yields $\hat{p}_1 (\hat{e}_B, \hat{e}_A) = -\hat{p}_2 (\hat{e}_A, \hat{e}_B)$ so that the first-order conditions can be rewritten as

\[
w_H - w_L = \frac{\partial}{\partial \hat{e}_A} (\hat{e}_A) = \frac{\partial}{\partial \hat{e}_B} (\hat{e}_B).
\]

Thus, we have a unique symmetric equilibrium $(\hat{e}_A, \hat{e}_B) = (\hat{e}, \hat{e})$ given by

\[
w_H - w_L = \frac{\partial}{\partial \hat{e}} (\hat{e}).
\]

Condition (9) shows that equilibrium efforts increase in the tournament prize spread $w_H - w_L$.\footnote{By strict concavity of $\hat{p}$ and convexity of $\hat{c}$, this condition is necessary and sufficient for $(\hat{e}, \hat{e})$ to be an equilibrium. Our assumptions do not rule out the existence of additional asymmetric equilibria. However, we restrict attention to the symmetric equilibrium, which is plausible in the given setting with homogeneous contestants. Asymmetric equilibria do not exist for the well-known Tullock or logit-form contest-success function, $\hat{p}(\hat{e}_A, \hat{e}_B) = \hat{e}_A / (\hat{e}_A + \hat{e}_B)$.} To simplify notation, we denote by $\Delta w(\hat{e})$ the prize spread that induces effort $\hat{e}$, i.e.,

\[
\Delta w(\hat{e}) := \frac{\partial}{\partial \hat{e}} (\hat{e})
\]

The firm maximizes $2\hat{p}(\hat{e}) - w_L - w_H$ subject to the incentive constraint (9), the participation constraint

\[
w_L + \frac{1}{2} (w_H - w_L) - \hat{c}(\hat{e}) \geq 0,
\]

and the limited-liability constraints $w_L, w_H \geq 0$. Note that, when choosing the equilibrium effort $\hat{e}$, a worker must obtain at least the same expected payment as if he exerted
zero effort, i.e.,
\[ w_L + \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq w_L + \hat{p}(0, \hat{e}) \Delta w(\hat{e}) - \hat{c}(0). \] (12)

Hence, \( \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \), implying that the firm chooses \( w_L^* = 0 \). Together with (9), it follows that \( w_H^* = \Delta w(\hat{e}) \). Thus, the firm implements effort \( \hat{e}^s > 0 \) given by\(^{19}\)

\[ \hat{e}^s = \arg \max_{\hat{e}} 2\hat{v}(\hat{e}) - \Delta w(\hat{e}). \]

The results of this subsection are summarized in the following proposition.

**Proposition 1** Under separate contracts, the firm implements the positive effort levels

\[ \hat{e}^s = \arg \max_{\hat{e}} \left\{ 2\hat{v}(\hat{e}) - \Delta w(\hat{e}) \right\}, \] (13)

\[ e^s = \arg \max_e \left\{ v(e) - r(e) - c(e) \right\}. \] (14)

The optimal contract elements are

\[ w_L^s = 0, \ w_H^s = \Delta w(\hat{e}^s), \ b_L^s = 0, \ b_H^s = \frac{e'(e^s)}{p'(e^s)}. \] (15)

where \( r(e) \) and \( \Delta w(\hat{e}) \) are given by (6) and (10), respectively.

From strict concavity of a level-1 worker’s objective function it follows that, at \( \hat{e} = \hat{e}^s > 0 \), inequality (12) is strict. We thus have \( \frac{1}{2} \Delta w(\hat{e}^s) - \hat{c}(\hat{e}^s) > 0 \), implying that workers earn strictly positive rents. Moreover, from Lemma 1, it follows that the manager on level 2 also earns a strictly positive rent \( r(e^s) \). Presuming that effort on higher hierarchy levels is more valuable to firms than effort on lower levels,\(^{20}\) rents increase across hierarchy levels, i.e., \( \frac{1}{2} \Delta w(\hat{e}^s) - \hat{c}(\hat{e}^s) < r(e^s) \). This suggests that the firm may benefit from establishing an internal labor market where the better performing level-1 worker is promoted to the next hierarchy level. The level-2 rent then provides additional effort incentives for the first hierarchy level. An approach that uses a strict internal promotion rule according to past performance corresponds to our combined contract, which we analyze in the following subsection.

---

\(^{19}\)The second-order condition \( 2\hat{v}''(\hat{e}) - \Delta w''(\hat{e}) < 0 \) is satisfied due to our assumption that \( \hat{e}'(\hat{e})/\hat{p}_1(\hat{e}, \hat{e}) \) is convex. An interior solution is guaranteed by \( \hat{e}''(0) = 0 \).

\(^{20}\)That is, the firm’s value function for effort increases more steeply on higher hierarchy levels.
3.2 Combined Contract

Under a combined contract, the firm offers two workers a contract \((w_L, w_H, b_L, b_H)\) at the start of the first period. The contract includes the commitment to promote the better performing level-1 worker to level 2 in the second period, i.e., worker \(i\) will be promoted if and only if \(\hat{s} = \hat{s}_i\). In the second period, the promoted worker will be paid according to the pre-specified bonus scheme. For simplicity, we assume that the worker who did not achieve promotion is dismissed. Furthermore, the worker selected for promotion can quit and realize his reservation value of zero in the second period. Figure 2 summarizes the timing under a combined contract.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>firm offers ((w_L, w_H, b_L, b_H))</td>
<td>level-1 firm</td>
<td>observes (\hat{s})</td>
<td>promotes better worker</td>
<td>level-2 worker</td>
</tr>
<tr>
<td></td>
<td>workers choose (\hat{e}_i)</td>
<td>chooses (\hat{e}); payments made</td>
<td>level-1 worker payments made</td>
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Figure 2: Timing under a combined contract.

The time schedule differs from the one under separate contracts only with respect to stages 1 and 4. Under a combined contract, at stage 1, the firm offers two workers a contract \((w_L, w_H, b_L, b_H)\) that covers the following two periods. At stage 4, the firm promotes the better level-1 worker to the next tier.

Again, the game is solved by backwards induction. In the second period, given the bonus payments \(b_L\) and \(b_H\), the promoted worker faces the same kind of decision problem as under separate contracts. Provided that his participation constraint (2) is satisfied, he chooses the effort level characterized by (1). In the first period, however, workers’ optimization problems fundamentally differ from the case of two separate contracts. Now, increasing effort also raises the chance of being promoted and, consequently, earning a rent under the bonus contract. Hence, worker \(A\)’s and \(B\)’s optimization problems, respectively,
are:

$$\max_{\hat{e}_A} w_L + \hat{p}(\hat{e}_A, \hat{e}_B) \cdot [w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}_A) \tag{16}$$

$$\max_{\hat{e}_B} w_L + [1 - \hat{p}(\hat{e}_A, \hat{e}_B)] \cdot [w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}_B). \tag{17}$$

Comparing the workers’ objective functions with those under separate contracts, (7) and (8), we can see that, under combined contracts, the “prize” for performing better on level 1 increases by the expected payment to the promoted worker, \(b_L + p(e)(b_H - b_L) - c(e)\). In analogy to the case of separate contracts, one can show that there is a unique symmetric equilibrium given by

$$\hat{p}_1(\hat{e}, \hat{e}) [w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] = \hat{c}(\hat{e}). \tag{18}$$

The first-period participation constraint thus is

$$w_L + \frac{1}{2}[w_H - w_L + b_L + p(e)(b_H - b_L) - c(e)] - \hat{c}(\hat{e}) \geq 0. \tag{19}$$

We can now state the firm’s optimization problem:

$$\max_{e, \hat{e}, w_L, w_H, b_L} [2\hat{c}(\hat{e}) - w_L - w_H] + [v(e) - b_L - p(e)(b_H - b_L)] \tag{20}$$

subject to (1), (2), (18), (19),

$$w_L, w_H, b_L, b_H \geq 0. \tag{21}$$

By solving this problem, we obtain the following result.

**Proposition 2** Under a combined contract, the firm implements the effort levels

$$(\hat{e}^c, e^c) \in \arg \max_{\hat{e}, e} \{2\hat{v}(\hat{e}) - \Delta w(\hat{e}) + v(e) - c(e)\} \tag{23}$$

subject to \(\Delta w(\hat{e}) - r(e) \geq 0. \tag{24}$$

---

21 The limited-liability constraints \(b_L, b_H \geq 0\) imply that a promoted worker cannot be held liable to the extent of his tournament prize \(w_H\). This assumption is justified when workers can use their tournament prizes for consumption before the second period ends or, alternatively, when workers are protected by a strict liability limit of zero after failure at the bonus stage. However, one can show that replacing (22) by \(w_L \geq 0, w_H + b_L \geq 0\) and \(w_H + b_H \geq 0\) would not alter our results.
The optimal contract elements are

\[ w_{cL}^e = 0, \quad w_{cH}^e = \Delta w(\hat{e}^c) - r(e^e), \quad b_{cL}^e = 0, \quad b_{cH}^e = \frac{c'(e^e)}{p'(e^e)}. \]  

(25)

where \( r(e) \) and \( \Delta w(\hat{e}) \) are given by (6) and (10), respectively.

