A Note on Revenue Sharing in Sports Leagues*

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Abstract

We discuss the optimal sharing of broadcasting revenues given a central marketing system. The league’s sports association wants to maximize the joint profits of the sports clubs’ owners. The results show that a winner-take-all scheme should be used, if the teams are homogeneous. However, if teams are sufficiently heterogeneous, equal sharing will be optimal.

Keywords: broadcasting revenues, equal sharing, sports leagues, revenue sharing, winner-take-all scheme.

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1 Introduction

In the theoretic literature on sport contests, clubs are typically assumed to buy playing talent in order to win matches. These expenditures for talent lead to a certain winning probability according to the well-known logit-form contest success function. Implicitly, the models abstract from incentive problems among the teams’ players, i.e. given a certain talent, each player is automatically assumed to fully use his talent.

This paper departs from the literature by assuming that players need to be given incentives to play hard. It focuses on endogenous incentives for players within a central marketing system in which the sport league (i.e. the respective sports association) chooses a sharing rule for distributing the league’s broadcasting revenues among the league’s sport clubs. We assume that a player’s income is positively related to the individual revenues a club receives from the league.\(^1\) In particular, we assume that a club’s individual revenues are divided between the club owner and the team according to their bargaining powers. Within this framework, we will analyze which kind of sharing rule is optimal for the league. The results of the model show that the league will choose a winner-take-all scheme (i.e. the winner of the

\(^1\)Hall et al. (2002) empirically test the causality between team performance and payment. At least for the free agents in Major League Baseball pay for performance seems to be an appropriate assumption. However, in the model described in the next section we do not assume strict pay for performance. We only assume that a player participates in the revenues of his club and that these revenues depend on the club’s success in the league.
league gets all the broadcasting revenues), if the teams are homogeneous. However, if teams are sufficiently heterogeneous, the league will optimally choose equal sharing so that each club receives the same fraction of the revenues independent of the club’s performance.

There is some work related to this paper. Szymanski (2003a) as well as Kesenne (2005) also consider the case of sharing broadcasting revenues. However, in both papers clubs invest in playing talents which then determine a team’s winning probability via the logit-form model; there is no modelling of players’ incentives. Szymanski (2003a, p. 22) tries to discuss revenue sharing on the basis of performance which would create incentives for clubs. Unfortunately, even within the standard logit-form setting an analytical solution cannot be derived. Palomino and Sakovics (2004) and Falconieri et al. (2004) also discuss the sharing of broadcasting revenues by a league which wants to maximize the clubs’ joint profits, but both papers abstract from the problem of incentives for individual players. Palomino and Sakovics show that competitive bidding for talent leads to an equal-sharing scheme in an isolated-league context, but in the case of inter-league competition for talent leagues may choose winner-take-all schemes. In the model of Falconieri et al., the league never chooses a winner-take-all scheme. The problem of individual incentives for players is addressed by Palomino and Rigotti (2000), Szymanski (2003b, pp. 1140–1146) and Gürtler (2006). In the Palomino-Rigotti paper, teams or clubs are modeled as collective players, i.e. the authors do not differentiate between owners and players of a sport club so that there is no distributional conflict. If the sport league wants to
maximize the teams’ total profits, then it unambiguously has to choose equal sharing. Szymanski and Gürtler analyze incentives for players, but they do not consider the connection of a player’s income, his club’s revenues and the league’s optimal sharing rule.

The note is organized as follows. In the next section, the model is introduced. The formal results on optimal sharing are given in Section 3. The last section concludes.

2 The Model

We consider a professional sports league which consists of two teams A and B. All individuals in the model are assumed to be risk neutral. In order to abstract from special team effects (e.g. free riding) we assume that each team is held by one owner and has exactly one player.\(^2\) When competing, team \(i\) \((i = A, B)\) can either be successful and generates a high outcome \(\pi_i = 1\), or be unsuccessful and realizes a low outcome \(\pi_i = 0\). Let \(\alpha_i \in [0, 1]\) denote the activities or efforts which are chosen by player \(i\) to win the match against the other team \(j\) \((i, j = A, B; i \neq j)\). These activities may be short term (e.g. direct efforts during the match) but also long term (e.g. training intensity, eating discipline). We assume that player \(i\) realizes \(\pi_i = 1\) with

