Tournaments versus Piece Rates under Limited Liability

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Abstract
We discuss two incentive schemes that are frequently used in practice – tournaments and piece rates. The existing literature on the comparison of these two incentive schemes has focused on the case of unlimited liability. However, real workers’ wealth is typically restricted. Therefore, this paper compares both schemes under the assumption of limited liability. The results show that piece rates will dominate tournaments if idiosyncratic risk is sufficiently high despite the partial insurance effect of tournament compensation.

Key words: incentives, piece rates, rank-order tournaments.
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1 Introduction

Since the seminal paper by Lazear and Rosen (1981) there has been a wide discussion of tournaments versus piece rates as alternative incentive schemes. In a tournament, at least two workers compete against each other for given winner and loser prizes. Under a piece-rate scheme, a worker’s payment consists of a fixed payment and a certain percentage – the piece rate – of the worker’s realized output in monetary terms.

There exist many examples for either incentive scheme in practice. Tournaments can be observed in sports (e.g., Ehrenberg and Bognanno 1990), in broiler production (Knoeber and Thurman 1994) and also in firms when people compete for job promotion (e.g., Baker, Gibbs and Holmstrom 1994). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the employees. Hence, forced-ranking or forced-distribution systems, in which supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes. Boyle (2001) reports that about 25 per cent of the so-called Fortune 500 companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, General Electric).

An extreme form of combining relative performance evaluation and tournament incentives is given when layoffs are tied to the lowest grade. The most prominent advocate of such layoff tournaments is the former General Electric CEO Jack Welch whose incentive philosophy demands dismissal of the bottom 10 percent of employees each year. A similar system is used by Enron. Enron employees are regularly rated on a scale of 1 to 5 and employees belonging to grade 5 are typically fired within six months. Of course, there are also lots of examples for piece-rate schemes in practice; see – among
many others – Lazear (2000) on the introduction of piece rates at the Safe-
lite Glass Corporations, and Freeman and Kleiner (1998) on the decline of
piece-rate systems in the American shoe industry. For an earlier discussion
of piece rates see, for example, Lazear (1986), Gibbons (1987), and Fama

Lazear and Rosen (1981) have shown that both incentive schemes lead to
first-best efforts given homogeneous and risk neutral workers with unlimited
liability. However, tournaments can dominate piece rates, since tournaments
only require an ordinal performance measure. Concerning risk averse workers,
there is no clear ranking between the two incentive schemes. On the one hand,
tournaments provide a crude form of insurance, since each agent receives at
least the given loser prize and at most the given winner prize. On the other
hand, tournaments have the drawback that in symmetric equilibrium the
probability mass is distributed equally on the high winner and the low loser
prize.

Green and Stokey (1983) emphasize that tournaments will dominate piece
rates if filtering of common noise is of major interest. In tournaments, com-
mon noise cancels out because of the relative comparison of the workers’
performance. Piece rates use an absolute performance measure and, there-
fore, cannot serve as a risk filter in a static context.

Malcomson (1984) points to an important advantage of tournaments com-
pared to piece rates. Since winner and loser prizes are fixed in advance (i.e.,
the employer commits himself to certain labor costs before the tournament
starts), tournaments can create incentives even if the workers’ performance
measure is unverifiable. However, an employer always needs a verifiable per-
formance measure for utilizing piece rates as an incentive scheme.

Although workers’ liability is often limited in practice, the existing com-
parison of tournaments and piece rates has been restricted to the case of
unlimited liability. By this assumption, loser prizes in tournaments and fixed
payments in piece-rate schemes are allowed to be arbitrarily negative. Hence,
not surprisingly given risk neutral workers first-best efforts are implemented
under either incentive scheme. In tournaments, the optimal spread between
winner and loser prize can always be chosen in order to induce first-best in-
centives, whereas the – possibly negative – loser prize is used by the employer
to extract the whole surplus so that the workers’ participation constraint al-
ways binds. In piece-rate schemes, optimal incentives are created by offering
a piece rate of 100%. However, this selling-the-firm strategy which induces
first-best incentives (i.e. maximizes welfare) is only optimal for the employer
if he is allowed to choose a negative fixed payment for extracting all welfare
gains from the workers.