**Proof.** See the Appendix.

### 3.3 Comparison of the Two Contracts

Given Propositions 1 and 2, we are now able to investigate the question which of the two contracts the firm prefers. Our conjecture was that the combined contract may have the advantage of partially substituting direct first-level incentives \( w_H - w_L \) for indirect incentives which arise due to the prospect of the expected second-period rent \( r(e) \). By comparing the optimal contract elements (15) and (25), it becomes clear that this is indeed the case because \( w_{cH}^e = \Delta w(\hat{e}^c) - r(e^e) \). When we examine the firm’s objective functions under separate contracts (see (13) and (14)) and a combined contract (see (23)), we can see that the substitution of incentives has the consequence that the firm’s costs of inducing a given pair of effort levels \((\hat{e}, e)\) are strictly lower under combined contracts: \( \Delta w(\hat{e}) + c(e) \) as opposed to \( \Delta w(\hat{e}) + r(e) + c(e) \).

However, in contrast to the case of separate contracts, the firm’s optimization problem under combined contracts exhibits a constraint, (24). At first sight, one might think that this constraint restricts the set of feasible effort pairs \((\hat{e}, e)\) under combined contracts and is thus detrimental. However, such a conclusion would be wrong. As the proof of Proposition 2 shows, constraint (24) arises because, for any given level-2 effort \( e \), the firm always wants to use the entire associated rent to enhance level-1 incentives. In other words, given \( e \), the firm wishes to implement at least the first-level effort \( \hat{e} \) that workers are willing to spend to win \( r(e) \), i.e., \( \Delta w(\hat{e}) \geq r(e) \). To induce a level-1 effort with \( \Delta w(\hat{e}) < r(e) \), the firm would have to punish good performance on the first tier by setting \( w_H < w_L \). This cannot be optimal because the firm would actually pay for reducing effort. Hence, a combined contract only has advantages over two separate contract and is, therefore, optimal for homogeneous workers.\(^{22}\)

\(^{22}\)This is formally shown in the proof of Proposition 3.
To characterize effort under the optimal combined contract, it is necessary to distinguish whether restriction (24) is binding or not at the optimum. First, assume the constraint is not binding. From (23) it then follows that effort on the second hierarchy level equals first-best effort, i.e., \( e^c = e^{FB} = \arg\max_e \{ v(e) - c(e) \} \). Hence, under the combined contract, level-2 effort is larger than under separate contracts. Concerning the first hierarchy level, however, a comparison of (13) and (23) points out that \( \hat{e}^c = \hat{e}^s \).

Thus, interestingly, the use of second-level rents for incentive purposes on hierarchy level 1 does not lead to higher effort on that hierarchy level. In particular, level-1 effort remains strictly below first-best, given by \( \hat{e}^{FB} = \arg\max_{\hat{e}} \{ \hat{v}(\hat{e}) - \hat{c}(\hat{e}) \} \). This result is due to the fact that raising incentives on the second tier increases efforts on both levels, but level-1 efforts are then decreased again by reducing \( w_H - w_L \). Direct first-level incentives stemming from relative performance pay are simply replaced by indirect ones. The observation that level-2 effort is first-best can be related to the concept of efficiency wages, which has been reconsidered by contract theorists in the last decade. According to Tirole (1999, p. 745) and Schmitz (2005a), efficiency wages occur if workers are protected by limited liability and earn positive rents under the optimal contract. In their models, the implemented effort level is inefficiently small. By contrast, in our setting the firm may implement the efficient effort level \( e^{FB} \) on hierarchy level 2 although this entails a strictly positive rent, which is monotonically increasing in effort. Hence, combining both hierarchy levels for creating optimal incentives allows for efficiency wages in a more literal sense.

If restriction (24) is binding at the optimal solution, the combined contract exhibits different characteristics. In this case, the rent \( r(e^{FB}) \) is so large that it exceeds the costs for inducing \( \hat{e}^s \) under separate contracts, \( r(e^{FB}) > \Delta w(\hat{e}^s) \). The firm then still finds it optimal to increase managerial effort and rents, \( e^c > e^s \), to enhance incentives for level-1 workers. However, the optimal managerial rent remains strictly below \( r(e^{FB}) \) and thus level-2 effort is inefficient, \( e^c < e^{FB} \). Nevertheless, the rent increase is large enough to raise effort on level 1 as well, \( \hat{e}^c > \hat{e}^s \). If the production technology on level 2 leads to very large managerial rents, we may even have \( \hat{e}^c > \hat{e}^{FB} \).

Altogether, we have the following results:

**Proposition 3** (i) The combined contract always dominates two separate contracts. (ii) If restriction (24) is non-binding, i.e., \( \Delta w(\hat{e}^s) > r(e^{FB}) \), effort levels under the two
contracts compare as follows: \( \hat{e}^c = \hat{e}^s \) and \( e^c = e^{FB} > e^s \). (iii) If restriction (24) is binding, the firm implements \( \hat{e}^c > \hat{e}^s \) and \( e^{FB} > e^c > e^s \).

**Proof.** See Appendix. ■

Proposition 3 shows that, when workers are homogeneous, an internal labor market with a job-promotion scheme (i.e., the combined contract) strictly dominates contractual solutions that allow external recruitment on higher hierarchy levels (i.e., separate contracts). According to the traditional view, an internal labor market has the major disadvantage of protecting employees’ rents by excluding competition with the external labor market. Our setting, however, shows that this argument is only partly true: Under the combined contract, a manager’s rent on hierarchy level 2 is indeed larger than under separate contracts. However, this actually reflects a comparative advantage of internal labor markets in contrast to alternative personnel policies that allow external hiring on higher levels: By utilizing a job-promotion scheme, higher-level rents are profitably used for incentive purposes on lower hierarchical tiers and should therefore be larger than under external recruitment.

Moreover, with an internal labor market, the firm recovers managerial rents at least partly by decreasing incentive pay for the lower hierarchy level. Thus, higher-level rents are not necessarily costly to the firm. This can be seen when we fully characterize all possible contracting environments using Proposition 3. In addition to a job-promotion scheme, the firm will implement both a bonus scheme and relative performance pay if the rent for implementing first-best effort on hierarchy level 2 is not too large (case (ii) of Proposition 3). In such a situation, the firm makes use of moderate relative performance pay on the first tier by choosing the tournament winner prize \( w^c_H = \Delta w(\hat{e}^c) - r(e^{FB}) > 0 \). Since \( \hat{e}^s = \hat{e}^c \), the winner prize \( w^c_H \) is smaller than the winner prize under separate contracts, \( w^s_H = \Delta w(\hat{e}^s) \). Moreover, the firm installs high-powered incentives via a bonus scheme on level 2 of the hierarchy. The optimal bonus is zero in case of an unfavorable performance signal (\( b^c_L = 0 \)). In case of a favorable signal, the worker receives the bonus \( b^c_H = \frac{\epsilon^c(e^{FB})}{\epsilon^c(e^s)} \), which is larger than the high bonus under separate contracts, \( b^s_H = \frac{\epsilon^c(e^s)}{\epsilon^s} \). However, in contrast to separate contracts, the firm completely recovers the managerial rent \( r(e^{FB}) \) by decreasing level-1 performance pay by exactly the same amount.

If the rent for implementing \( e^{FB} \) is rather large (case (iii) of Proposition 3), the firm
does not implement relative performance pay on the first tier and thus solely relies on indirect level-1 incentives through the second-period rent. The managerial rent is now larger than the decrease in level-1 incentive pay relative to separate contracts. However, the firm additionally benefits from this large rent because it entails higher level-1 effort compared to separate contracts.

Our results are nicely in line with two empirical observations by case studies on internal labor markets that contradict standard models on job-promotion tournaments. As a first result, they find considerable variation in pay on each hierarchy level. This finding contradicts the important prerequisite of standard job-promotion tournaments that wages must be attached to jobs and, therefore, to hierarchy levels in order to generate incentives. In our model, under the combined contract we have a job-promotion tournament with pay variation because the promoted worker may or may not receive a bonus on hierarchy level 2. Furthermore, according to standard job-promotion tournaments, hierarchical wage differences should be completely explained by promotion premiums paid to workers when moving to higher levels in the hierarchy. However, the empirical studies point out that hierarchical wage differences are significantly larger than the corresponding promotion premiums. This second observation is also in line with our modified tournament model. Under the combined contract, a promoted worker does not only earn the promotion premium \( w_H - w_L \) but may also receive a bonus. In particular, the higher the expected rent on hierarchy level 2, the smaller will be the optimal promotion premium. The reason is that indirect incentives replace direct ones. Presuming that effort on higher hierarchy levels is more valuable to firms than effort on lower levels, we will have considerable rents on higher tiers, thus reducing corresponding promotion premiums.

So far, we have shown that an internal labor market structure in form of a combined contract with a job-promotion scheme is optimal when workers are homogeneous. However, in practice workers typically differ in their abilities. In that case, selection of high-ability workers becomes an issue. In the next section, we therefore consider worker heterogeneity and investigate whether an internal labor market is still optimal.

\[ \Delta w(\hat{e}) = r(\hat{e}) \implies w_H = 0. \]

\[ \text{See Treble et al. (2001), who analyze the internal structure and the wage policy of a British firm. Considerable wage variation within job levels is also documented by the empirical studies of Seltzer and Merrett (2000), Dohmen et al. (2004), Gibbs and Hendricks (2004) and Grund (2005). Moreover, Dohmen et al. (2004) show that promotion and wage increase are often not simultaneous, which gives further evidence that salaries are also determined by bonuses and not solely by promotion premiums.} \]
4 Heterogeneous Workers

In this section, we extend our basic model to two situations where workers potentially differ in their abilities. In one situation, we assume that a worker’s ability is revealed to the firm after one period of interaction. Such characteristics could be soft skills such as social competence, the capability to lead and motivate people, or to oversee complex production processes. However, other characteristics of a worker’s ability may not be revealed for some time, which is the other situation that we analyze. We start with the latter case of unobserved heterogeneity.