\(^2\)This simplifying assumption is typically used in the theoretic models on professional team sport. An alternative interpretation would be that a team collectively decides on the activity level so that players should be interpreted as collective players. Two papers explicitly model teams that consist of at least two players: Gürtler and Kräkel (2003), Gürtler (2006). Of course, in both papers the well-known free-rider effect applies.
probability $a_i$, and $\pi_i = 0$ with probability $1 - a_i$. In other words, each activity level $a_i \in (0, 1)$ leads to a different lottery $(1, a_i; 0, 1 - a_i)$, with $(1, a''; 0, 1 - a'')$ dominating $(1, a'; 0, 1 - a')$ within the meaning of first-order stochastic dominance if $a'' > a'$. It is assumed that spending activities $a_i$ leads to a disutility or costs (e.g. opportunity costs for time consuming training) for player $i$ which are described in monetary terms by the convex function $c(a_i) = \frac{a_i^2}{2\pi_i}$. Here, $t_i$ denotes player $i$’s talent; the higher a player’s talent the lower will be his costs $c(a_i)$.

As pointed out in the introduction, the model focuses on the sharing of a league’s broadcasting rights between both teams. We assume that broadcasting revenues can be described by a function of both teams’ performances:

$$B(\pi_i, \pi_j) = \pi_i + \pi_j - x(\pi_i - \pi_j)^2.$$  \hspace{1cm} (1)

Eq. (1) shows that broadcasting revenues increase in the league’s aggregate performance and in competitive balance.\(^5\) The higher both teams’ outcomes (e.g. the larger the number of overall goals) and the closer the match (i.e.\(^3\) this cost function can be either interpreted as disutility of effort or as opportunity costs of time. As usual in principal-agent models, the cost function is assumed to be convex. The quadratic form is used for simplification. Since it is easier for more talented players to choose a high activity level, the function monotonically decreases in talent. Division by 2 is used as a normalization so that this number disappears when calculating the first-order condition.

\(^4\)For a similar objective function see Falconieri et al. (2004).

\(^5\)Typically, competitive balance is measured via the teams’ winning probabilities. For reasons of analytical tractability, here we use a slightly different approach. Note that considering outcomes instead of probabilities does not make a great difference.
the less the difference between teams’ performances), the higher will be the league’s revenues from selling broadcasting rights. The parameter $x$ measures the impact of competitive balance. We assume $x \in (0, 1)$ so that revenues cannot become negative.

In the following, we consider the distribution of broadcasting revenues $B(\pi_i, \pi_j)$ within the league. Let $\alpha \in \left[\frac{1}{2}, 1\right]$ denote the share of the league winner in $B(\pi_i, \pi_j)$, whereas $1 - \alpha$ describes the share of the loser in $B(\pi_i, \pi_j)$.\(^6\) If both teams perform identically (i.e. $\pi_i = \pi_j$), each team will receive $B(\pi_i, \pi_j)/2$. Therefore, expected revenues of club $i$ are given by $\pi_i + \pi_j - x(\pi_i - \pi_j)^2$

$$R_i(\alpha) = \alpha B(1, 0) a_i (1 - a_j) + (1 - \alpha) B(0, 1) a_j (1 - a_i)$$
$$+ \frac{B(1, 1)}{2} a_i a_j + \frac{B(0, 0)}{2} (1 - a_i)(1 - a_j)$$
$$= \alpha (1 - x) a_i (1 - a_j) + (1 - \alpha) (1 - x) a_j (1 - a_i) + a_i a_j$$
$$= (1 - x) (a_j + \alpha(a_i - a_j)) + xa_i a_j. \quad (2)$$

We assume that $\alpha$ is chosen by the league in order to maximize the team owners’ profits.\(^7\) For example, if the league chooses $\alpha = 1$ the teams will face a winner-take-all distribution scheme which is highly competitive. However, by choosing equal sharing $\alpha = \frac{1}{2}$, the broadcasting revenues of a club are independent of the club’s performance within the league.

It is assumed that a player’s income is significantly influenced by the success of his club and his bargaining power. In particular, we assume that


\(^7\)The same assumption can be found in Palomino and Sakovics (2004) and Falconieri et al. (2004). Owners’ profits are defined in the next paragraph.
player $i$ receives the fraction $\lambda_i \in (0, 1)$ of club $i$’s broadcasting revenues $R_i(\alpha)$, whereas owner $i$ obtains the fraction $1-\lambda_i$. Hence, player $i$’s objective function is given by

$$EU_i(a_i) = \lambda_i R_i(\alpha) - \frac{a_i^2}{2t_i}$$

$$= \lambda_i ((1-x)(a_j + \alpha (a_i - a_j)) + xa_i a_j) - \frac{a_i^2}{2t_i}$$

(3)

and the objective function of owner $i$ by

$$\Pi_i = (1-\lambda_i) R_i(\alpha)$$

$$= (1-\lambda_i) ((1-x)(a_j + \alpha (a_i - a_j)) + xa_i a_j).$$

(4)

The timing of the game is as follows: At the first stage, the league determines the revenue-sharing scheme by choosing $\alpha$ in order to maximize the club owners’ joint profits $\Pi_A + \Pi_B$. At the second stage, the teams or players compete by choosing their respective activities $a_i$.