Of course, if negative payments such as negative loser prizes or negative
fixed payments are not feasible since workers are restricted in wealth – i.e.
they are protected by limited liability – these standard solutions to the in-
centive problem under risk neutral workers are not possible. But then the
employer does not want to implement first-best efforts any longer because
this would be too expensive for him. This paper addresses the open question
which incentive scheme, a tournament or piece rates, is preferable by the
employer in those situations with limited liability. The analysis of this paper
offers several interesting results. In particular, it can be shown that if idio-
syncratic risk is sufficiently high, piece rates will dominate tournaments. Of
course, under limited liability a high idiosyncratic risk is detrimental under
either incentive scheme since there is a significant probability that a worker
only realizes a very low output. However, the findings show that implement-
tion of first-best efforts will be less likely in tournaments than under a
piece-rate scheme, if such risk is large. Moreover, if in this situation only a second-best solution can be implemented and workers earn positive rents, efforts will be larger under the piece-rate than under the tournament scheme.

Note that the given risk in the model is assumed to be idiosyncratic. In practice, this means that the risk is mainly determined by a worker’s individual characteristics and by the special tasks which have been delegated to him. Another interpretation of the idiosyncratic error term is that there are talent or ability characteristics of the worker which are unknown to all players at that moment including the worker himself (see Lazear and Rosen (1981), p. 843). Given this interpretation of the error term, a tournament has perfect selection properties: workers are ex-ante homogeneous (i.e. there is symmetric uncertainty about talent) and choose identical efforts in a symmetric equilibrium; hence that worker will win whose unknown ability turns out to be highest. As the paper will show, from an incentive perspective tournaments with idiosyncratic noise and limited liability may be problematic compared to piece rates if the idiosyncratic risk is large. However, if there is also a common error term (e.g. the economic situation of the firm or the industry, future working conditions which will be chosen by the employer for his workforce, the general monitoring ability of the supervisor if the common error term denotes measurement errors) and the impact of this error term dominates the influence of the idiosyncratic risk, then the employer will prefer tournaments to piece rates even under limited liability. This point will be discussed more closely in Section 4.

The results of the paper offer a further implication for practice. It will be shown that the limited-liability problem should be less severe the higher a worker’s reservation value. In practice, often the reservation value is determined by the employee’s qualification or his position in the corporate hier-
archy (see also Kim 1997, p. 910). Moreover, also the wealth of a manager is typically greater than that of a worker belonging to a lower hierarchical level. Hence, the relative disadvantages of tournaments due to limited liability should decrease upwards the corporate hierarchy.

It is important to emphasize that we do not look for the optimal individual contract under limited liability in a principal-agent setting. Such question has been addressed by Sappington (1983) and Kim (1997), for example. The results of Kim (1997) show that the optimal incentive contract is a kind of bonus scheme. However, in this paper we compare two compensation schemes that are frequently used in practice in order to show under which circumstances tournaments will dominate piece rates and vice versa. We do also not address the case of a principal who is characterized by limited liability which is considered by Kahn and Scheinkman (1985), Tsoulouhas and Vukina (1999) and Tsoulouhas and Marinakis (2006). Lewis and Sappington (2000a, 2000b, 2001) extend the discussion of incentives for wealth-constrained agents by introducing private information about the agents’ wealth and/or their abilities. Optimal incentives under limited liability in organizations are discussed by Schmitz (2005a, 2005b).

The paper is organized as follows. The next section introduces the model. The results of the model are presented in Section 3. Section 4 concludes.

2 Model

To compare tournaments with piece rates, a model with one employer and two workers is considered. All players are assumed to be risk neutral. Worker $i$’s ($i = A, B$) output is described by the production function $q_i = e_i + \varepsilon_i$ with $e_i$ denoting $i$’s effort choice and $\varepsilon_i$ idiosyncratic noise which is distributed over
$[-\bar{\varepsilon}_L, \bar{\varepsilon}_H]$ with mean $\bar{\varepsilon}$ and $\bar{\varepsilon}_L, \bar{\varepsilon}_H > 0$. As usual in tournament models, $\varepsilon_A$ and $\varepsilon_B$ are assumed to be identically and independently distributed (i.i.d.). Let $G(\cdot)$ denote the cumulative distribution function and $g(\cdot)$ the density of the composed random term $\varepsilon_j - \varepsilon_i$ ($i, j = A, B; i \neq j$). The output $q_i$ is verifiable, whereas the employer neither observes $e_i$ nor $\varepsilon_i$. Worker $i$’s effort costs are described by the convex function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$, $c''(e_i) > 0$ and $c''(e_i) \geq 0$. Each worker is assumed to have a reservation value $\bar{u} \geq 0$, and, in any given case, the employer wants to hire the two workers (e.g., because of their human capital). The employer maximizes expected total output minus labor costs (i.e., wages), whereas each worker maximizes expected wages minus effort costs.