4.1 Unobserved Worker Heterogeneity

4.1.1 Modifications of the Basic Model

Neither the firm nor the workers themselves can observe abilities during the two periods of employment, i.e., there is symmetric uncertainty about worker quality. We assume that this aspect of unobserved ability persists over time and hierarchy levels, i.e., it is not task specific. Let each worker have either high or low unobservable talent, \( t_1 \) or \( t_0 \) respectively, with \( t_1 > t_0 > 0 \). All players have the same prior distribution about worker talent. For simplicity, let each talent be equally likely so that talent can be described by a random variable \( t \) that takes values \( t_0 \) and \( t_1 \), both with probability \( \frac{1}{2} \), and has mean \( E[t] = (t_0 + t_1)/2 \). To save notation, we normalize \( E[t] \) to 1.

On each hierarchy level, a worker’s talent influences both the value of effort for the firm and the probability of generating a favorable signal. In particular, the value of worker \( i \) to the firm is \( t \cdot \hat{v}(\hat{e}_i) \) on level 1 and \( t \cdot v(e) \) on level 2. In analogy, the probability of a favorable signal on level 2 is \( t \cdot p(e) \), with \( t_1 \cdot p(e) \leq 1 \) for all \( e \). At level 1, if both workers are equally talented, \( A \)'s probability of winning the tournament is still \( \hat{p}(\hat{e}_A, \hat{e}_B) \). Otherwise, if \( A \) has talent \( t \), his winning probability is \( \hat{p}(\hat{e}_A, \hat{e}_B; t) \) and \( B \)'s winning probability is \( 1 - \hat{p}(\hat{e}_A, \hat{e}_B; t) \).

We assume that the new probability function \( \hat{p}(\hat{e}_A, \hat{e}_B; t) \) has properties analogous to (i)–(iii) of function \( \hat{p}(\cdot, \cdot) \) (see Section 2). For example, in the basic model we have \( \hat{p}_1(\hat{e}_j, \hat{e}_i) = -\hat{p}_2(\hat{e}_i, \hat{e}_j) \), which follows from the symmetry assumption (i). In analogy, we
assume that \( \hat{p}(\hat{e}_i, \hat{e}_j; t_1) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i; t_0) \), implying

\[
\hat{p}_1 (\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_2 (\hat{e}_j, \hat{e}_i; t_0) \quad \text{and} \quad \hat{p}_2 (\hat{e}_i, \hat{e}_j; t_1) = -\hat{p}_1 (\hat{e}_j, \hat{e}_i; t_0)
\]  \hspace{1cm} (26)

for \( i, j = A, B; \ i \neq j \). We make the plausible assumption that talent has an impact on a worker’s absolute winning probability and his marginal one. In particular, we assume that, for given effort levels, a more talented worker has a higher winning probability than a less talented one, i.e.,

\[
\hat{p}(\hat{e}_i, \hat{e}_j; t_1) > \hat{p}(\hat{e}_i, \hat{e}_j; t_0).
\]  \hspace{1cm} (27)

Furthermore, effort and talent are complements in the sense of

\[
\hat{p}_1 (\hat{e}_i, \hat{e}_j; t_1) > \hat{p}_1 (\hat{e}_i, \hat{e}_j; t_0) \quad \text{and} \quad -\hat{p}_2 (\hat{e}_i, \hat{e}_j; t_0) > -\hat{p}_2 (\hat{e}_i, \hat{e}_j; t_1), \]  \hspace{1cm} (28)

i.e., marginally increasing effort is more effective under high talent than under low one. Properties (ii) and (iii) from the basic model also hold analogously for heterogeneous workers. Note that property (iii) together with symmetry here implies that \( \hat{p}_{12} (\hat{e}, \hat{e}; t_1) = -\hat{p}_{12} (\hat{e}, \hat{e}; t_0) \). Finally, we assume analogous regularity conditions to hold as in the basic model (see Section 2).

In the following, we again compare separate and combined contracts with a new focus on the selection quality of the two contracts.

### 4.1.2 Separate Contracts

We first consider the case of separate contracts. The equilibrium on hierarchy level 1 is now described by the first-order conditions

\[
(w_H - w_L) \frac{1}{4} \left( \hat{p}_1 (\hat{e}_A, \hat{e}_B; t_1) + \hat{p}_1 (\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}_1 (\hat{e}_A, \hat{e}_B) \right) = \hat{c}' (\hat{e}_A),
\]

\[
(w_H - w_L) \frac{1}{4} \left( -\hat{p}_2 (\hat{e}_A, \hat{e}_B; t_1) - \hat{p}_2 (\hat{e}_A, \hat{e}_B; t_0) - 2\hat{p}_2 (\hat{e}_A, \hat{e}_B) \right) = \hat{c}' (\hat{e}_B).
\]

Using \( \hat{p}_1 (\hat{e}_B, \hat{e}_A) = -\hat{p}_2 (\hat{e}_A, \hat{e}_B) \) and (26) shows that there exists a symmetric equilibrium in which each worker chooses \( \hat{e} \) characterized by

\[
w_H - w_L = \Delta \hat{w}(\hat{e}) \quad \text{with} \quad \Delta \hat{w}(\hat{e}) := \frac{4\hat{c}' (\hat{e})}{\hat{p}_1 (\hat{e}, \hat{e}; t_1) + \hat{p}_1 (\hat{e}, \hat{e}; t_0) + 2\hat{p}_1 (\hat{e}, \hat{e})} \]

(29)
and $\Delta \hat{w}'(\hat{e}) > 0$. Since $E[t] = 1$, the firm maximizes $2\hat{v}(\hat{e}) - w_L - w_H$ subject to the participation constraint (11), the limited-liability constraints $w_L, w_H \geq 0$, and the incentive constraint (29). The optimal tournament prizes are, therefore, given by $w_L^* = 0$ and $w_H^* = \Delta \hat{w}(\hat{e})$, and the firm implements the effort level $\hat{e}_h^*$ that solves

$$\max_{\hat{e}} 2\hat{v}(\hat{e}) - \Delta \hat{w}(\hat{e}).$$

(30)

On hierarchy level 2, the firm’s optimization problem is identical to the one in Subsection 3.1 because $E[t] = 1$. The firm thus implements $\hat{e}_h^* = e^*$.  

### 4.1.3 Combined Contract

Now we turn to the combined contract. Solving the game by backwards induction, we first consider the actions on level 2. Here, based on the realization of the relative performance signal $\hat{s}$, all players update their beliefs about the unknown talent of the promoted worker. Let $E[t|\hat{s}]$ denote the expected talent of the promoted worker. Note that at any prior point in time the workers as well as the firm already know that they have to update their beliefs in light of the promotion decision. Hence, when designing the optimal combined contract, the firm has to include the incentive constraint

$$b_H - b_L = \frac{c'(e)}{E[t|\hat{s}] p'(e)}$$

(31)

and the participation constraint

$$b_L + E[t|\hat{s}] p(e) (b_H - b_L) - c(e) \geq 0 \leftrightarrow b_L + r(e) \geq 0,$$

(32)

where the last inequality follows from (6) and (31).

---

25 We obtain $\Delta \hat{w}'(\hat{e}) > 0$ from $\frac{\partial}{\partial e} (\hat{p}_1 (\hat{e}, \hat{e}; t_1) + \hat{p}_1 (\hat{e}, \hat{e}; t_0) + 2\hat{p}_1 (\hat{e}, \hat{e})) = \hat{p}_{11} (\hat{e}, \hat{e}; t_1) + \hat{p}_{12} (\hat{e}, \hat{e}; t_1) + \hat{p}_{11} (\hat{e}, \hat{e}; t_0) + \hat{p}_{12} (\hat{e}, \hat{e}; t_0) + 2\hat{p}_{11} (\hat{e}, \hat{e}) + 2\hat{p}_{12} (\hat{e}, \hat{e}) < 0$. Concerning the relative size of $\Delta \hat{w}(\hat{e})$ and $\Delta w(\hat{e})$, note that the introduction of symmetric ability uncertainty increases the randomness of the tournament outcome, which usually implies that the prize spread required to induce a given effort level increases. In our model, however, talent-effort complementarities may lead to an opposite effect so that $\Delta \hat{w}(\hat{e})$ is not necessarily larger than $\Delta w(\hat{e})$.