3 Results

We start by considering the second stage. Here, each player $i$ chooses $a_i$ in order to maximize his expected net income according to Eq. (3). The first-order condition yields the following reaction function:

$$a_i = \lambda_i t_i ((1-x)\alpha + xa_j).$$

Note that the second-order condition is always satisfied since player $i$’s objective function is strictly concave.
Together with j’s reaction function \( a_j = \lambda_j t_j \left( (1 - x) \alpha + x a_i \right) \) we obtain the Nash equilibrium\(^9\)

\[
a^*_i = \frac{(1 - x) \lambda_i t_i \alpha (1 + x \lambda_j t_j)}{1 - x^2 \lambda_i \lambda_j t_i t_j} \quad \text{and} \quad a^*_j = \frac{(1 - x) \lambda_j t_j \alpha (1 + x \lambda_i t_i)}{1 - x^2 \lambda_i \lambda_j t_i t_j}. \tag{5}
\]

Obviously, each activity level monotonically increases in the sharing parameter \( \alpha \) in equilibrium. In other words, the league can directly influence incentives by choosing an appropriate value of \( \alpha \). Furthermore, comparative-statics immediately show that \( \frac{\partial a^*_i}{\partial \lambda_i t_i} > 0 \) and \( \frac{\partial a^*_j}{\partial \lambda_j t_j} > 0 \). The intuition is the following one: Broadcasting revenues and, therefore, players’ incomes are determined by overall performance so that we have a kind of team production in which both players’ activities are complements. In this situation, increased bargaining power and increased talent do not only enhance one’s own incentives but also the incentives of the other player.

At the first stage, the league maximizes owners’ joint profits \( \Pi_i + \Pi_j \). According to (4), we have

\[
\Pi_i + \Pi_j = (1 - \lambda_i) \left( (1 - x) (a_j + \alpha (a_i - a_j)) + xa_i a_j \right) \\
+ (1 - \lambda_j) \left( (1 - x) (a_i + \alpha (a_j - a_i)) + xa_i a_j \right) \\
= (1 - x) (a_i (1 - \lambda_j) + a_j (1 - \lambda_i) - (\lambda_i - \lambda_j) \alpha (a_i - a_j)) \\
+ (2 - \lambda_i - \lambda_j) xa_i a_j. \tag{6}
\]

By inspection of (6) we obtain the first result:

\(^9\)We have to assume that parameter values guarantee \( a^*_i, a^*_j \in [0, 1] \). Hence, we assume \( x^2 \lambda_i \lambda_j t_i t_j \) < 1 and \( (1 - x) \lambda_j t_j \alpha (1 + x \lambda_i t_i) \) < 1 - \( x^2 \lambda_i \lambda_j t_i t_j \) and \( (1 - x) \lambda_j t_j \alpha (1 + x \lambda_i t_i) < 1 - x^2 \lambda_i \lambda_j t_i t_j \).
Proposition 1 If players are homogeneous in bargaining power and talent (i.e. $t_i\lambda_i = t_j\lambda_j$), then the league will choose a winner-take-all scheme $\alpha^* = 1$.

Proof. If $t_i\lambda_i = t_j\lambda_j$, according to Eq. (5) both players will choose the same activity level in equilibrium so that $- (\lambda_i - \lambda_j) \alpha (a_i - a_j)$ cancels out in the league’s objective function. The remaining part $(1 - x) (a_i (1 - \lambda_j) + a_j (1 - \lambda_i)) + (2 - \lambda_i - \lambda_j) xa_i a_j$ strictly increases in both players’ activity levels. The league then chooses the upper bound for $\alpha$ since activities are monotonically increasing in $\alpha$. ■

Given homogeneity among the players, the sharing parameter $\alpha$ only has an impact on players’ incentives. Since broadcasting revenues depend on joint performance, the club owners and, hence, the league prefer to induce maximum incentives to the players.\(^{10}\)

It seems somewhat natural that the owners and the league are interested in high incentives for the players. Hence, the question remains whether there are any situations, in which the league does not choose a winner-take-all scheme in the given setting. The following proposition gives an answer to this question:

Proposition 2 If players are sufficiently heterogeneous in bargaining power and talent, then the league will choose equal sharing $\alpha^* = \frac{1}{2}$.