If the employer organizes a tournament between the two workers, at the first stage of the game he will choose a winner prize $w_1$ (which will be given to the worker with the highest realized output) and a loser prize $w_2$ prior to the tournament to induce incentives. Let $\Delta w = w_1 - w_2$ denote the prize spread. Then, for given tournament prizes, the two workers choose their optimal efforts at the second stage. In order to model limited liability, the loser and the winner prize are not allowed to become negative ($w_1, w_2 \geq 0$). However, since positive incentives require $w_1 > w_2$, the limited-liability constraint actually reduces to $w_2 \geq 0$.

Under a piece-rate scheme, at the first stage of the game the employer uses a linear incentive formula $w_i = \alpha + \beta q_i$ ($i = A, B$) with $q_i$ as worker $i$’s realized output, $\alpha$ as a fixed payment and $\beta \in [0, 1]$ as the piece rate (see, e.g., Lazear and Rosen (1981), Lazear (1986), Gibbons (1987), Fama (1991)). Again, the limited-liability assumption for the workers requires wages $w_i$ to be non-negative ($w_i \geq 0$). At the second stage, each worker chooses his effort $e_i$ for a given pair $(\alpha, \beta)$. 

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Most of the assumptions follow the standard tournament model by Lazear and Rosen (1981). However, there are two exceptions which should be highlighted. First, Lazear and Rosen do not address the problem of limited liability which is focussed on in this paper. Second, Lazear and Rosen do not assume that the distribution of the error term has a finite support. For this paper, the assumption of a finite support is important as limited liability is defined as guaranteeing the workers non-negative wages in any situation. Hence, given the case of an infinite support piece rates would be clearly dominated by tournaments since it would be too expensive for the employer to compensate the workers for possibly infinite losses. As an alternative, the employer could define a lower bound for possibly negative piece-rate payments $\beta q_i$. However, then we could also think about variants of the tournament model which may lead to further improvements. Altogether, in order to guarantee a "fair" comparison of piece rates and tournaments under realistic assumptions, a bounded influence of the error term is assumed throughout the paper.

3 Results

As a benchmark result, the first-best effort $e_{FB}$ can be calculated. This effort maximizes $E [q_i] - c(e_i)$, which yields

$$c'(e_{FB}) = 1 \quad (i = A, B).$$

(1)

Under the tournament scheme, at the second stage of the game worker $i$ maximizes

$$EU_i (e_i) = w_2 + \Delta w \cdot G(e_i - e_j) - c(e_i).$$

(2)
Hence, if an equilibrium in pure strategies exists at the tournament stage (for a model where existence is guaranteed see Kräkel (2004)), it will be unique and symmetric with each worker choosing effort implicitly defined by

\[ \Delta w_T (0) = c' (e_T) . \]  

(3)

At the first stage, the employer chooses \( w_1 \) and \( w_2 \) to maximize net profit \( 2e_T - w_1 - w_2 \) subject to the workers’ incentive constraint (3) and their participation constraint

\[ \frac{w_1 + w_2}{2} - c(e_T) \geq \bar{u}. \]  

(4)

Since the employer’s net profit as well as the workers’ incentives (see (3)) strictly decrease in the loser prize, the employer will choose that value of \( w_2 \) which makes the workers’ participation constraint just bind. Without restriction on the loser prize, the employer chooses \( \Delta w \) in order to maximize welfare (i.e. he implements \( e^{FB} \) for both workers) and extracts all rents from the workers by the optimally chosen loser prize. Incentive constraint and binding participation constraint together lead to

\[ w_1^{FB} = c(e^{FB}) + \bar{u} + \frac{c'(e^{FB})}{2g(0)} \quad \text{and} \]

\[ w_2^{FB} = c(e^{FB}) + \bar{u} - \frac{c'(e^{FB})}{2g(0)}. \]  

(5)  

(6)

Note, however, that due to the limited-liability assumption \( w_2 \geq 0 \) this solution will only be feasible if

\[ c(e^{FB}) + \bar{u} \geq \frac{c'(e^{FB})}{2g(0)} \iff c(e^{FB}) + \bar{u} \geq \frac{1}{2g(0)}. \]
Under the piece-rate scheme, the workers’ incentive constraint is given by
\[ \beta = c'(e_{PR}) \]  \hfill (7)
and their participation constraint by
\[ \alpha + \beta (e_{PR} + \tilde{\varepsilon}) - c(e_{PR}) \geq \bar{u}. \]  \hfill (8)

Therefore, the employer can implement \( e_{FB} \) and make the participation constraint bind by choosing
\[ \beta_{FB} = 1 \quad \text{and} \quad \alpha_{FB} = c(e_{FB}) + \bar{u} - e_{FB} - \tilde{\varepsilon}. \]

Due to limited liability, the workers’ wages still have to be non-negative in the worst case, i.e. \( w_i = \alpha + \beta (e_{PR} - \varepsilon_L) \geq 0 \). Hence, first-best implementation will be feasible if
\[ c(e_{FB}) + \bar{u} \geq \varepsilon + \varepsilon_L. \]

Comparing tournaments with piece rates and using Eq. (1), we have the following results:

**Proposition 1** (i) The higher the workers’ reservation value, \( \bar{u} \), the more likely \( e_{FB} \) is implemented under either incentive scheme. (ii) If \( \frac{1}{2g(0)} > (\leq) \varepsilon_L + \varepsilon \), implementation of \( e_{FB} \) will be more (less) likely under a piece-rate than under a tournament scheme.