26 Here and in the following, the subscript "h" for optimal efforts indicates heterogeneity of workers.
At level 1, worker $A$ and worker $B$ maximize

$$w_L + (w_H - w_L + b_L + E [t|\hat{s}] p(e) (b_H - b_L) - c(e))$$

$$\times \frac{1}{4} (\hat{p}(\hat{e}_A, \hat{e}_B; t_1) + \hat{p}(\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}(\hat{e}_A, \hat{e}_B)) - \hat{c}(\hat{e}_A) \quad \text{and}$$

$$w_L + (w_H - w_L + b_L + E [t|\hat{s}] p(e) (b_H - b_L) - c(e))$$

$$\times \frac{1}{4} ((1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_1)) + (1 - \hat{p}(\hat{e}_A, \hat{e}_B; t_0)) + 2 (1 - \hat{p}(\hat{e}_A, \hat{e}_B))) - \hat{c}(\hat{e}_B),$$

respectively. Equations (6) and (31) together with the first-order conditions, $\hat{p}_1(\hat{e}_B, \hat{e}_A) = -\hat{p}_2(\hat{e}_A, \hat{e}_B)$, and (26) yield

$$(w_H - w_L + b_L + r(e)) \frac{\hat{p}_1(\hat{e}_A, \hat{e}_B; t_1) + \hat{p}_1(\hat{e}_A, \hat{e}_B; t_0) + 2\hat{p}_1(\hat{e}_A, \hat{e}_B)}{4} = \hat{c}'(\hat{e}_A)$$

$$(w_H - w_L + b_L + r(e)) \frac{\hat{p}_1(\hat{e}_B, \hat{e}_A; t_0) + \hat{p}_1(\hat{e}_B, \hat{e}_A; t_1) + 2\hat{p}(\hat{e}_B, \hat{e}_A)}{4} = \hat{c}'(\hat{e}_B).$$

Thus, in the symmetric equilibrium each worker exerts $\hat{c}$ described by

$$w_H - w_L + b_L + r(e) = \Delta \hat{w}(\hat{c})$$

with $\Delta \hat{w}(\hat{c})$ being defined in (29).

Now we can state the firm’s problem. It maximizes

$$2\hat{v}(\hat{c}) - 2w_L - (w_H - w_L) + E [t|\hat{s}] v(e) - b_L - E [t|\hat{s}] p(e) (b_H - b_L)$$

$$(6),(31),(33) \quad 2\hat{v}(\hat{c}) - \Delta \hat{w}(\hat{c}) + E [t|\hat{s}] v(e) - 2w_L - c(e)$$

subject to the limited-liability constraints (22), the incentive constraints (31) and (33), the participation constraint for the second hierarchy level (32), and the participation constraint for the first level,

$$w_L + \frac{1}{2} (w_H - w_L + b_L + E [t|\hat{s}] p(e) (b_H - b_L) - c(e)) - \hat{c}(\hat{e}) \geq 0$$

$$(6),(31),(33) \quad w_L + \frac{1}{2} \Delta \hat{w}(\hat{c}) - \hat{c}(\hat{e}) \geq 0.$$

Moreover, the firm has to take into account that $E [t|\hat{s}]$ depends on the workers’ equilib-
rium efforts chosen on hierarchy level 1:

\[
E [t|s] = \frac{1}{4} t_1 + \frac{1}{4} t_0 + \frac{1}{4} \left( \hat{p}(\hat{e}, \hat{e}; t_1) t_1 + (1 - \hat{p}(\hat{e}, \hat{e}; t_1)) t_0 \right) \\
+ \frac{1}{4} \left( \hat{p}(\hat{e}, \hat{e}; t_0) t_0 + (1 - \hat{p}(\hat{e}, \hat{e}; t_0)) t_1 \right) \\
= 1 + \frac{\Delta t (\hat{p}(\hat{e}, \hat{e}; t_1) - \hat{p}(\hat{e}, \hat{e}; t_0))}{4} \quad (27)
\]

with \( \Delta t := t_1 - t_0 \). Thus, the posterior expectation is larger than the prior one because the more talented worker is promoted with higher probability in case of an asymmetric pairing in the tournament. Furthermore, the posterior mean strictly increases in level-1 equilibrium efforts as talent and effort are complements:

\[
\frac{\partial E [t|s]}{\partial \hat{e}} = \frac{\Delta t}{4} (\hat{p}_1(\hat{e}, \hat{e}; t_1) + \hat{p}_2(\hat{e}, \hat{e}; t_1) - \hat{p}_1(\hat{e}, \hat{e}; t_0) - \hat{p}_2(\hat{e}, \hat{e}; t_0)) \\
\quad = \frac{\Delta t}{2} (\hat{p}_1(\hat{e}, \hat{e}; t_1) - \hat{p}_1(\hat{e}, \hat{e}; t_0)) \quad (28) > 0.
\]

Applying the same two-step procedure as in the basic model yields that the firm implements the effort pair \((\hat{e}_h^e, \hat{e}_h^c)\) with\(^{27}\)

\[
(\hat{e}_h^e, \hat{e}_h^c) \in \arg \max_{\hat{e}, e} \left\{ 2\hat{e}(\hat{e}) + E [t|s] v(e) - \Delta \hat{w}(\hat{e}) - c(e) \right\} \quad (36)
\]

subject to \( \Delta \hat{w}(\hat{e}) - r(e) \geq 0 \).\(^{28}\) (37)

When comparing optimal efforts under the combined contract with those under two separate contracts, we have to distinguish whether the restriction (37) is binding or not at the optimum. In case of a non-binding restriction, optimal efforts \((\hat{e}_h^e, \hat{e}_h^c)\) are described by the first-order conditions

\[
2\hat{e}'(\hat{\hat{e}}) + \frac{\partial E [t|s]}{\partial \hat{e}} v(e) = \Delta \hat{w}'(\hat{\hat{e}}) \quad \text{and} \quad E [t|s] v'(e) = e'(e). \quad (38)
\]

Comparing the first equation with (30) clearly shows that \( \hat{e}_h^e > \hat{e}_h^s \) as \( \partial E [t|s] / \partial \hat{e} > 0 \). The comparison of the second equation with (14) points out that \( \hat{e}_h^c > e_h^s \), due to Lemma 1 and the fact that \( E [t|s] > E [t] = 1 \). Now, we have to consider the case of a binding restriction (37). Using this restriction, we can express level-2 effort as a function of

\(^{27}\)The computations can be requested from the authors.
level-1 effort, \( e(\hat{e}) \), with 
\[
\frac{\partial e}{\partial \hat{e}} = \frac{\Delta \hat{w}'(\hat{e})}{r'(e)} > 0.
\]
The firm’s objective function under a combined contract can be rewritten as 
\[
2\hat{v}(\hat{e}) + E[t|s]v(e(\hat{e})) - \Delta \hat{w}(\hat{e}) - c(e(\hat{e})).
\]
Inserting for \( \partial e/\partial \hat{e} \) in the respective first-order condition leads to
\[
2\hat{v}'(\hat{e}) + \frac{\partial E[t|s]}{\partial \hat{e}}v(e(\hat{e})) + \frac{E[t|s]v'(e(\hat{e})) - c'(e(\hat{e})) - r'(e(\hat{e}))}{r'(e(\hat{e}))}\Delta \hat{w}'(\hat{e}) = 0.
\]
Since the first two expressions as well as \( r'(e(\hat{e})) \) and \( \Delta \hat{w}'(\hat{e}) \) are positive, the numerator of the last expression is negative. As this numerator is a strictly concave function of \( e(\hat{e}) \) and since \( E[t|s] > E[t] = 1 \), we obtain from the comparison with (14) that \( e^*_c > e^*_s \).

Finally, we have to consider optimal effort implementation on level 1. Since (37) is binding, the effort \( \hat{e} \) that would maximize level-1 profit corresponds to a level-2 effort that is below the effort \( e \) that maximizes level-2 profit \( E[t|s]v(e) - c(e) \). Hence, the firm may be interested in further raising \( \hat{e} \). As both profit functions are strictly concave, we can apply the same argument as in the proof of Proposition 3: The firm would, thus, never implement a smaller \( \hat{e} \) than the optimal effort under a non-binding restriction. Since that effort was larger than the optimal level-1 effort under separate contracts, we have proved that \( \hat{e}^*_c > \hat{e}^*_s \) also holds under a binding restriction.

**Proposition 4** Irrespective of whether restriction (37) is binding or not at the optimum, we have \( \hat{e}^*_c > \hat{e}^*_s \) and \( e^*_c > e^*_s \).

Proposition 4 points out that, under a combined contract, the firm always implements strictly larger effort on level 1 than under separate contracts. This result contrasts with our findings on homogeneous workers in Proposition 3. Intuitively, in case of unobservable talent, the firm has an additional motive for implementing large effort on level 1: The larger \( \hat{e} \) the higher is the probability that the worker of higher unobserved talent is promoted to level 2 in case of a heterogeneous pairing, i.e., \( \hat{p}(\hat{e}, \hat{e}; t_1) > 0 \). This, in turn, increases the posterior expected talent of the promoted worker: \( \partial E[t|s]/\partial \hat{e} > 0 \) according to (35) since \( E[t|s] \) monotonically increases in \( \hat{p}(\hat{e}, \hat{e}; t_1) \). In other words, if agents have unobservable characteristics that persist across hierarchy levels, higher incentives on level 1 improve worker selection for level 2. The reason is that incentives and selection are strictly interlinked.