Proof. Let player $i$ be a very strong player (i.e. $\lambda_i$ and $t_i$ are very large) and player $j$ be very weak in the sense of $\lambda_j \to 0$ and $t_j \to 0$. According to

\[^{10}\text{Note that homogeneous players always receive a positive expected utility: } EU_i (a_i^*) = \left( \frac{\lambda_i (1-x)((2-\alpha)-2x\lambda_i t_i(1-\alpha))}{2(1-x\lambda_i t_i)} \right) a_i^* > 0 \text{ since } x\lambda_i t_i < 1 \text{ so that equilibrium efforts are positive.}\]
(5), we then have \( a_j \to 0 \) so that the league’s objective function reduces to

\[
\Pi_i + \Pi_j = (1 - x)(1 - \lambda_i \alpha) a_i
\]  

(7)

and player \( i \)'s equilibrium activity to

\[
a_i^* = (1 - x) \lambda_i t_i \alpha.
\]  

(8)

Inserting \( a_i^* \) into (7) yields the function

\[
\Pi(\alpha) = (1 - x)^2 (1 - \lambda_i \alpha) \lambda_i t_i \alpha
\]

which describes a parabola open to the bottom with a global maximum at

\[
\alpha^* = \frac{1}{2 \lambda_i}.
\]

Since player \( i \) is very strong in the sense of \( \lambda_i \to 1 \) we obtain \( \alpha^* = \frac{1}{2} \).

An intuition can be given by inspection of the league’s objective function

\[
\Pi_i + \Pi_j = (1 - \lambda_i) ((1 - x)(a_j + \alpha (a_i - a_j)) + x a_i a_j) + (1 - \lambda_j) ((1 - x)(a_i + \alpha (a_j - a_i)) + x a_i a_j).
\]

Very strong heterogeneity is detrimental for the league, because the positive externalities due to team production – i.e. \( a_j a_i \) – decrease in players’ heterogeneity. If player \( j \) becomes very weak in the sense of \( a_j^* \to 0 \), externalities even completely diminish so that

\[
\Pi_i + \Pi_j = (1 - \lambda_i) (1 - x) \alpha a_i + (1 - \lambda_j) ((1 - x)(1 - \alpha) a_i).
\]

Note that the first term describes expected profits of club owner \( i \) whereas the second term stands for club owner \( j \)'s expected profits. When player \( i \) is very
strong (i.e. \( \lambda_i \to 1 \)) and player \( j \) very weak (i.e. \( \lambda_j \to 0 \)), the league’s profits become identical with club owner \( j \)’s profits: \( (1 - x)(1 - \alpha) a_i \). Hence, we have a *distribution effect* in the sense that the sharing parameter \( \alpha \) should be fixed at its lower bound \( \frac{1}{2} \) in order to give club \( j \) as much as possible from the broadcasting revenues. However, there is also an *incentive effect* of \( \alpha \) since players’ activities monotonically increase in the sharing parameter. This trade-off becomes clear by comparison of Eqs. (7) and (8). Since the distribution effect dominates the incentive effect, the league optimally chooses equal sharing.

4 Conclusion

In this paper, we have analyzed the connection of revenue sharing and players’ incentives within professional team sports. For this purpose, we assumed that a player’s income is positively related to the broadcasting revenues which his sport club receives from the league via central marketing. Alternatively, we could assume that a player directly receives bonuses that depend on team success or failure. However, note that the model used in this paper can also be interpreted in this direction, because it would be strategically equivalent whether a player receives certain lump-sum payments dependent on team performance or he receives a certain fraction of the club’s revenues which depend on success/failure.\(^{11}\)

The results of the model show that the optimal sharing arrangement

crucially depends on the degree of homogeneity among the sports teams. In this paper, we have assumed that the spectators are especially interested in aggregate performance of the teams (e.g. in the number of goals) and in competitive balance (i.e. in a close match). These interests explain the positive relation to the amount of broadcasting revenues. However, there may be other possible specifications of the league’s objective function. For example, spectators may be interested in maximum performances of single players, but this aspect should be more important in connection with individual sports (e.g. beating a world record) and not with team sport.

Finally, the two-team model could be extended to competition between more than two teams. In this case, competition for becoming the league winner would be stronger compared to the two-team case. Of course, the strength of competition influences players’ incentives but the qualitative results of this paper concerning the impact of homogeneity and heterogeneity of the players should remain the same.
References