**Proof.** See appendix. \( \blacksquare \)

Note that "more likely" means "under more parameter constellations". The intuition for result (i) comes from the fact that workers can be given stronger incentives the higher their wealth. If workers have high reservation
values, the employer will have to compensate the workers for these foregone values by a large lump-sum payment when they sign the contract. By this, the workers’ wealth increases significantly so that it is more likely that the employer wants to create sufficiently high incentives which lead to first-best effort (see for bonus schemes Kim (1997), p. 910.). Hence in this context, large reservation values of the workers are strictly welfare enhancing.

Result (ii) shows that the larger $\bar{L}$ and the smaller $1/g(0)$, the more advantageous tournaments will be relative to piece rates. In particular, if

$$\frac{1}{2g(0)} < \bar{L},$$

then first-best effort $e^{FB}$ will be more likely implemented under the tournament scheme. This result can also be explained intuitively: $\bar{L}$ characterizes the worst case under the piece-rate scheme, in which the workers’ compensation must be still non-negative. Under the tournament scheme,

$$w_2^{FB} = c(e^{FB}) + \bar{u} - \frac{c(e^{FB})}{2g(0)} = c(e^{FB}) + \bar{u} - \frac{1}{2g(0)},$$

which will become negative if $\frac{1}{2g(0)}$ is too large. Note that the marginal winning probability, $g(\cdot)$, determines incentives in the tournament and, hence, optimal prizes. If $g(\cdot)$ is flat (i.e., the outcome of the tournament is mainly determined by luck) – and, therefore, $g(0)$ is small – effort incentives will be rather low (see the incentive constraint (3)). Following Lazear (1995, p. 29), we can interpret $1/g(0)$ as a measure of luck or risk in the tournament. Alternatively, we may interpret $g(0)$ as a measure of the principal’s monitoring precision. Hence, in a situation with considerable luck or a low monitoring precision, the employer has to choose a sufficiently high prize spread $\Delta w$ to restore incentives. This means, however, that the loser prize $w_2$ has to be rather small, and that $w_2^{FB}$ may become negative. Furthermore, the more biased $q_i$ is as a signal for realized effort (i.e. the higher $\tilde{e}$), the less likely first-best effort is
implemented under piece rates. Altogether, small values of $\tilde{\varepsilon}_L$ and $\tilde{\varepsilon}$ relax
the limited-liability constraint under the piece-rate scheme, whereas a small
$\frac{1}{2g(0)}$ relaxes the one under the tournament scheme, which drives result (ii)
of Proposition 1.

Now we can examine whether risk harms tournament incentives more
than piece-rate incentives. First, note that – contrary to tournaments – risk
will not influence piece-rate incentives given risk neutral workers, if there
is unlimited liability. However, under limited liability maximum bad luck
clearly influences inequality (9): if $\tilde{\varepsilon}_L$ and, therefore, risk is large, piece rates
will be disadvantageous. As mentioned above, $\frac{1}{2g(0)}$ can also be used as a
measure of risk. If $\frac{1}{2g(0)}$ is large, tournaments will become disadvantageous,
too. Hence, we have to examine which of these two effects is dominant. Of
course, for $\tilde{\varepsilon}_L \to \infty$ (e.g., if the error terms are normally distributed) piece
rates become prohibitively expensive for the employer, but tournaments still
work. They offer workers a partial insurance, since minimum and maximum
income are determined by the loser and the winner prize, respectively.