It is interesting to compare the selection quality of a combined contract with that of alternative recruiting forms for the level-2 job. Under external recruiting and the resulting
random choice of a level-2 worker, the probability of assigning a high-ability worker to the management task is 1/2. With a combined contract, however, the probability of promoting the more talented worker from a heterogenous match is \( \hat{p}(\hat{e}_h^c, \hat{e}_i^c; t_1) > 1/2 \) because of \( \hat{p}(\hat{e}_i, \hat{e}_j; t_1) = 1 - \hat{p}(\hat{e}_j, \hat{e}_i; t_0) \) and (27). That promotion tournaments improve selection is well-known in the literature (e.g., Gifford and Kenney 1986; Rosen 1988, p. 83). Several papers analyze in detail the selection properties of such standard tournaments. Lazear and Rosen (1981) show that workers of different ability will not self-sort if a firm offers alternative tournaments for the respective workers. However, combining prize spreads and monitoring precisions for different contests (O’Keeffe et al. 1984), or comparing workers across instead of within hierarchy levels (Bhattacharya and Guasch 1988) can lead to self-sorting. Like our paper, Meyer (1991) analyzes a setting with symmetric ability uncertainty, but does not allow for endogenous effort choices. She shows how selection can be improved by splitting a one-period tournament into two successive contests. Clark and Riis (2001, 2007) depart from a standard job-promotion tournament with a strict wages-attached-to-job scheme. They emphasize that endogenous prizes that depend on performance lead to a considerably better selection in a one-period setting. In our model, the separate contract for level 1 corresponds to a standard promotion scheme: The relative performance pay \( w_H \) can also be interpreted as a fixed wage attached to the next hierarchy level. Since level-1 effort is higher under the combined contract, \( \hat{e}_h > \hat{e}_h^* \), our combined contract also improves upon the selection quality of a standard job-promotion tournament, as shown in the following corollary.

**Corollary 1** Combining job-promotion with incentive pay on the next hierarchy level always improves the selection quality of a job-promotion tournament.

**Proof.** \( \hat{p}(\hat{e}_h^c, \hat{e}_i^c; t_1) > \hat{p}(\hat{e}_h^*, \hat{e}_h^*; t_1) \) since \( \frac{\partial}{\partial \hat{e}} \hat{p}(\hat{e}, \hat{e}; t_1) = \hat{p}_1(\hat{e}, \hat{e}; t_1) + \hat{p}_2(\hat{e}, \hat{e}; t_1) \) \( \overset{(26)}{=} \hat{p}_1(\hat{e}, \hat{e}; t_1) - \hat{p}_1(\hat{e}, \hat{e}; t_0) > 0 \). □

In the Introduction and at the end of Section 3, we mentioned empirical puzzles that contradict standard tournament theory but can be explained in our model. One of these puzzles was that wages are not attached to jobs and, therefore, to hierarchy levels. As Corollary 1 shows, the selection quality of standard job-promotion tournaments can be significantly improved by replacing wages that are attached to jobs with incentive pay such as a bonus scheme.
4.2 Observed Worker Heterogeneity

In this section, we extend the basic model to analyze how observed heterogeneity of workers affects optimal contracting. In contrast to Section 4.1, where unobserved worker heterogeneity persists across hierarchy levels, we now assume that all workers share the same abilities in the production task, but differ in their managerial talents. On the first hierarchy level, we maintain the assumptions of the basic model. On level 2, however, the firm’s valuation of the manager’s effort now depends on the latter’s ability. In particular, if the firm assigns worker $i$ to the management task, $i$’s effort choice $e$ yields the value $v(e) + \mu_i$, $i = A, B$. Here, $\mu_A$ and $\mu_B$ are independent draws from a probability distribution of the same random variable, which reflects workers’ different talents for the management position. At the beginning of the first period, nobody knows $\mu_i$. However, during the course of that period, the firm gets to know the workers and, finally, can assess who is better suited for the managerial task. Hence, the firm observes $\mu_i$ at the end of the first period. No other party is able to assess the workers’ suitability for level 2 and, as a consequence, $\mu_i$ is non-verifiable. Instead of promoting a worker from level 1 to the management position, the firm can also hire an external candidate of unknown talent. The expected monetary value of managerial effort then is $v(e) + E[\mu_i]$.

For simplicity, we assume that ability can be either high or low, $\mu_i \in \{0, \mu\}$, where $\mu > 0$. The probability that $\mu_i$ is high is $1/2$. Accordingly, at the end of period 1, there are four possible situations with respect to the outcome of the tournament:

(i) both winner and loser have high ability,
(ii) only the winner has high ability,
(iii) both have low ability,
(iv) the winner has low ability whereas the loser’s ability is high.

In cases (i) and (ii), the firm cannot do better than promoting the tournament winner, because he is of the highest possible ability. However, in case (iii), from a pure selection perspective, the firm would be better off by hiring an external candidate for the management position, receiving an expected ability of $\mu/2$ instead of zero. In case (iv), the firm prefers to promote the loser of the tournament instead of the winner.

Hence, the firm faces the following trade-off: If it ex-ante announces to always promote internally, thereby strengthening level-1 effort incentives, it forgoes the possibility to hire a better suited external candidate for the management job when both workers are of
low ability. Moreover, in case (iv), the firm encounters a commitment problem: Since the relative performance signal from level 1 is not verifiable, the firm cannot commit to promote the tournament winner when he is less suited for the management position than his colleague. We assume that the firm can contract to exclusively promote internally, but has no means to commit to promoting the tournament winner in case (iv).

Consequently, the firm can choose between two contractual forms denoted by $C_1$ and $C_2$. Under contract $C_1$, the firm announces to always promote internally. In particular, in cases (i)–(iii), the winner of the tournament becomes the new manager whereas, in case (iv), the tournament loser is promoted. By contrast, under contract $C_2$, the firm leaves open the possibility to hire from the external labor market. It announces to promote the winner of the tournament only if he is of high ability, i.e., in cases (i) and (ii). In case (iii), however, the firm recruits the new manager from the external labor market. In case (iv), analogously to $C_1$, the tournament loser is promoted.

By choosing between the contracts $C_1$ and $C_2$, the firm decides on the probability with which the tournament winner is promoted, which we denote by $\alpha_C$, $C = C_1, C_2$. Furthermore, $\Gamma_C$ denotes the expected ability of the worker that is assigned to level 2 under contract $C$. The values of $\alpha_C$ and $\Gamma_C$ are given in Table 1.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\multicolumn{2}{|c|}{$C_1$} & \multicolumn{1}{c|}{$C_2$} \\
\hline \hline $\alpha_C$ & $\frac{3}{4}$ & $\frac{1}{2}$ \\
$\Gamma_C$ & $\frac{3}{4}\mu$ & $\frac{7}{8}\mu$ \\
\hline
\end{tabular}
\caption{Contracts under observed heterogeneity}
\end{table}

Accordingly, contract $C_1$ uses the level-2 rent more effectively for level-1 incentive provision because the tournament winner is promoted with higher probability. However, contract $C_2$ implies a higher expected managerial ability because it allows for external recruitment in case the production workers are of low ability.

Now assume the firm has chosen contract $C$ and consider the workers’ decision problems. On level 2, the worker faces the same decision problem as in the basic model.

---

28 This assumption is in line with Waldman (2003), who shows that internal labor markets are a remedy for the time-inconsistency problem arising in promotion decisions due to the simultaneous presence of incentive and selection issues.

29 Hence, we do not consider the possibility to engage in relational contracting, where such a promotion rule can be self-enforcing when the firm cares about its future reputation.
Thus, assume that \( e \) satisfies the second-level incentive constraint (1) and, moreover, the second-level participation constraint (2) holds. Then, on the first level, worker \( A \) chooses \( \hat{e}_A \) to maximize

\[
\begin{aligned}
& w_L + \hat{p}(\hat{e}_A, \hat{e}_B) (w_H - w_L) + \left( \alpha_C \cdot \hat{p}(\hat{e}_A, \hat{e}_B) + \frac{1 - \hat{p}(\hat{e}_A, \hat{e}_B)}{4} \right) [b_L + r(e)] - \hat{c}(\hat{e}_A),
\end{aligned}
\]

with \( r(e) \) given by (6), whereas, by choosing \( \hat{e}_B \), worker \( B \) maximizes

\[
\begin{aligned}
& w_L + (1 - \hat{p}(\hat{e}_A, \hat{e}_B)) (w_H - w_L) + \left( \alpha_C \cdot (1 - \hat{p}(\hat{e}_A, \hat{e}_B)) + \frac{\hat{p}(\hat{e}_A, \hat{e}_B)}{4} \right) [b_L + r(e)] - \hat{c}(\hat{e}_B).
\end{aligned}
\]

Thus, the symmetric equilibrium on level 1 is implicitly given by

\[
\hat{p}_1 (\hat{e}, \hat{e}) \left( w_H - w_L + \left( \alpha_C - \frac{1}{4} \right) [b_L + r(e)] \right) = \hat{c}' (\hat{e}). \tag{39}
\]

The first-level participation constraint is

\[
\begin{aligned}
& w_L + \frac{1}{2} (w_H - w_L) + \frac{1}{2} \left( \alpha_C + \frac{1}{4} \right) [b_L + r(e)] - \hat{c}(\hat{e}) \geq 0. \tag{40}
\end{aligned}
\]

Hence, given contract \( C_1 \), the firm has to solve

\[
\max_{\text{e,e}, \text{w}_L, \text{w}_H, \text{b}_H, \text{b}_L}
\begin{aligned}
& 2\hat{c}(\hat{e}) - w_L - w_H + [v(e) - b_L - p(e)(b_H - b_L)] + \Gamma_{C_1}
\end{aligned}
\]

subject to (1), (2), (22), (39), (40).