When looking at less extreme cases, the comparison may lead to a dif-
erent result. Assume, for example, that the i.i.d. error terms $\varepsilon_i$ and $\varepsilon_j$
follow a normal distribution $N(0, \sigma^2)$ that has been truncated on the left at
$-\tilde{\varepsilon}_L = -\tilde{\varepsilon}$ and on the right at $\tilde{\varepsilon}_H = \tilde{\varepsilon}$. This implies that the convolution $g(\cdot)$
for $\varepsilon_j - \varepsilon_i$ is also a truncated normal distribution with mean zero. However,
the variance of this new normal distribution is given by $2\sigma^2$, and the com-
piled random variable $\varepsilon_j - \varepsilon_i$ is distributed over the interval $[-2\tilde{\varepsilon}, 2\tilde{\varepsilon}]$. We
obtain following result:

**Proposition 2** Let $\varepsilon_i$ and $\varepsilon_j$ follow a normal distribution $N(0, \sigma^2)$ truncated
at $-\tilde{\varepsilon}$ and $\tilde{\varepsilon}$. If $\sigma^2 < (> \frac{\pi^2}{\ln 2}$, the left-hand side of (9) will increase less (more)
rapidly in $\tilde{\varepsilon}$ than the right-hand side. If $\sigma^2 \to 0 (\sigma^2 \to \infty)$, inequality (9)
will always (never) hold.

Proof. See appendix. ■

Proposition 2 shows that for low variances of the initial normal distribution, inequality (9) is more likely to hold, whereas for high values of $\sigma^2$ the opposite is true. If the variance tends to zero, the inequality will always be satisfied, whereas for sufficiently high variances it will always be violated. Hence, tournaments will only dominate piece rates, if risk – i.e. the variance of $\varepsilon_i$ and $\varepsilon_j$ – is not too large. Following Lazear and Rosen (1981) and Lazear (1995), $\varepsilon_i$ and $\varepsilon_j$ can be interpreted in different ways. For example, they can (a) measure the exogenous risk of the given production technologies, (b) the individual measurement errors when workers are evaluated, or (c) the ex ante unknown abilities of the workers in case of symmetric uncertainty. This means that, given limited liability, tournaments will only be attractive for the employer compared to piece rates, if workers use quite safe production technologies, the supervisors’ monitoring precision is not too low, or initial uncertainty about the workers’ talents is sufficiently reduced by introducing appropriate recruiting techniques.

Next, the employer’s complete optimization problems under limited liability at the first stage of the game are considered. When organizing a tournament the employer maximizes

$$\pi_T = 2e_T - w_1 - w_2$$

subject to incentive constraint (3), participation constraint (4) and limited-liability constraint $w_2 \geq 0$. In case of a piece-rate system, the employer maximizes

$$\pi_{PR} = 2(1 - \beta)(e_{PR} + \hat{e}) - 2\alpha$$
subject to incentive constraint (7), participation constraint (8) and limited-liability constraint $w_i \geq 0$. Let $e_T^*$ ($e_{PR}^*$) denote the workers’ equilibrium effort under the tournament (piece-rate) scheme. The solution to the employer’s optimization problems gives the following proposition:

**Proposition 3** Let $c''(\cdot) > 0$. In the employer’s optimization problems at least one constraint is binding: (i) If only the participation constraint is binding, we will have $e_T^* = e_{PR}^* = e^{FB}$. (ii) If both constraints are binding, equilibrium efforts will be described by

$$
\frac{d'(e_T^*)}{2g'(0)} = \bar{u} + c'(e_T^*) \quad \text{and} \quad d'(e_{PR}^*)(\bar{\varepsilon}_L + \hat{\varepsilon}) = c'(e_{PR}^*) + \bar{u}. \quad (12)
$$

(iii) If only the limited-liability constraint is binding, equilibrium efforts will be characterized by

$$
2g(0) = c''(e_T^*) \quad \text{and} \quad \frac{1}{\bar{\varepsilon}_L + \hat{\varepsilon}} = c''(e_{PR}^*). \quad (13)
$$

Let $c(e_i) = \eta e_i^\delta$ with $\delta > 2$ and $\eta > 0$. Given $\bar{u} = 0$ and the limited-liability constraint is binding, if $\frac{1}{2g(0)} > (\leq) \bar{\varepsilon}_L + \hat{\varepsilon}$, the workers will more (less) likely receive a positive rent under the tournament than under the piece-rate scheme.

**Proof.** See appendix. ■

Result (i) is not surprising. As we know from the standard tournament model with risk neutral workers, if there are no limited-liability problems (i.e., the limited-liability constraint is not binding), first-best effort is implemented under either incentive scheme. However, if the limited-liability constraint is binding, we will either have an interior solution given that $\bar{u}$ is not too large, so that the participation constraint does not become binding (result (iii)) or
a corner solution otherwise (result (ii)).

In principal-agent models with a wealth constrained agent, typically the agent earns a positive rent. This case is described by Proposition 3(iii). In this situation, the two workers will exert more (less) effort under the piece-rate scheme than under the tournament scheme if

\[
\frac{1}{2g(0)} > (\lesssim) \bar{\varepsilon}_L + \hat{\varepsilon}. \tag{14}
\]

Interestingly, this condition is identical with the one in Proposition 1(ii). Together with the findings of Proposition 2, we can conclude that tournaments will be highly problematic if exogenous risk becomes large – besides the fact that the first-best solution is less likely to be implemented, Proposition 3(iii) shows that in the case of a second-best solution the workers spend less effort compared to the piece-rate scheme. Moreover, according to the parametric example of Proposition 3(iii), given condition (14) the workers will more (less) likely receive a positive rent under the tournament than under the piece-rate scheme.

Recall that result (iii) corresponds to a situation in which workers earn positive rents. If the employer is able to control the exogenous risk to some extent (e.g. by increasing the monitoring precision), he can create additional incentives for the workers. Since these extra incentives only reduce the workers’ rents, they are – at first sight – free for the employer. However, reducing risk often leads to additional costs for the employer (e.g. higher monitoring costs) so that we obtain a trade-off between incentives and risk-reducing costs although workers are risk neutral.
4 Discussion

In this Section, some further points will be addressed which have not been discussed so far in the paper. First, one might question the realism of assuming limited liability for the workers. It is important to stress that the limited-liability assumption used in this paper is simply assuming that wages are not allowed to be negative. Typically, this assumption should be satisfied in practice. Consider, for example, the case of job-promotion tournaments. There, the winner prize is the wage at the higher position in the hierarchy after being promoted whereas the loser stays at his current job and receives his current wage as loser prize. Of course, both winner prize and loser prize should be positive in this case. I can imagine only two examples where workers may receive negative wages. An example for a piece-rate scheme with a negative fixed payment may be the case of a lawyer or a physician who wants to buy into a partnership. An example for tournaments with negative loser prizes might be the layoff tournaments mentioned in the introduction. However, even in the last case tournament losers often receive severance payments when being laid off.

Another assumption which may be questioned is the existence of tournaments in the strict form as defined in Section 2. The main assumption here was that the employer has fixed prizes in advance before the tournament starts. This assumption is crucial for inducing incentives to the workers. Naturally, in sports contests like in golf or tennis tournaments, prizes are set prior to the tournament. However, there are also several examples for pre-specified prizes in corporate tournaments. For example, in contests between salesmen each contestant knows in advance how large the premium will be when being declared "best salesman of the month". In large corporations, a considerable part of the workers’ wages is attached to jobs so that
we typically have pre-specified prizes in job-promotion tournaments. Finally, the monetary consequences after being rated according to a forced-ranking system are usually known by the workers before the evaluation starts.

The production technology described by \( q_i = e_i + \varepsilon_i \) (with \( \varepsilon_A \) and \( \varepsilon_B \) being i.i.d. given an underlying probability distribution with noise \( \sigma^2 \)) in this paper is identical with the one that is used in most of the paper by Lazear and Rosen (1981): individual output is linear in effort and idiosyncratic noise which are perfect substitutes in production. However, on pages 856–857 Lazear and Rosen discuss the case in which a common error term, say \( \rho \), is added to the production function considered so far so that output is now given by

\[
q_i = e_i + \varepsilon_i + \rho. \quad (15)
\]

If we compare piece rates and tournaments under the new production function (15), now the employer may strictly prefer a tournament. By using a tournament, the common shock \( \rho \) is filtered out which is not the case when using piece rates. Note that income risk under a piece-rate scheme is \( \beta^2 \sigma^2 + \beta^2 \text{Var} [\rho] \), whereas for a tournament scheme the relative performance \( q_i - q_j \) becomes crucial which has variance \( 2\sigma^2 \) (in other words, note that the support of the distribution for \( \varepsilon_i - \varepsilon_j \) is two times the support of the underlying distribution for \( \varepsilon_i \)). Altogether, if there exists common noise and if the impact of the common risk is sufficiently large relative to \( \sigma^2 \), then tournaments will always outperform piece rates given limited liability. This important advantage of a tournament in the presence of a common shock has also been highlighted by the seminal work of Holmstrom (1982).

However, tournaments may even be advantageous if there does not exist a common error term. The results of this paper have shown under which conditions tournaments outperform piece rates when workers are protected.
by limited liability and there only exists idiosyncratic noise. Tournaments
will also be preferable to piece rates from the employer’s viewpoint if mea-
measurement costs are an issue since they only need an ordinal scale (Lazear
and Rosen (1981)). Also important for practice is the point raised by Mal-
comson (1984): Often workers’ performances or performance signals are not
verifiable by a third party (e.g. a court). In such situations, piece rates
and other pay methods that are based on verifiable individual output do not
work since the employer can save labor costs by claiming bad performances
ex post. However, tournaments do work even in such setting.