The restrictions (1) and (2) are the incentive and participation constraints, respectively, for level 2. Conditions (39) and (40) are the new incentive constraint and participation constraint, respectively, for level 1. Finally, (22) are the limited-liability constraints. Analogously to the problem of determining the optimal combined contract for homogeneous workers (Proposition 2), we can simplify the firm’s problem to\(^{30}\)

\[
\max_{\text{e,e}} \pi^{C_1}(e, \hat{e}) + \Gamma_{C_1} \quad \text{s.t.} \quad \Delta w(\hat{e}) - \frac{1}{2} r(e) \geq 0, \tag{41}
\]

where \( \pi^{C_1}(e, \hat{e}) = 2\hat{c}(\hat{e}) - \Delta w(\hat{e}) + \left[ v(e) - c(e) - \frac{1}{2} r(e) \right] \)

\(^{30}\)Detailed computations can be requested from the authors.
Given contract $C_2$, the firm solves

$$\max_{e, \hat{e}, \es, \wL, \wH, \bL, \bH} \ 2\hat{\nu}(\hat{e}) - \wL - \wH + \frac{3}{4} \left[ \nu(e) - \bL - p(e) (\bH - \bL) \right] + \frac{1}{4} E + \Gamma_{C_2}$$

subject to (1), (2), (22), (39), (40).

Here, $E$ denotes the firm’s level-2 profit when an external candidate is hired for the job on the second hierarchy tier. To such a candidate, the firm offers the optimal separate contract for level 2. Consequently, we have $E = v(e^*) - r(e^*) - c(e^*)$. Proceeding in analogy to contract $C_1$, the firm’s problem boils down to

$$\max_{e, \hat{e}} \ \pi^{C_2}(e, \hat{e}) + \Gamma_{C_2} \ \text{s.t.} \ \Delta \wL (\hat{e}) - \frac{1}{4} r(e) \geq 0,$$

where $\pi^{C_2}(e, \hat{e}) = 2\hat{\nu}(\hat{e}) - \Delta \wL (\hat{e}) + \frac{3}{4} \left[ \nu(e) - c(e) - \frac{2}{3} r(e) \right] + \frac{1}{4} E$

Let $(\hat{e}_C, e_C)$ denote the effort levels that the firm induces under contract $C$. Then, the firm allows for external recruitment if and only if

$$\pi^{C_1}(\hat{e}_C, e_C) - \pi^{C_2}(\hat{e}_C, e_C) \leq \Gamma_{C_2} - \Gamma_{C_1}. \quad (42)$$

The left-hand side of inequality (42) corresponds to the incentive effect of establishing an exclusively internal labor market (contract $C_1$) instead of sometimes recruiting on the external labor market (contract $C_2$). By contrast, the right-hand side of (42) characterizes the beneficial selection effect of external recruitment. Since the left-hand side is independent of $\mu$ while the right-hand side is increasing in $\mu$, we obtain the following result.

**Proposition 5** The firm recruits externally with positive probability if workers differ sufficiently strongly in their abilities, i.e., if $\mu$ is sufficiently large.

According to Proposition 5, when the firm is able to observe some worker characteristics that affect productivity on the second hierarchy level, the firm may want to leave open the possibility of external recruitment for the level-2 job. External recruitment

\[ \text{Under contract } C_2, \text{ for a given effort pair } (\hat{e}, e), \text{ the firm minimizes implementation costs } \wL + \wH + \frac{3}{4} [\bL + p(e) (\bH - \bL)]. \]
occurs with positive probability whenever the firm will suffer a sufficiently large productivity loss if it assigns a low-ability type to the management position. Then, in order to improve selection, the firm sacrifices part of the incentive effect of the level-2 rent for level-1 workers.

As mentioned in the introduction, empirical studies on internal labor markets document external recruiting on higher hierarchy levels. This observation contradicts the strict version of an internal labor market as well as traditional models on job-promotion tournaments since external hiring destroys internal career incentives. According to Proposition 5, the observation of entry ports on higher levels is in line with our model. Such a weaker form of an internal labor market will be favored by the firm if it learns workers’ abilities during their careers and selection of appropriate employees for management positions is an important issue relative to incentive provision.

5 Conclusion

We analyzed a two-tier hierarchy where workers compete in a rank-order tournament on level 1. On the second tier, a worker carries out a managerial task leading to an individual performance signal, which can be used in a bonus contract. Employees are protected by limited liability. We have shown that interlinking the incentive schemes from the two tiers by a job-promotion tournament, thereby creating an internal labor market, has two advantages: First, rents from level 2 can be used to create incentives for level 1. As a consequence, the firm may even implement first-best effort on the second hierarchy level although the worker earns a strictly positive rent on this level. Second, in case of unobserved worker heterogeneity, a combined contract has the additional advantage of improving the tournament’s selection quality in promoting the most talented internal worker. If, however, some characteristics of a worker’s ability are observable, the firm may prefer to hire level-2 employees from the external labor market when it turns out that the internal candidates are of low ability. Thus, if selection of high-ability workers is sufficiently important, it may be optimal to have ports of entry on higher hierarchy levels. In this case, level-2 employees hired from the outside market obtain a different contract than internally promoted workers. In particular, external candidates receive

lower bonuses and, consequently, lower rents than internal ones.

Our paper yields two testable implications. First, the effect of top-tier rents on improving overall incentives implies that firms offering large rents for top executives should rely more on internal promotions than on external recruiting compared to firms with lower top-tier rents. Recall from the introduction that the seminal paper by Rosen (1986) leads to a similar implication. Empirically, we can distinguish between both approaches by separating bonus payments from base salaries when measuring top executive income. Whereas Rosen assumes that fixed wages are attached to jobs so that internal promotions should be positively correlated with top executives’ base salaries, our approach predicts a positive correlation between internal promotions and top executives’ bonus payments. Note that the former correlation is documented by Baker et al. (1994), whereas the latter one is shown by Grund and Kräkel (2012). Gibbs (1995) presents evidence for both correlations. Moreover, departing from our model by assuming that more than two workers compete for promotion, the rents a firm offers on upper tiers should be increasing in the number of lower-tier employees competing for promotion. The reason is that higher rents counteract diminished effort incentives due to fiercer competition for promotion.

As a second testable prediction, our model claims that, if firms utilize internal labor markets to exploit higher-level rents for incentive provision on lower tiers, they should offer lower-powered incentive contracts to external hires than to internal ones. When testing this result empirically, however, one has to take account for an opposing effect. In a situation with firm competition for high-ability workers and (partly) observed heterogeneity, firms have to pay high wage premiums to external employees with high abilities to attract them. Since these wage premiums may be offered in form of both base salaries and bonuses, there also exists a tendency to offer external hires a larger income than internally promoted employees, which is documented by Agrawal et al. (2006).

From a theoretical perspective, combining a promotion tournament with a bonus scheme on the next tier of the hierarchy might lead to further advantages if there is the possibility of sabotage among heterogeneous workers. For example, Münster (2007) shows that more able workers may be deterred from participating in a tournament if contestants can sabotage each other. The advantage of higher talent is then completely erased since more able workers are sabotaged more heavily than less able ones, thereby equalizing the winning probabilities of the heterogeneous workers. However, if the winner prize of
the tournament is a bonus contract that entails higher rents for more able workers, the problem of adverse participation may be mitigated.

6 Appendix

6.1 Proof of Proposition 2

We solve problem (20)-(22) in two steps: First, we derive the firm’s minimum cost for inducing given effort levels \((\hat{e}, e)\). We then use the optimal cost function to solve the profit maximization problem and determine the optimal effort levels \((\hat{e}^c, e^c)\). The cost minimization problem for implementing \((\hat{e}, e)\) reads as

\[
\min_{\omega_L, \omega_H, b_L, b_H} 2\omega_L + (\omega_H - \omega_L) + b_L + p(e)(b_H - b_L)
\]

subject to (1), (2), (18), (19), \(\omega_L, \omega_H, b_L, b_H \geq 0\).

From (1), \(b_H - b_L = \frac{c'(e)}{p'(e)}\). Together with the incentive constraint (18), we obtain

\[
\omega_H - \omega_L = \frac{c'(\hat{e})}{p'(\hat{e})} - b_L - p(e)\frac{c'(e)}{p'(e)} + c(e) = \Delta w(\hat{e}) - b_L - r(e),
\]

where \(\Delta w(\hat{e})\) is given by (10) and \(r(e)\) by (6).\(^{33}\) Using (43), the first-level participation constraint (19) boils down to

\[
\omega_L + \frac{1}{2} \Delta w(\hat{e}) - \cdot(\hat{e}) \geq 0.
\]

Furthermore, the second-level participation constraint (2) becomes

\[
b_L + p(e)\frac{c'(e)}{p'(e)} - c(e) = b_L + r(e) \geq 0.
\]

\(^{33}\)Recall that \(\Delta w(\hat{e})\) is the prize spread necessary to induce \(\hat{e}\) under separate contracts. However, note that \(\Delta w(\hat{e})\) will usually be different from \(w^e_H - w^e_L\).
Thus, substituting for the tournament prize spread $w_H - w_L$ and the bonus spread $b_H - b_L$, the cost minimization problem can be simplified to

$$\min_{w_L, b_L} 2w_L + \Delta w(\hat{e}) + c(e) \quad \text{subject to (44), (45), and}$$

$$\Delta w(\hat{e}) - b_L - r(e) + w_L, \quad w_L, \quad b_L \geq 0. \quad (46)$$

By Lemma 1, we obtain $b_L^* = 0$ for the optimal low bonus: This satisfies the participation constraint for the second hierarchy level (45) and is also best for ensuring that $w_H = \Delta w(\hat{e}) - b_L - r(e) + w_L \geq 0$. Hence, we can skip constraint (45). By (12), $\frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0$, and thus (44) can also be skipped. The cost-minimizing $w_L$ is therefore given by

$$w_L = \max \{0, r(e) - \Delta w(\hat{e})\}.$$

We now have to distinguish two cases. The first case is $w_H - w_L = \Delta w(\hat{e}) - r(e) \geq 0$. Then, $w_L = 0$ and $w_H = \Delta w(\hat{e}) - r(e)$. In the second case, $w_H - w_L = \Delta w(\hat{e}) - r(e) < 0$. Hence, $w_L = r(e) - \Delta w(\hat{e})$ and $w_H = 0$. The firm’s expected labor costs are $\Delta w(\hat{e}) + c(e)$ in the first scenario and $2r(e) - \Delta w(\hat{e}) + c(e)$ in the second scenario.