5 Conclusion

In this paper, tournaments and piece rates have been compared under the
assumption of limited liability. The comparison has shown that idiosyncratic
risk or luck has a large impact on the profitability of both incentive schemes.
While tournaments offer a partial insurance, piece rates may not work any
longer if potential losses become very large. However, if risk is sufficiently
high, piece rates will dominate tournaments, because first-best implementa-
tion will be more likely under a piece-rate scheme, and because efforts will
be larger under piece rates when workers earn positive rents.

The theoretical findings can be used to derive some implications for prac-
tice. According to the paper’s results the employer should prefer piece rates
to tournaments if idiosyncratic risk is high. Therefore, the employer will
benefit from favoring piece rates (a) if there is large symmetric uncertainty
about the workers’ talents, (b) when incentives should generated for specific
risky tasks, or (c) when a worker’s performance is highly volatile. Further-
more, the limited-liability constraint should be rather binding for workers
on low hierarchy levels than for managers at higher levels of the corporate hierarchy. Hence, the relative dominance of piece rates over tournaments diminishes when moving upward the corporate hierarchy.

Appendix

Proof of Proposition 2:

Defining \( z := \varepsilon_j - \varepsilon_i \) the density of the truncated normal distribution can be written as

\[
g(z) = \frac{1}{\sqrt{2\pi \sigma^2}} \frac{\phi \left( \frac{z}{\sqrt{2\sigma^2}} \right)}{1 - 2\Phi \left( \frac{-\bar{\varepsilon}}{\sqrt{2\sigma^2}} \right)}
\]

with \( \phi (\cdot) \) denoting the density and \( \Phi (\cdot) \) the cumulative distribution function of the standardized normal distribution. We obtain

\[
\frac{1}{2g(0)} = \frac{\sqrt{2\pi}}{2 \sigma} \frac{1 - 2\Phi \left( \frac{-\bar{\varepsilon}}{\sqrt{2\sigma^2}} \right)}{\phi (0)} = \sigma \sqrt{\pi} \left( 1 - 2 \int_{-\infty}^{\frac{-\bar{\varepsilon}}{\sqrt{2\sigma^2}}} \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{x^2}{2} \right\} dx \right) =: \Psi (\sigma^2, \bar{\varepsilon}),
\]

Differentiating this expression (and therefore the left-hand side of (9)) with respect to \( \bar{\varepsilon} \) gives

\[
\frac{\partial \Psi (\sigma^2, \bar{\varepsilon})}{\partial \bar{\varepsilon}} = 2 \exp \left\{ -\frac{\bar{\varepsilon}^2}{\sigma^2} \right\}.
\]

Hence, the left-hand side of (9) will increase less rapidly in \( \bar{\varepsilon} \) than the right-hand side, if

\[
2 \exp \left\{ -\frac{\bar{\varepsilon}^2}{\sigma^2} \right\} < 1 \iff \sigma^2 < \frac{\bar{\varepsilon}^2}{\ln 2}.
\]

The second result of Proposition 2 becomes obvious by inspection of \( \Psi (\sigma^2, \bar{\varepsilon}) \).

For \( \sigma^2 \to 0 \), the upper limit of the integral tends to \(-\infty\) and the whole integral tends to zero so that \( \Psi (\sigma^2, \bar{\varepsilon}) \) goes to infinity. If \( \sigma^2 \to \infty \), the upper limit of the interval tends to zero so that the whole interval goes to \( \frac{1}{2} \) and,
therefore, the term in brackets to zero. However, the expression $\sigma \sqrt{\pi}$ in front of the brackets grows more rapidly to infinity.

**Proof of Proposition 3:**
Let $e^*_T(\Delta w)$ ($e^*_PR(\beta)$) denote a worker’s optimal effort choice characterized by incentive constraint (3) ((7)), then the employer’s optimization problems are described by the two Lagrangians

\[
L_T(w_1, w_2) = 2e^*_T(\Delta w) - w_1 - w_2 + \lambda_1 \left[ \frac{w_1 + w_2}{2} - c(e^*_T(\Delta w)) - \bar{u} \right] + \lambda_2 w_2
\]  
(A1)

and

\[
L_{PR}(\alpha, \beta) = 2 (1 - \beta) (e^*_PR(\beta) + \varepsilon) - 2\alpha 
+ \lambda_1 [\alpha + \beta (e^*_PR(\beta) + \varepsilon) - c(e^*_PR(\beta)) - \bar{u}] + \lambda_2 [\alpha + \beta (e^*_PR(\beta) - \bar{\varepsilon}_L)] .
\]  
(A2)