We can now turn to the second step of the solution procedure, i.e., the firm’s profit maximization problem. The optimal effort pair $(\hat{e}^c, e^c)$ solves

$$\max_{e^e} \begin{cases} 
2\hat{e}(\hat{e}) + v(e) - \Delta w(\hat{e}) - c(e) & \text{if } \Delta w(\hat{e}) - r(e) \geq 0 \\
2\hat{e}(\hat{e}) + v(e) - [2r(e) - \Delta w(\hat{e}) + c(e)] & \text{otherwise.}
\end{cases}$$

We can see that in case 2 (i.e., the second line of the maximization problem) the objective function is monotonically increasing in $\hat{e}$. Hence, for each $e$, the firm chooses the maximum possible $\hat{e}$, which makes the given restriction just binding, i.e., $\Delta w(\hat{e}) = r(e)$. This implies that case 2 becomes a special case of case 1. Thus, the firm never wants to induce effort levels $(\hat{e}, e)$ such that $\Delta w(\hat{e}) < r(e)$. Doing so would imply that $0 = w_H^c < w_L^c$.

Intuitively, this means that, by implementing an adverse relative performance scheme, the firm pays for reducing first-level incentives that stem from the second-level rent $r(e)$. Such a contract cannot be optimal. The firm would be better off by setting $0 = w_H^c = w_L^c$, thereby increasing first-level effort and reducing workers’ first-period rents. Hence, we

\[34\text{Note that the optimal high bonus, } b_H = \frac{\hat{c}(\hat{e})}{\hat{r}(\hat{e})} + b_L, \text{ is non-negative due to } b_L \geq 0.\]
are always in the first case where \( w_L = 0 \) and the results of the proposition follow.

6.2 Proof of Proposition 3

(i) Let \( \pi^s \) and \( \pi^c \) denote the firm’s respective profits under the two contracts, i.e.,

\[
\pi^s := 2\hat{v}(\hat{e}^s) - \Delta w(\hat{e}^s) + v(e^s) - r(e^s) - c(e^s),
\]

\[
\pi^c := 2\hat{v}(\hat{e}^c) - \Delta w(\hat{e}^c) + v(e^c) - c(e^c).
\]

Under a combined contract, the firm can induce the same level-2 effort as under the optimal separate contracts by offering the bonuses \( b_L^*, b_H^* \). If the corresponding rent \( r(e^s) > 0 \) does not exceed \( \Delta w(\hat{e}^s) \), setting \( w_H = \Delta w(\hat{e}^s) - r(e^s) \) implements \( \hat{e}^s \) on level 1. Thus, if \( r(e^s) \leq w_H^* \), there is a combined contract that replicates the effort choices under the optimal separate contracts at strictly lower costs. If \( r(e^s) > \Delta w(\hat{e}^s) \), the combined contract that induces \( e^s \) on level 2 entails \( \hat{e} > \hat{e}^s \) on level 1. Thus, profit under the combined contract \((0, 0, b_L^*, b_H^*)\) is strictly larger than \( \pi^s \). Hence, under the optimal combined contract, we must also have \( \pi^c > \pi^s \).

(ii) \( \hat{e}^c = \hat{e}^s \) immediately follows from examining the objective functions (13) and (23). \( e^c > e^s \) follows from \( r'(e) > 0 \), which we have proven in Lemma 1, and \( r''(e) > 0 \), which follows from our regularity assumptions and is straightforward to check.

(iii) Let \( \gamma \) denote the Lagrange multiplier of (24). The firm’s problem (23) and (24) yields the first-order conditions

\[
2\hat{v}'(\hat{e}^c) - \Delta w'(\hat{e}^c) + \gamma \Delta w'(\hat{e}^c) = 0,
\]

\[
v'(e^c) - c'(e^c) - \gamma \Delta r'(e^c) = 0.
\]

Since (24) is binding, \( \gamma > 0 \). Recall that \( \Delta w'(\hat{e}) > 0 \) and \( r'(e) > 0 \). We thus obtain

\[
2\hat{v}'(\hat{e}^c) - \Delta w'(\hat{e}^c) < 0 \text{ and hence } \hat{e}^c > \hat{e}^s.
\]

Furthermore, \( v'(e^c) - c'(e^c) > 0 \) and thus \( e^c < e^{FB} \). It remains to show that \( e^c > e^s \). Due to the binding restriction (24), we can consider \( e \) as an implicitly defined function of \( \hat{e} \), i.e., \( e(\hat{e}) \) with \( \frac{\partial e}{\partial \hat{e}} = \frac{\Delta w'(\hat{e})}{r'(e)} > 0 \). The firm’s objective function (23) can then be rewritten as

\[
2\hat{v}(\hat{e}) + v(e(\hat{e})) - \Delta w(\hat{e}) - c(e(\hat{e})).
\]
The respective first-order condition is

\[ 2 \delta'(\hat{e}) - \Delta w'(\hat{e}) + [v'(e(\hat{e})) - c'(e(\hat{e}))] \frac{\partial e}{\partial \hat{e}} = 0. \]

Inserting \( \partial e/\partial \hat{e} \) yields

\[ 2 \delta'(\hat{e}) + \frac{v'(e(\hat{e})) - c'(e(\hat{e})) - r'(e(\hat{e}))}{r'(e(\hat{e}))} \Delta w'(\hat{e}) = 0. \]

The optimal effort, \( e^c \), must therefore satisfy \( v'(e^c) - c'(e^c) - r'(e^c) < 0 \). Under separate contracts, we have \( v'(e^s) - c'(e^s) - r'(e^s) = 0 \). Thus, since \( v'(e) - c'(e) - r'(e) \) is strictly concave, it follows that \( e^c > e^s \).

References


7 Not for Publication

7.1 Modified Limited-Liability Constraints

In this section, we reconsider the problem (20)–(21) of a combined contract in the basic model where the limited-liability constraints (22) are replaced by \( w_L \geq 0, w_H + b_L \geq 0 \) and \( w_H + b_H \geq 0 \). We will show that these modifications do not change our results. Again, we start with minimizing the firm’s cost for inducing a given pair of effort levels \((\hat{e}, e)\):

\[
\min_{w_L, w_H, b_L, b_H} \quad 2w_L + (w_H - w_L) + b_L + p(e)(b_H - b_L)
\]

subject to (1), (2), (18), (19), \( w_L, b_L + w_H, b_H + w_H \geq 0 \).

From the incentive constraint (1) we obtain \( b_H - b_L = c'(e) / p'(e) \), which, in combination with the incentive constraint (18), yields

\[
w_H - w_L = \frac{c'(e)}{p_1(\hat{e}, \hat{e})} - b_L - p(e) \frac{c'(e)}{p'(e)} + c(e) = \Delta w(\hat{e}) - b_L - r(e),
\]

where \( \Delta w(\hat{e}) \) is given by (10) and \( r(e) \) by (6). Using this expression, (19) can be rewritten as \( w_L + \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \). Furthermore, (2) becomes \( b_L + r(e) \geq 0 \). In addition, we have

\[
b_L + w_H = b_L + \Delta w(\hat{e}) - b_L - r(e) + w_L = \Delta w(\hat{e}) - r(e) + w_L
\]

\[
b_H + w_H = b_H + \Delta w(\hat{e}) - b_L - r(e) + w_L = \frac{c'(e)}{p'(e)} + \Delta w(\hat{e}) - r(e) + w_L.
\]

Substituting for \( w_H - w_L \) and \( b_H - b_L \) in the objective function, the cost minimization problem can be summarized as follows:

\[
\min_{w_L, b_L} \quad 2w_L + \Delta w(\hat{e}) + c(e)
\]

subject to \( w_L + \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \)

\[
b_L + r(e) \geq 0
\]

\[
w_L, \Delta w(\hat{e}) - r(e) + w_L, \frac{c'(e)}{p'(e)} + \Delta w(\hat{e}) - r(e) + w_L \geq 0.
\]
The last non-negativity constraint is less strong than the second one and can thus be skipped. Since \( b_L \) does not appear in the objective function but only in the second-level participation constraint, we can set \( b_L = 0 \) (or any other \( b_L \geq -r(e) \)). Moreover, since \( \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \), the first constraint is satisfied whenever \( w_L \geq 0 \) and can, therefore, also be skipped. Altogether, we obtain the same cost minimization problem as in Subsection 6.1 (Proof of Proposition 2), where we assumed \( w_L, w_H, b_L, b_H \geq 0 \). The intuition is as follows. If \( w_L, b_L + w_H, b_H + w_H \geq 0 \), a negative bonus \( b_L \) can be used to decrease rents on the second tier. However, all these rents serve as indirect incentives for the first tier. Hence, these rents do not constitute costs for the firm so that it cannot benefit from lowering them. Finally, if the rents are so high that they provide too strong incentives for the first tier, \( w_H = 0 \) anyway.