We obtain the following optimality conditions for $w_1$ and $w_2$ (for brevity, the conditions for the multipliers and the restrictions are omitted):

\[
2e^{\alpha'}_T - 1 + \lambda_1 \left[ \frac{1}{2} - c'(e^*_T) e^{\alpha'}_T \right] = 0 \quad (A3)
\]

\[
-2e^{\alpha'}_T - 1 + \lambda_1 \left[ \frac{1}{2} + c'(e^*_T) e^{\alpha'}_T \right] + \lambda_2 = 0 \quad (A4)
\]

with $e^*_T := e^*_T(\Delta w)$. From (A3) and (A4) we get $\lambda_1 + \lambda_2 = 2$. Hence, at least one constraint must be binding in equilibrium. Concerning the piece-rate scheme, the optimality conditions for $\alpha$ and $\beta$ are:

\[
-2 + \lambda_1 + \lambda_2 = 0 \quad (A5)
\]

\[
-2 (e^*_PR + \varepsilon) + 2 (1 - \beta) e^{\alpha'}_PR 
+ \lambda_1 [e^*_PR + \varepsilon + \beta e^{\alpha'}_PR - c'(e^*_PR) e^{\alpha'}_PR] + \lambda_2 [e^*_PR - \bar{\varepsilon}_L + \beta e^{\alpha'}_PR] = 0. \quad (A6)
\]
According to (A5), again at least one constraint must be binding. Note that from (3) and (7) we have

\[ e_T^* = \frac{g(0)}{c''(e_T)} \quad \text{and} \quad e_{PR}^* = \frac{1}{c''(e_{PR})}. \]  

(A7)

(i) Substituting \( \lambda_1 = 2 \) and \( \lambda_2 = 0 \) into the optimality conditions immediately replicates the benchmark result of Lazear and Rosen (1981) for the case of unlimited liability. (ii) Combining the binding limited-liability constraints, the binding participation constraints, and the incentive constraints (3) and (7) leads to (12). (iii) Inserting \( \lambda_1 = 0 \) into (A3) together with (A7) yields \( e_T^* \). Using \( \lambda_1 = 0 \) and \( \lambda_2 = 2 \) in (A6) together with (A7) gives \( e_{PR}^* \).

Finally, consider the case of a parameterized cost function \( c(e_i) = \eta e_i^\delta \) with \( \delta > 2 \) and \( \eta > 0 \). Given a binding limited-liability constraint and \( \bar{u} = 0 \), workers will receive a positive rent under the tournament scheme, if \( \frac{w_1}{2} - c(e_T^*) > 0 \) with \( e_T^* \) being described by \( 2g(0) = c''(e_T^*) \). Substituting for \( w_1 \) according to the incentive constraint \( w_1 g(0) = c'(e_T^*) \) leads to \( \frac{c'(e_T^*)}{2g(0)} - c(e_T^*) > 0 \iff \frac{1}{2g(0)} > \frac{\frac{c'(e_T^*)}{c(e_T^*)}}{\frac{c'(e_T^*)}{c(e_T^*)}} \). Using the specific form of the cost function yields

\[ e_T^* = c''^{-1}(2g(0)) = \left( \frac{2g(0)}{\eta \delta (\delta - 1)} \right)^{\frac{1}{\delta - 2}} \]

and the inequality becomes

\[ \frac{\delta}{2g(0)} > \left( \frac{2g(0)}{\eta \delta (\delta - 1)} \right)^{\frac{1}{\delta - 2}}. \]  

(A8)

Under the piece-rate scheme, workers will get a positive rent, if \( \alpha + \beta (e_{PR}^* + \hat{\varepsilon}) > c(e_{PR}^*) \) with \( e_{PR}^* \) being characterized by \( \frac{1}{\bar{\varepsilon}_{L+\hat{\varepsilon}}} = c''(e_{PR}) \), i.e.

\[ e_{PR}^* = \left( \frac{1}{\eta \delta (\delta - 1)(\bar{\varepsilon}_L + \hat{\varepsilon})} \right)^{\frac{1}{\delta - 2}} \]
Because of the binding limited-liability constraint $\alpha + \beta (e_{PR}^* - \bar{e}_L) = 0$ and the incentive constraint $\beta = c'(e_{PR}^*)$ the inequality can be rewritten as $\bar{e}_L + \hat{e} > \frac{c'(\hat{e}_2)}{c'(\hat{e}_1)}$. By using the parametric form of the cost function and the concrete expression for the equilibrium effort $e_{PR}^*$ we obtain

$$\left(\bar{e}_L + \hat{e}\right) \delta > \left(\frac{1}{\eta \delta (\delta - 1) (\bar{e}_L + \hat{e})}\right)^{\frac{1}{1-s}} \quad (A9)$$

Comparing (A8) with (A9) completes the proof.

References