7.2 Combined Contract under Unobserved Heterogeneity

Step 1: Minimizing costs

Since \( b_H \geq 0 \) is ensured by the incentive constraint for hierarchy level 2 in combination with \( b_L \geq 0 \) the problem of minimizing implementation costs reduces to

\[
\begin{align*}
\min_{w_L, w_H, b_L} & \quad \Delta \tilde{w}(\hat{e}) + 2w_L + c(e) \\
\text{subject to} & \quad b_L + r(e) \geq 0 \\
& \quad w_L + \frac{1}{2} \Delta \tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \\
& \quad w_H - w_L + b_L + r(e) = \Delta \tilde{w}(\hat{e}) \\
& \quad w_H, w_L, b_L \geq 0.
\end{align*}
\]

Replacing \( w_H \) yields:

\[
\begin{align*}
\min_{w_L, b_L} & \quad \Delta \tilde{w}(\hat{e}) + 2w_L + c(e) \\
\text{s.t.} & \quad b_L + r(e) \geq 0 \\
& \quad w_L + \frac{1}{2} \Delta \tilde{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \\
& \quad \Delta \tilde{w}(\hat{e}) - b_L - r(e) + w_L, w_L, b_L \geq 0.
\end{align*}
\]

From Lemma 1 we know that \( r(e) \geq 0 \) so that \( b_L = 0 \) and the minimization problem
further reduces to

\[
\begin{align*}
\min_{w_L} & \Delta \hat{w}(\hat{e}) + 2w_L + c(e) \\
\text{s.t.} & \quad w_L + \frac{1}{2} \Delta \hat{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \\
& \quad \Delta \hat{w}(\hat{e}) - r(e) + w_L, \quad w_L \geq 0.
\end{align*}
\]

Hence, \( w_L = \max \{ 0, \hat{c}(\hat{e}) - \frac{1}{2} \Delta \hat{w}(\hat{e}), \ r(e) - \Delta \hat{w}(\hat{e}) \} \). We know that \( \frac{1}{2} \Delta \hat{w}(\hat{e}) - \hat{c}(\hat{e}) \geq 0 \); otherwise, \( \hat{e} \) would not be an equilibrium strategy. Thus, \( w_L = \max \{ 0, \ r(e) - \Delta \hat{w}(\hat{e}) \} \).

We have to distinguish two cases. First, \( w_H - w_L = \Delta \hat{w}(\hat{e}) - r(e) \geq 0 \). Then, \( w_L = 0 \) and \( w_H = \Delta \hat{w}(\hat{e}) - r(e) \). Second, \( w_H - w_L = \Delta \hat{w}(\hat{e}) - r(e) < 0 \). Then, \( w_L = r(e) - \Delta \hat{w}(\hat{e}) \) and \( w_H = 0 \). In the first case, the firm’s expected labor costs are \( \Delta \hat{w}(\hat{e}) + 2w_L + c(e) = \Delta \hat{w}(\hat{e}) + c(e) \) and in the second they amount to \( \Delta \hat{w}(\hat{e}) + 2w_L + c(e) = 2r(e) - \Delta \hat{w}(\hat{e}) + c(e) \).

**Step 2: Maximizing expected profits**

Therefore, the optimal effort pair \((\hat{e}^*, e^*)\) solves

\[
\max_{\hat{e}, e} \left\{ \begin{array}{ll}
2\hat{v}(\hat{e}) + E [t|\hat{s}] v(e) - \Delta \hat{w}(\hat{e}) - c(e) & \text{if } \Delta \hat{w}(\hat{e}) - r(e) \geq 0 \\
2\hat{v}(\hat{e}) + E [t|\hat{s}] v(e) - 2r(e) + \Delta \hat{w}(\hat{e}) - c(e) & \text{otherwise.}
\end{array} \right.
\]

In analogy to the basic model, again the firm’s objective function in the second line is monotonically increasing in \( \hat{e} \) (recall that \( \partial E [t|\hat{s}] / \partial \hat{e} > 0 \) according to (35)). Hence, for each \( e \) the firm chooses the maximum possible \( \hat{e} \) that makes the given restriction just bind so that the second line becomes a special case of the problem in line 1. The firm chooses \( w^*_L = 0 \) and implements the effort pair \((\hat{e}^*_h, e^*_h)\) with

\[
(\hat{e}^*_h, e^*_h) \in \arg \max_{\hat{e}, e} \{ 2\hat{v}(\hat{e}) + E [t|\hat{s}] v(e) - \Delta \hat{w}(\hat{e}) - c(e) \}
\]

subject to \( \Delta \hat{w}(\hat{e}) - r(e) \geq 0 \).

### 7.3 Simplification of the Firm’s Problem under Contract \( C_1 \)

The solution procedure is analogous to the one in Proposition 2. First, we consider the firm’s problem of minimizing implementation costs for a given pair of effort levels \((\hat{e}, e)\).

\[
\begin{align*}
\min_{w_L, w_H, b_H, b_L} & \quad w_L + w_H + b_L + p(e)(b_H - b_L) \\
\text{s.t.} & \quad (1), (2), (22), (39), (40).
\end{align*}
\]
By using (10), the incentive constraint (39) can be rewritten as
\[ w_H - w_L = \Delta w(\hat{e}) - \left( \alpha C_1 - \frac{1}{4} \right) [b_L + r(e)] \] (49)
so that the first-level participation constraint (40) boils down to
\[ w_L + \frac{1}{2} \Delta w(\hat{e}) + \frac{1}{4} [b_L + r(e)] - \hat{c}(\hat{e}) \geq 0. \] (50)
Recall that the second-level participation constraint can be written as
\[ b_L + r(e) \geq 0. \] (51)
Using (49) together with \( \alpha C_1 = \frac{3}{4} \) and \( p(e)(b_H - b_L) = r(e) + c(e) \), the cost minimization problem reads as\(^{35}\)
\[
\min_{w_L,b_L} 2w_L + \Delta w(\hat{e}) + c(e) + \frac{1}{2} [b_L + r(e)] \quad \text{s.t.} \quad (50), (51) \text{ and } \Delta w(\hat{e}) - \frac{1}{2} r(e) + w_L, \ w_L, \ b_L \geq 0.
\]
Again, \( b_L = 0 \) is optimal. Hence, we get rid of constraint (51) and obtain
\[
\min_{w_L} 2w_L + \Delta w(\hat{e}) + c(e) + \frac{1}{2} r(e) \quad \text{s.t.} \quad (50) \text{ and } \Delta w(\hat{e}) - \frac{1}{2} r(e) + w_L, \ w_L \geq 0.
\]
The cost-minimizing \( w_L \) is thus given by
\[ w_L = \max \left\{ 0, \frac{1}{2} \Delta w(\hat{e}) - \frac{1}{4} r(e), \frac{1}{2} r(e) - \Delta w(\hat{e}) \right\} . \]
From (12), we know that \( \frac{1}{2} \Delta w(\hat{e}) - \hat{c}(\hat{e}) \leq 0 \). Hence, \( w_L = \max \{ 0, \frac{1}{2} r(e) - \Delta w(\hat{e}) \} \). We now have to distinguish two cases. The first case is \( w_H - w_L = \Delta w(\hat{e}) - \frac{1}{2} r(e) \geq 0 \). Then, \( w_L = 0 \) and \( w_H = \Delta w(\hat{e}) - \frac{1}{2} r(e) \). In the second case, \( w_H - w_L = \Delta w(\hat{e}) - \frac{1}{2} r(e) < 0 \). Hence, \( w_L = \frac{1}{2} r(e) - \Delta w(\hat{e}) \) and \( w_H = 0 \). In the first case, the firm’s expected labor costs are \( 2w_L + \Delta w(\hat{e}) + c(e) + \frac{1}{2} r(e) = \Delta w(\hat{e}) + c(e) + \frac{1}{2} r(e) \), and in the second scenario the firm’s costs amount to \( 2w_L + \Delta w(\hat{e}) + c(e) + \frac{1}{2} r(e) = \frac{3}{2} r(e) - \Delta w(\hat{e}) + c(e) \). We can now turn to the final step of the solution procedure, the solution of the firm’s profit\(^{35}\)
Note that \( b_H \geq 0 \) is guaranteed.
maximization problem. The optimal effort combination solves

\[
\max_{\hat{e}, \bar{e}} \left\{ \begin{array}{ll}
2\hat{v}(\hat{e}) + v(e) - \Delta w(\hat{e}) - c(e) - \frac{1}{2}r(e) + \Gamma_A & \text{if } \Delta w(\hat{e}) - \frac{1}{2}r(e) \geq 0 \\
2\hat{v}(\hat{e}) + v(e) - [\frac{3}{2}r(e) - \Delta w(\hat{e}) + c(e)] + \Gamma_A & \text{otherwise.}
\end{array} \right.
\]

With the same argumentation as in the proof of Proposition 2, it follows that the firm will never implement effort levels \((\hat{e}, \bar{e})\) such that \(\Delta w(\hat{e}) - \frac{1}{2}r(e) < 0\). Thus, \(w_L = 0\) and the firm’s optimization problem is given by (41).